The Influence of Crack Tip Plasticity in Dynamic Fracture

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ABSTRACT

Dynamic fracture resistance of structural materials is commonly described in terms of the relationship between a measure of the crack driving force and the crack tip speed. In this article, analytical modelling directed toward establishing a basis for such a relationship in terms of crack tip plastic fields is described. The discussion is based mainly on analysis of steady crack growth through an elastic-plastic or elastic-viscoplastic material under small scale yielding conditions. Some observations on crack arrest, fracture mode transition and dynamic ductile hole growth are included in the discussion.

KEYWORDS

Dynamic fracture; dynamic crack propagation; elastic-plastic fracture; high strain rate fracture; crack arrest; ductile hole growth.

INTRODUCTION

Consider growth of a crack in an elastic-plastic material under conditions that are essentially two dimensional. The process depends on the configuration of the body in which the crack grows and on the details of the applied loading, in general, as well as on the properties of the material. However, if the region of active plastic flow is confined to the crack tip region, and if the elastic fields surrounding the active plastic zone are adequately described in terms of an elastic stress intensity factor, then it is commonly assumed that the prevailing stress intensity factor controls the crack tip inelastic process. This viewpoint mimics the small-scale-yielding hypothesis of elastic-plastic fracture mechanics but the basis for it in the study of rapid crack growth is less well established.

Experimental data on rapid crack growth in elastic-plastic materials is commonly interpreted on the basis of an extension of the Irwin crack growth criterion in those cases in which a stress intensity factor fields exists. If the applied stress intensity factor is K_a

then a common constitutive assumption is that there exists a material parameter or material function, say $K_m(v,T)$, depending on crack speed, and possibly on temperature, such that the crack grows with $K_a = K_m$. Indeed, in the jargon of fracture dynamics, such a condition provides an equation of motion for the position of the crack tip as a function of time.

Rapid crack growth in metals subjected to quasistatic loading or stress wave loading of modest intensity seems to follow this constitutive assumption to a sufficient degree so that a systematic study of its physical underpinnings is warranted. Thus, in recent years considerable effort has been devoted to developing models to explain the reasons why K_m depends on crack speed, and possibly on temperature, as it does for real materials. The approach is quite straightforward. It is assumed that a crack grows at some speed v in an elastic-plastic or elastic-viscoplastic material under the action of the input K_a applied remotely from the crack tip region. The potentially large stresses near the edge of the crack are relieved through inelastic deformation in an active plastic zone, and a permanently deformed layer is left in the wake of the active plastic zone along each crack face as the crack advances through the material. A solution of this problem is then obtained for arbitrary K_a and v in the form of stress and deformation fields that satisfy the field equations in some sense. With this solution in hand, a crack growth criterion motivated by the physics of the process is imposed on the solution to yield a relationship between K_a and v that must be satisfied for the crack to steadily advance. This relationship is, in fact, the material function $K_m(v,T)$ for the model problem.

The way in which K_m depends on v and T depends critically on the details of the fracture separation process, that is, whether it is a void nucleation and ductile hole growth mechanism or a cleavage mechanism, whether there is a strain rate induced elevation of flow stress or not, whether the material strain hardens significantly or flows with little hardening, and so on. The processes that must be analyzed are complex due to inertial effects and inherent nonlinear. A few general results which have been obtained are described in the following sections.

RATE INDEPENDENT MATERIAL RESPONSE

Experimental data on the dependence of dynamic fracture toughness versus crack speed for AISI 4340 steel and other materials that are commonly considered as elastic-plastic in their bulk mechanical response have some common features. Here, attention is limited to situations in which crack growth occurs by the single mechanism of void nucleation and ductile hole growth to coalescence, and in which the extent of plasticity is sufficiently limited to permit interpretation of the fields surrounding the crack tip region on the basis of a stress intensity factor. The toughness is found to be relatively insensitive to variations in speed for very low speeds (less than 20% of the shear wave speed) but increase dramatically with crack speed for greater speeds. The speed dependence of the surrounding elastic field is not nearly great enough to account for this dependence, so an explanation must be sought in the plastically deforming region itself. The most likely reasons for this toughness-speed behavior are material inertia and material rate sensitivity. While the strain rate in the crack tip region is necessarily very high, the same general behavior has been observed in materials that are relatively rate insensitive in their bulk response up to strain rates of about $10^3\ sec^{-1}$, so the role of material inertia has

been examined separately from the role of rate effects. The general idea is to generate a theoretical fracture toughness versus crack speed relationship to determine the role of inertia on the scale of crack tip plastic zone on the observed dynamic crack growth response.

