The Study of the Feasibility of Fully Plastic Solution of Weldment for Plane Stress Problem

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ABSTRACT

The feasibility of fully plastic solutions of welded center-cracked strip for plane stress problem was investigated with the fully plastic finite element method. The fracture mechanics parameters such as J-integral, crack opening displacement, and load-point displacement of weldment were computed and discussed. It was introduced for engineering approach of assessing the fracture mechanics parameters of weldment that there exist an equivalent yielding stress and an equivalent strain hardening exponent in the vicinity of the crack tip keeping the assessment of fracture mechanics parameters of weldment like the homogeneous material. The engineering approach was given for estimating the fracture mechanics parameters of weldment with mechanical heterogeneity in elastic-plastic range.

KEYWORDS

Fracture parameter; Weldment; Heterogeneity; FEM.

INTRODUCTION

Welding structures are widely used in important engineering structures such as boiler, pressure vessles, off-shore structure,and naval vessels. Weldments are more sensitive parts of a structure with regard to crack growth and failure because of the inherent characteristics of weldments. The majority of important engineering structures is made of low-carbon,low alloy steels which behave both of high strength and good ductile. A large scale plasticity will be developed at the crack tip which exists in the welded structure prior to crack initiation. After the initiation of the crack there will be a slow crack growth before instability occurs. Hence, elastic-plastic fracture mechanics is required to assess the integrity of structures containing cracks. Recently, some engineering approaches have been developed such as two-criteria method (Dowling et al.,1975) and EPRI approach (Kumer et al., 1981) to assess the elastic-plastic fracture behavior of structures. These methods have been used successfully in pressure vessels and piping including the nuclear system for fracture analysis. However, approaches mentioned above are not avaluable for weldment with mechanical heterogeneity. A path independence of J-integral in weldment with mechanical heterogeneity was proved recently (Ma et al., 1986). In this paper, the feasibility of fully plastic solution of weldment with mechanical heterogeneity was studied using the fully plastic finite element method. The engineering approach was given for estimating elastic-plastic fracture mechanics parameters of weldment.

THE THEORETICAL BACKROUND OF "EPRI" APPROACH

The theoretical backround of the EPRI approach includes both the HRR singularity near the crack tip (Hutchinson, 1968, Rice et al., 1968) and the J-controlled crack growth (Hutchinson et al., 1979, Shih et al., 1979). On the basic of the theory of HRR singularity field, the stresses and strains near the crack tip under yielding conditions varying from small scale to fully plastic may be represented by

$$\sigma_{ij} = \sigma_o \left[\frac{EJ}{\alpha \sigma_o^2 \, \mathrm{In}\, r} \right]^{1/n+1} \tilde{\sigma}_{ij}(\theta, n) \qquad (1)$$

$$\varepsilon_{ij} = \alpha \varepsilon_o \left[\frac{EJ}{\alpha \sigma_o^2 \, \mathrm{In}\, r} \right]^{n/n+1} \tilde{\varepsilon}_{ij}(\theta, n) \qquad (2)$$

where J expressed the path independent line J-integral. E is the Young's modulus; θ, r crack tip coordinates; $\tilde{\sigma}_{ij}$, $\tilde{\varepsilon}_{ij}$ dimensionless function of the circumferential position and the strain hardening exponent n. In is a constant which is a function of n, and ε_o, σ_o the yielding strain and yielding stress, complying with the pure power stress-strain law

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha \left(\frac{\sigma}{\sigma_o} \right)^n \qquad (3)$$

The condition in which crack growth will be J-controlled is the following

$$dJ/J >> d\, a/r \qquad (4)$$

where dJ is the increment of J-integral, da the crack growth increment.

The approximate formulas for quantities such as J, δ, and Δ can be written as following according to the EPRI approach,

$$J = J_e(a_e) + J_p(a,n) \qquad (5)$$

$$\delta = \delta_e(a_e) + \delta_p(a,n) \qquad (6)$$

$$\Delta_c = \Delta_{ce}(a_e) + \Delta_{cp}(a,n) \qquad (7)$$

where a_e is an effective crack length, $J_e(a_e)$, $\delta_e(a_e)$, and $\Delta_{ce}(a_e)$ are the elastic contributions based on an adjusted crack length; $J_p(a,n)$, $\delta_p(a,n)$, and $\Delta_{cp}(a,n)$ the plastic contributions based on the strain hardening exponent n and crack length a.

A simple summation of the tabulated fully plastic solutions has be made based on a lot of computations (Kumar et al., 1981). Several simplified graphical methods for fracture analysis are employed combining the resistance curve, J_R, of material. These permit the prediction of the crack growth initiation, the extent of stable crack growth and possible instability conditions.