A rough estimate of the conditions under which material inertia has a significant influence on the development of fields within the active plastic zone is obtained as follows. For steady quasistatic growth of a crack in the plane strain opening mode in an elastic-ideally plastic material, Rice, Drugan and Sham (1980) have constructed an asymptotic field consisting of a constant state region ahead of the crack tip, followed by a fan sector with singular plastic strain, an elastic unloading region, and finally a small plastic re-loading zone along the crack flanks. Within the region of singular plastic strain, the distribution of particle velocity and shear strain in crack tip polar coordinates is

$$\dot{u}_r, \dot{u}_\theta \sim v\epsilon_o \ln\left(\frac{r_o}{r}\right), \quad \epsilon^p_{r\theta} \sim \epsilon_o \ln\left(\frac{r_o}{r}\right) \tag{1}$$

where ϵ_o is the tensile yield strain, r_o is the maximum extent of the plastic zone, and r is the radial distance from the crack tip. If an expression for the kinetic energy density and the stress work density are derived for this deformation field, then the ratio of the kinetic energy density to the stress work density as a function of r is

$$KE/SW \approx 10\frac{v^2}{c^2}\ln\left(\frac{r_o}{r}\right) \tag{2}$$

where $c = \sqrt{E/\rho}$ is an elastic wave speed. Based on this simple estimate, it might be expected that inertial effects will be significant when the ratio in (2) is greater than one-tenth. For example, if $v/c = 0.1$ then the ratio is greater than one-tenth if $r/r_o \leq 0.3$.

In retrospect, such an estimate would have provided a warning that asymptotic analysis of this problem would have some subtle difficulties, as has been discovered more recently. The estimate suggests that for any nonzero v/c there is a range of r/r_o for which material inertial effects are important, but that the size of that region diminishes very rapidly as v/c approaches zero. Indeed, the estimate suggests that inertial effects are important only over a region for which $(r/r_o)^{v/c} \ll 1$ which is similar in form to the restriction on the domain of validity of the dynamic asymptotic solution given below in (4).

Analysis of Steady Growth

The steady-state growth of a crack at speed v in the the antiplane shear mode, or mode III in fracture mechanics terminology, under small scale yielding conditions was analyzed by Freund and Douglas (1982) and by Dunayevsky and Achenbach (1982). The field equations governing this process include the equation of momentum balance, the strain-displacement relations, and the condition that the stress distribution far from the crack edge must be the same as the near tip stress distribution in a corresponding elastic problem. For elastic-ideally plastic response of the material, the stress state is assumed to lie on the Mises yield locus, a circle of radius τ_o in the plane of rectangular stress components, and the stress and strain are related through the incremental Prandtl-Reuss flow rule. The material is linearly elastic with shear modulus μ in regions where the stress state does not satisfy the yield condition.

With a view toward deriving a theoretical relationship between the crack tip speed and the imposed stress intensity factor required to sustain this speed according to a critical plastic strain crack growth criterion, attention was focussed on the strain distribution on the crack line within the active plastic zone, and on the influence of material inertia on this stress distribution. It was found that the distribution of shear strain on this line, say $\gamma_{yz}(x,0)$ in crack tip rectangular x, y coordinates, could be determined *exactly* in terms of the plastic zone size r_o in the parametric form

$$\gamma_{yz}(x,0) = \frac{\mu}{\tau_o}\left\{1 - \left(\frac{1-m^2}{2m^2}\right)\ln\left(\frac{1-m^2h^2}{1-m^2}\right)\right\}$$
$$x = r_o\frac{I(-h)}{I(m)}, \qquad I(t) = \int_0^{(1-t)/(1+t)} \frac{s^{(1-m)/2m}}{(1+s)}\,ds \tag{3}$$

where $m = v/c_s$ and c_s is the elastic shear wave speed. While the integral $I(t)$ has a representation in terms of elementary functions only for very special values of its argument, it is easily evaluated by numerical methods for any nonzero value of m.