MATERIALS AND COMPUTATION

According to the deformation theory and assumption of incompressive material, a fully plastic finite element program in plane stress was made. Singular elements were used in the crack tip (Henshell et al., 1975). In addition, mechanical heterogeneity in weldment was taken into account.

In the finite element analysis, it was assumed that the weldment consisted of both base metal and weld metal bahaving in simple tension according to the pure power hardening law neglecting the elastic strain

$$\varepsilon = \alpha \varepsilon_o (\sigma / \sigma_o)^n \qquad (8)$$

The formulas of fully plastic fracture mechanics parameters such as J_p, δ_p, and Δ_{cp} can be written as the following equations in the manner consistent with the EPRI approach,

$$J_p = \alpha \varepsilon_o \sigma_o a(1-a/W) h_1(a/W,n)(P/P_o)^{n+1} \qquad (9)$$

$$\delta_p = \alpha \varepsilon_o a h_2(a/W,n)(P/P_o)^n \qquad (10)$$

$$\Delta_{cp} = \alpha \varepsilon_o a h_3(a/W,n)(P/P_o)^n \qquad (11)$$

where W was the specimen width, h_{1-3} the function of parameter a/W and strain hardening exponent n, and $P=2w\sigma^\infty$, $P_o=2(w-a)\sigma_o$, where P was the load per unit thickness and P_o the reference load per unit thickness. σ^∞ was the nominal stress of specimen in tension.

The welded center-cracked specimen configuration and finite element mesh for the upper-right of the specimen were shown in Fig.1 and 2,

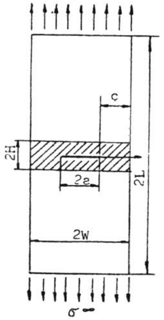

Fig.1 Configuration of Welded Center-cracked Specimen

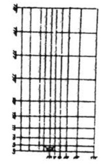

Fig.2 Finite Element Mesh

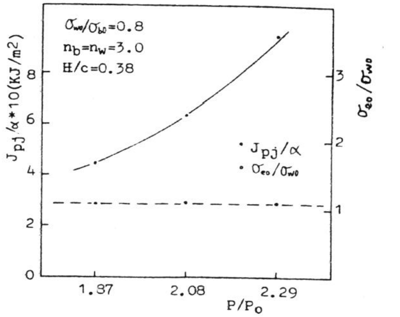

Fig.3 The J-integral of weldment and equivalent yielding stress versus applied load.

respectivily. In the large strain gradient zone the mesh was refined. In computation, the following data were inputted, the yielding stress of the base metal σ_{BO}=490 MPa, the strain hardening exponent of the base metal n_B=3.0, the yielding stress of weld metal σ_{WO}=390, and 490 MPa, the strain hardening exponent of weld metal n_w=2, and 4. The fully plastic solutions were made in three different values of loading. The ratio of crack length to specimen width a/W=0.4. The ratio of weld metal width to ligament length H/c=0.38. The numerical evaluation of the J-integral was conducted according to the reference (Shih et al., 1984). In the present work, six paths of the J-integral were taken. Two of the paths Located within the weld metal region. The others traversed both base metal and weld metal. with the mesh having 124 elements and 850 nodes the time per an exponent n or an yielding stress was about 5 min on a Honeywell DPS/8-52 computer.

RESULTS AND DISCUSSIONS

If the strain hardening exponent of the weld metal is equal to that of base metal, the difference of the fracture mechanics parameters between the homogeneous material and the welded joint is mainly caused by the difference of the yielding stresses between weld metal and base metal. Therefore, it may be assumed that there is an equivalent yielding stress in the vicinity of the crack tip in the weld metal. The equivalent yielding stress can be get by each one in eq.(9-11). Based on Eq.(9),

$$\sigma_{eo} = \left\{ \left(\frac{EJ_{pj}W}{\alpha a c h_1(a/W,n)} \right) \left(\frac{2c}{P} \right)^{n+1} \right\}^{1/1-n} \qquad (12)$$

where σ_{eo} was the equivalent yielding stress in the vicinity of crack tip, and c=W-a. J_{pj} was the plastic part of J-integral of the weldment and computed by means of the fully plastic finite element method.

In Fig.3, the solid line expresses the regression curve of the J-integral of weldment versus the applied load, and the dotted line is the regression curve of the modified values of the equivalent yielding stress near crack tip obtained from the Eq.(12) according to the values of J_{pj} versus the applied load. It can been seen from the Fig.3 that the change of applied

Table 1 The computation of equivalent yielding stress near crack tip for undermatched weldment.

P/P_o	1.87	2.08	2.29
J_{pj}/α	4.473	6.640	9.602
σ_{eo}/σ_{wo}	1.093	1.1050	1.112
δ_{PJ}/α	0.09142	0.12270	0.16184
σ_{eo}/σ_{wo}	1.090	1.103	1.108
Δ_{cpj}/α	0.05391	0.07178	0.09673
σ_{eo}/σ_{wo}	1.093	1.110	1.105

Note: H/c=0.38 n_w/n_B=1.0 σ_{wo}/σ_{Bo}=0.8

load will not affect the equivalent yielding stress. Almost the same value of equivalent yielding stress can be obtained from differet values of J_{pj}. The values of equivalent yielding stress based on Eq.(9-11) were listed in table 1. It was indicated that the values of were quite similar in spite of the computations taking from J_{pj}, δ_{pj}, and Δ_{cpj}. Thus, in the condition of this heterogeneity with different yielding stresses between base metal and weld metal, there exists an equivalent yielding stress in the vicinity of crack tip to keep the forms of estimating the fracture mechanics parameters of weldment like those of homogeneous materials. The formulas may be written as follows,

$$J_{pj} = \alpha \varepsilon_{eo} \sigma_{eo} a(1-a/W) h_1(a/W,n)(P/P_e)^{n+1} \qquad (13)$$

$$\delta_{pj} = \alpha \varepsilon_{eo} a\, h_2(a/W,n)(P/P_o)^n \qquad (14)$$

$$\Delta_{cpj} = \alpha \varepsilon_{eo} a\, h_3(a/W,n)(P/P_o)^n \qquad (15)$$

where $\varepsilon_{eo} = \sigma_{eo}/E$, $P_e=2(W-a)\sigma_{eo}$

If the strain hardening exponent of weld metal is different from that of base metal with the same yielding stress in the weldment. It may be assumed that there is an equivalent strain hardening exponent near the crack tip to reflect the differences of fracture mechanics parameters between weldment and homogeneous material. From Eq.(9), the n_e can be obtained from Eq.(16),

$$n_e = \left\{ [\log(\frac{WJ_{pj}}{\alpha \varepsilon_o \sigma_o a c h_1})]/[\log(\frac{P}{P_o})] \right\} -1 \qquad (16)$$

The other values of δ_{pj} and Δ_{cpj} may also be used to calculate the equivalent strain hardening exponent. The table 2 gives the results computed from J_{pj}, δ_{pj}, and Δ_{cpj}, respectivily. It can be seen from the table 2 that the changes of loading may not affect the equivalent strain hardening exponent, and almost the same value of n_e was given in spite of computing from J_{pj}, δ_{pj}, and Δ_{cpj}. In the Fig.4, the solid line expressed

Table 2 The Computation of Equivalent strain Hardening Exponent for Evenmatched Weldment.

P/P_o	1.87	2.08	2.29
J_{pj}/α	4.253	6.920	10.718
n_e/n_w	0.953	0.942	0.935
δ_{pj}/α	0.08536	0.12389	0.17277
n_e/n_w	0.952	0.957	0.944
Δ_{cpj}/α	0.14578	0.20668	0.28402
n_e/n_w	0.943	0.935	0.928

Note: H/c=0.38 n_w/n_B=1.33 σ_{Bo}/σ_{wo}=1.0

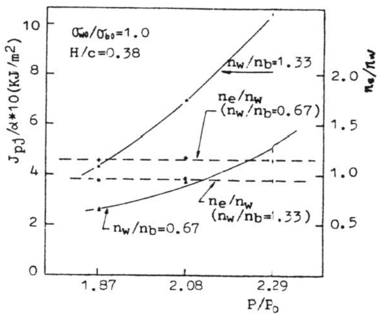

Fig.4 The J-integral of weldment and equivalent strain hardening exponent versus applied load.

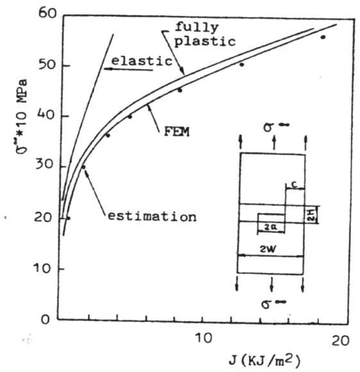

Fig.5 The comparsion between the values from elastic-plastic FEM and those from estimation procedure

the relationship of the J_{pj} versus the applied load. The dotted line is the curve of equivalent strain hardening exponent abtained from Eq.(13) according to J_{pj}. The strain hardening exponent of base metal can affect on the equivalent strain hardening exponent near the crack tip. When the exponent of weld metal is larger (or smaller) than that of base metal, the equivalent exponent near crack tip is lower (or larger) than that of weld metal because of the strengthening (or softening) of base metal. In this condition, the fracture mechanics parameters could be calculated from the Eq.(17-19)