The exact result (3) resolved a long standing paradox concerning mode III crack tip fields. Rice (1968) showed that the near tip distribution of strain $\gamma_{yz}(x,0)$ for steady growth of a crack under equilibrium conditions was singular as $\ln^2(x/r_o)$ as $x/r_o \to 0$. On the other hand, Slepyan (1976) showed that the asymptotic distribution for any $m > 0$ was of the form $(m^{-1} - 1)\ln(x/r_o)$ as $x/r_o \to 0$. These two features could be verified by examining the behavior of the exact solution for dynamic growth (3) under the condition $m \to 0$ for any nonzero value of x/r_o and under the condition that $x/r_o \to 0$ for any nonzero value of m, respectively. The resolution of the paradox was found, however, in the observation that Slepyan's asymptotic solution is valid only if

$$(x/r_o)^{2m/(1+m)} \ll 1\,. \tag{4}$$

Thus, the apparent inconsistency arises from the fact that the asymptotic result due to Slepyan is valid over a region that becomes vanishingly small as $m \to 0$.

Graphs of the plastic strain distribution on the crack line in the active plastic zone are shown in Fig. 1 for $m = 0, 0.3, 0.5$. The plastic strain is singular in each case, as has already been noted. The most significant observation concerns the influence of material inertia on the strain distribution. An increase in crack speed results in a substantial reduction of the level in plastic strain for a fixed fractional distance from the crack tip to the elastic-plastic boundary. Therefore, if a local ductile crack growth criterion is imposed, then it would appear that the fracture resistance or toughness would necessarily increase with increasing crack tip speed. To quantify this idea, the fracture criterion proposed by McClintock and Irwin (1965) was adopted. According to this criterion, a crack will grow with a critical value of plastic strain at a point on the crack line at a characteristic distance ahead of the tip. The crack will not grow for levels of plastic strain at this point below the critical level, and levels of plastic strain greater than the critical level are inaccessible. To make a connection between the plastic strain in the active plastic zone and the remote loading, a relationship between the size of the plastic zone and the remote applied stress intensity factor is required. This can be provided

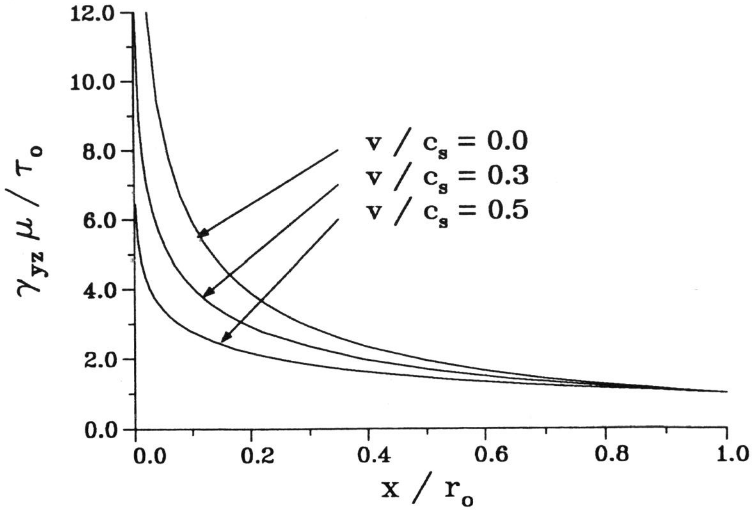

Fig. 1. Strain on the crack line in the active plastic zone for steady dynamic growth of a mode III crack in an elastic-ideally plastic material from (3).

only through a complete solution of the problem, and it was obtained for the case of mode III by Freund and Douglas (1982) through a full field numerical solution of the governing equations. The resulting theoretical fracture toughness K_{IIId} versus crack speed is shown in Fig. 2 for continuous variation of the critical plastic strain from $\gamma_c = 0$ to $\gamma_c = 20\tau_o/\mu$. The critical distance has been eliminated in favor of K_{IIIc}, the level of applied stress intensity required to satisfy the same criterion for a stationary crack in the same material under equilibrium conditions. The variable intercept at $m = 0$ indicates an increasing amount of plasticity with increasing critical plastic strain, and the intercept values correspond to the so-called steady state toughness values of the theory of stable crack growth, that is, with the plateau level of the resistance curve.

The plot in Fig. 2 illustrates some typical features. The ratio of K_{IIId}/K_{IIIc} is a monotonically increasing function of crack speed m for fixed critical strain, and this function takes on large values for moderate values of m. Although there is no unambiguous way to associate a terminal velocity with these results, a maximum attainable velocity well below the elastic wave speed of the material is suggested. It is emphasized that the variation of toughness with crack speed in Fig. 2 is due to inertial effects alone. The material response is independent of rate of deformation, and the crack growth criterion that is enforced involves no characteristic time. If inertial effects were neglected, the calculated toughness would be *completely independent* of speed. The question of the influence of material rate sensitivity on this relationship is a separate issue.

The equivalent plane strain problem of dynamic crack growth in an elastic-ideally plastic material has not been so fully developed. However, a numerical calculation leading

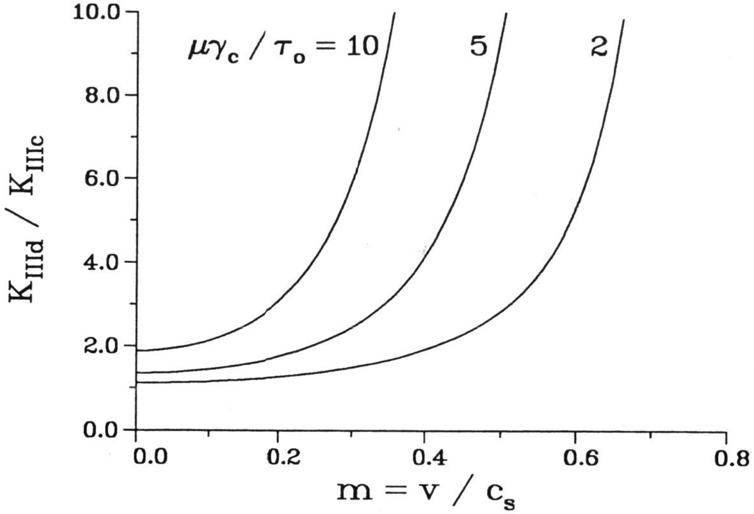

Fig. 2. Theoretical fracture toughness versus crack speed for steady growth of a mode III crack according to the critical plastic strain at a characteristic distance criterion, for three levels of critical plastic strain γ_c equal to 2, 5 and 10 times the yield strain τ_o/μ.

to a fracture toughness versus crack speed relationship, analogous to Fig. 2, has been described by Lam and Freund (1985). They adopted the critical crack tip opening angle growth criterion and derived results for mode I on the basis of the Mises yield condition and J_2 flow theory of plasticity that are quite similar in general form to those shown for mode III. The nature of the elastic-plastic fields deep within the active plastic zone were difficult to discern from the finite element results, and an analytical study of the asymptotic field was undertaken by Leighton et al (1987) in order to examine this feature. It was found that the plastic strain components had bounded limiting values at the crack tip for any nonzero crack speed $m = v/c_s$, but that these limits depended on crack speed as $1/m$. Both Slepyan (1976) and Achenbach and Dunayevsky (1981) reported studies of this problem in which they took elastic compressibility into account. They were able to extract solutions valid very close to the crack tip in the limit of vanishing crack speed. It should be noted that in both studies the Tresca yield condition was used together with the Mises flow rule, so that the fields described are consistent with normality of the plastic strain rate to the yield surface only in the limit of incompressibility. In addition, both studies imposed restrictions on the out-of-plane deformation that arise from assuming normality of the plastic strain rate to the Tresca yield surface. In a study of the same problem in the incompressible limit, Gao and Nemat-Nasser (1983) reported a solution with jumps in stress and particle velocity across radial lines emanating from the crack tip for all crack speeds between zero and the elastic shear wave speed, where the jump magnitudes were subject to the appropriate jump conditions. It is shown by Leighton

et al (1987), however, that if the sequence of deformation states throughout the jump must be admissible plastic states, consistent with the theory of mechanical shocks, then discontinuities in the angular variation of stress and particle velocity components around the crack edge can be ruled out.