$$J_{pj} = \alpha\, \varepsilon_o\, \sigma_o a(1-a/W) h_1(a/W,n_w)(P/P_o)^{n_e+1} \quad (17)$$

$$\delta_{pj} = \alpha\, \varepsilon_o a\; h_2(a/W,n_w)(P/P_o)^{n_e} \quad (18)$$

$$\Delta_{cpj} = \alpha\, \varepsilon_o a\; h_3(a/W,n_w)(P/P_o)^{n_e} \quad (19)$$

If both the yielding stress and the strain hardening exponent are different between the weld metal and base metal, first the Eq.(12) may be used to calculate the equivalent yielding stress, and then the Eq.(16) to calculate the equivalent strain hardening exponent in the vicinity of crack tip individually. Thus the fully plastic fracture mechanics parameters of weldment with different yielding stress and strain hardening exponent between weld metal and base metal can be written as follows,

$$J_{pj} = \alpha\, \varepsilon_{eo}\, \sigma_{eo} a(1-a/W)\; h_1(a/W,n_w)(P/P_e)^{n_e+1} \quad (20)$$

$$\delta_{pj} = \alpha\, \varepsilon_{eo}\; a\; h_2(a/W,n_w)(P/P_e)^{n_e} \quad (21)$$

$$\Delta_{cpj} = \alpha\, \varepsilon_{eo}\; a\; h_2(a/W,n_w)(P/P_e)^{n_e} \quad (22)$$

For the weldment with mechanical heterogeneity, the fracture mechanics parameters computed from the fully plastic finite element method and those from the estimating Eq.(20-22) were scheduled in table 3. The good relations can be seen from this table.

ENGINEERING APPROACH PROCEDURE

If stress-strain relation of weld metal and base metal is satisfied by Eq.(3) with different yielding stress and strain hardening exponent, the fracture mechanics parameters of weldment with mechanical heterogeneity such as J-integral the crack opening displacement, and load-point displacement may be written as the following forms similar to those by EPRI approach for homogeneous material,

$$J_j = J_{ej}(a_e) + J_{pj}(a/W,\ H/W,\ n_e,\ \sigma_{eo}) \quad (23)$$

$$\delta_j = \delta_{ej}(a_e) + \delta_{pj}(a/W,\ H/W,\ n_e,\ \sigma_{eo}) \quad (24)$$

$$\Delta_{cj} = \Delta_{cej}(a_e) + \Delta_{cpj}(a/W,\ H/W,\ n_e,\ \sigma_{eo}) \quad (25)$$

where $J_{ej}(a_e)$, δ_{ej}, and Δ_{cej} are the elastic part of J-integral, crack opening displacement, and load-point displacement and computed based on small scale yielding theory. J_{pj}, δ_{pj}, and Δ_{cpj} were estimated from Eq.(20-22).

For the purpose of verifying the results calculated from fully plastic solutions, the same mesh is used to calculate the fracture mechanics parameters with the elastic-plastic finite element method, then the solution computed by elastic-plastic finite element method is compared with the one from estimation procedure, Eq.(23), as shown in Fig.5 The error between them is within 6 percent.

CONCLUTIONS

According to the calculation and discussion previous mentioned, it is proved that the fully plastic solutions of weldment are available, and reliable by using the fully plastic finite element method for plane stress problem. For the weldments with mechanical heterogeneity, it can be assumed that there exist an equivalent yielding stress and equivalent strain hardening exponent in the vicinity of crack tip to keep the forms of fully plastic fracture mechanics parameters of weldment with mechanical

Table 3 The Fully Plastic Solutions of Fracture Mechanics Parameters of Weldment

P/P_o		1.87	2.08	2.29
J_{pj}/α	*	6.532	10.782	15.129
	* *	6.143	10.181	16.078
δ_{pj}/α	*	0.1251	0.1832	0.2521
	* *	0.1186	0.1769	0.2540
Δ_{cpj}/α	*	0.08415	0.1247	0.1858
	* *	0.08533	0.1273	0.1827
Note:	H/c=0.38	n_w/n_B=1.33	σ_{wo}/σ_{Bo}=0.8	

* Computations   * * Estimations

heterogeneity like those of homogeneous materials suggested by the EPRI approach. The width of weld metal, the yielding stress and strain hardening exponent of base metal will affect on the equivalent yielding stress and equivalent strain hardening exponent near the crack tip located in weld metal.

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