Experimental Observations

Some data on the dynamic fracture toughness of metals during rapid crack growth are available. Rosakis, Duffy and Freund (1984) used the optical shadow spot method in reflection mode to infer the prevailing stress intensity factor during rapid crack growth in 4340 steel hardened to $R_C = 45$. This is a relatively strain rate insensitive material with very little strain hardening, so that the material may presumably be modeled as elastic-ideally plastic. The observed toughness varied little with crack speed for speeds up to about 600 to 700 m/s, and thereafter the toughness increased sharply with increasing crack tip speed. The general form of the toughness versus speed data was similar to the theoretical prediction based on the numerical simulation reported by Lam and Freund (1985), lending support to the view that material inertia on the scale of the crack tip plastic zone has an important influence on the perceived dynamic fracture toughness. Similar data were reported by Kobayashi and Dally (1979) who made photoelastic measurements of the crack tip stress field by means of a birefringent coating on the specimen. Data on crack propagation and arrest in steels were reported by Dahlberg, Nilsson and Brickstad (1980).

Dynamic Ductile Hole Growth

The examination of fracture surfaces of a wide range of metals and other materials following tensile crack growth under essentially plane strain conditions leads to the conclusion that the process of crack advance is essentially the nucleation of voids or cavities at material inhomogenieties, and the subsequent ductile growth of these voids to coalescence. Certain features of the elastic-plastic crack tip field, which provides the environment in which the mechanism operates, are important for this process. Among these are the high triaxial stress condition within the small strain region ahead of the crack tip that serves to nucleate voids and the zone of large plastic straining directly ahead of the tip that accommodates the ductile expansion of voids in this region necessary for coalescence with the main crack. The mechanical process of the ductile growth of cylindrical and spherical voids in plastic materials has been described by McClintock (1968) and Rice and Tracey (1969), respectively, who showed the strong influence of the mean normal stress component on the rate of growth of an isolated void.

McClintock (1968) and Rice and Johnson (1970) developed models of the void growth process within the crack tip field with a view toward relating microstructural features to fracture mechanics parameters for fracture initiation and stable crack growth. While these models can be viewed only as rough approximations, they do appear to capture the essence of the hole growth mechanism. It should be emphasized that the assumption of a fully ductile separation mechanism on the microscale does not necessarily imply extensive plastic flow in the body containing the crack. Indeed, it is quite possible to observe fully ductile separation at a crack tip in a material for which the plastic zone size is small and conditions of small scale yielding are satisfied. Likewise, extensive plastic flow in the body does not imply that the separation mechanism is a ductile mechanism. It could as well be cleavage induced by material rate effects or strain hardening as a result of the plastic flow.

In order to examine the influence of material inertia on a small scale on the ductile void growth process, a simple spherically symmetric model introduced by Carroll and Holt (1972) is adopted. This model was further developed within the framework of dynamic spall fracture by Johnson (1981). Consider a thick spherical shell of incompressible material with inner and outer radii $r = a(t)$ and $r = b(t)$, respectively, where r is the Eulerian coordinate representing distance from the fixed center of the shell. The material is initially at rest with void radius $a_o = a(0)$. At time $t = 0$, a uniform normal traction of magnitude σ_b begins to act on the outer surface. The inner surface is traction free. It is assumed that elastic effects are negligible and that the magnitude of σ_b is sufficient to produce ductile expansion of the shell.

With a view toward making a connection with the process of ductile void growth and coalescence as a mechanism of crack advance, the parameter a_o is identified with a representative physical dimension of the void at nucleation, and $2b_o = 2b(0)$ is identified with the spacing of void nucleation sites. If a body with a periodic array of such voids is subjected to a mean normal stress σ_b resulting in spherical growth of the voids, then the body 'fractures' at some later time, say $t = t^*$, for which $a(t^*) = b_o$. The objective of this simple model calculation is to determine the relationship between σ_b and t^*. With reference to a crack growth process, $2b_o/t^*$ and σ_b provide crude estimates of crack speed and crack tip field intensity, respectively.

The velocity in the radial direction of a particle with instantaneous radial coordinate r is $v_r(r,t) = \dot{a}a^2/r^2$. Furthermore, the inner and outer radii are related by $b^3 - b_o^3 = a^3 - a_o^3$ identically in time. Integration of the momentum equation with respect to r from a to b yields

$$\sigma_b = \rho\left[\tfrac{1}{2}\dot{a}^2\left(\frac{a^4}{b^4} - 1\right) - (a\ddot{a} + 2\dot{a}^2)\left(\frac{a}{b} - 1\right)\right] - 2\int_a^b \frac{\sigma_r - \sigma_t}{r}\, dr\,. \qquad (5)$$

This result is independent of the constitutive description of the material, except for the assumption of incompressibility. It is tacitly assumed in writing (5), however, that the normal stress difference $\sigma_r - \sigma_t$ is completely determined by the velocity field by means of a constitutive description for flow. This description can be for rate independent or rate dependent material response.

To examine the influence of inertia on void growth, suppose that the material is perfectly plastic with tensile flow stress of magnitude σ_o. Thus, $\sigma_r - \sigma_t = -\sigma_o$ in this case and

$$2\int_a^b \frac{\sigma_r - \sigma_t}{r}\, dr = -2\sigma_o \ln\frac{b}{a}\,. \qquad (6)$$

The minimum applied stress σ_b required to produce flow in the sphere is $(\sigma_b)_{min} = 2\sigma_o \ln(b_o/a_o)$. For any given value of σ_b, the relation (5) provides a second order ordinary differential equation for $a(t)$ subject to the initial conditions that $a(0) = a_o$ and $\dot{a}(0) = 0$. This equation can be solved numerically for any values of $b_o/a_o > 1$ and $\sigma_b/(\sigma_b)_{min} > 1$. A particular result is shown in Fig. 3 for the cases of $b_o/a_o = 20$ and $b_o/a_o = 100$, in the form of a graph of $\sigma_b/(\sigma_b)_{min}$ versus b_o/kt^* where $k = \sqrt{\rho/\sigma_o}$. Recall that t^* is defined

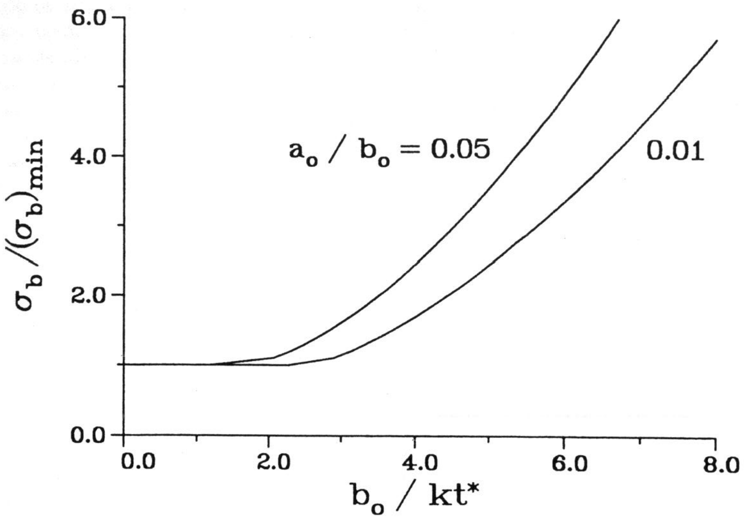

Fig. 3. The stress required to expand a thick spherical ideally plastic shell to a specified size versus the time required to do so for two values of initial hole size.

by the condition that $a(t^*) = b_o$. It is evident from the result that any applied stress only slightly larger than $(\sigma_b)_{min}$ will result in growth of the void to critical size in a time $t^* > b_o/2k$. However, if the void must be grown to critical size in a shorter time, then a larger stress σ_b is required to overcome the inertial resistance.

Based on this simple model calculation, the time $b_o/2k$ appears to have particular significance for assessing the influence of material inertia on the physical scale of the ductile hole growth mechanism. This time can be estimated for high strength steel or aluminum alloys for which the crack growth mechanism is predominantly ductile hole growth. Of greater interest, perhaps, is the 'speed' $2b_o/t^*$ which, as noted above, is a crude estimate of crack propagation speed if the initial void nucleation site spacing is $2b_o$ and the time required to grow the voids to the critical size is t^*. Thus, microscale inertia is significant for $b_o/a_o = 20$ if

$$2b_o/t^* \geq 4\sqrt{\sigma_o/\rho}\,. \qquad (7)$$

If the parameter $k = \sqrt{\sigma_o/\rho}$ has a value of $300\,m/s$, which is typical for high strength alloys, then (7) implies that the crack speed at which local inertial effects result in an observable influence on macroscopic dynamic fracture toughness is about $1200\,m/s$. This speed is somewhat greater than the observed crack speed at which the measured or inferred dynamic fracture toughness for such materials begins to show a dramatic increase with speed, which is usually in the range of $500 - 1000\,m/s$. Material strain rate dependence has been neglected in this simple analysis, and it is possible that the influence of rate effects could be equal to or greater than the influence of inertial effects, especially for small voids (Curran et al, 1987).

An estimate of the plastic strain rate near the tip of an advancing crack can be obtained as follows. Suppose that the yield stress in shear is τ_y and that the elastic shear modulus is μ, so that the yield strain is τ_y/μ. As a rough estimate of the plastic strain rate, consider the yield strain divided by the time required for the crack tip to traverse a region that is the size of the active plastic zone at speed v. Following McClintock and Irwin (1965), if the size of the plastically deforming region is interpreted as the largest extent in an elastic field of the locus of points on which the maximum shear stress is τ_y, then the estimate of strain rate is

$$(\dot{\gamma}^p)_{est} \sim 7v\tau_y^3/\mu^2 G \tag{8}$$

where energy release rate G is the characterizing parameter for the elastic field. Clearly, for rapid growth of a crack in a low toughness material, the strain rate estimate can be enormous, in excess of $10^6\ sec^{-1}$.

Viscoplastic Material Response

The particular material model known as the over-stress power law model has been considered by Lo (1983), Brickstad (1983) and a number of other authors. According to this idealization, the plastic strain rate in simple shear $\dot{\gamma}^p$ depends on the corresponding shear stress τ through

$$\dot{\gamma}^p = \dot{\gamma}_t + \dot{\gamma}_o\{(\tau - \tau_t)/\mu\}^n \quad \text{for} \quad \tau \geq \tau_t \tag{9}$$

where $\dot{\gamma}_t$ is the threshold strain rate for this description, or the plastic strain rate when $\tau = \tau_t$. The description also includes the elastic shear modulus μ, the viscosity parameter $\dot{\gamma}_o$, and the exponent n. A common special case is based on the assumption that the slow loading response of the material is elastic-ideally plastic and that *all* inelastic strain is accumulated according to (9). For this case, $\dot{\gamma}_t = 0$ and τ_t is the slow loading flow stress τ_o. For other purposes, it is assumed that (9) provides a description of material response only for high plastic strain rates, in excess of the transition plastic strain rate $\dot{\gamma}_t$ and for stress in excess of the corresponding transition stress level τ_t. For low or moderate plastic strain rates, the variation of plastic strain rate with stress is weaker than in (9), and a common form for the dependence is (cf. Frost and Ashby, 1982)

$$\dot{\gamma}^p = g_1(\tau)\exp\{-g_2(\tau)\} \tag{10}$$

where g_1 and g_2 are algebraic functions. The marked difference between response at low or moderate plastic strain rates and at high strain rates may be due to a change in fundamental mechanism of plastic deformation with increasing rate, or it may be a structure induced transition. For present purposes, it is sufficient to regard the difference as an empirical observation. The two forms of constitutive laws (9) and (10) can lead to quite different results in analysis of crack tip fields and, indeed, the form (9) leads to fundamentally different results for different values of the exponent n.

Lo (1983) extended some earlier work on the asymptotic field for steady quasistatic crack growth in an elastic-viscoplastic material by Hui and Riedel (1981) to include inertial

effects. In both cases, the multiaxial version of (9) with $\dot{\gamma}_t = 0$ and $\tau_t =$ was adopted to describe inelastic response, with no special provision for unloading. They showed that for values of the exponent n less than 3, the asymptotic stress field is the elastic stress field. For values of n greater than 3, on the other hand, Lo constructed an asymptotic field including inertial effects having the same remarkable feature of complete autonomy found by Hui and Riedel, that is, it revealed no dependence on the level of remote loading. For steady antiplane shear mode III crack growth, Lo found the radial dependence of the inelastic strain on the crack line ahead of the tip to be

$$\gamma^p_{yz}(x,0) \approx (n-1)(v/\dot{\gamma}_o x)^{1/(n-1)}\,T_L(v/c_s) \tag{11}$$

where the dependence of the amplitude factor T_L on crack speed is given graphically by Lo, who also analyzed the corresponding plane strain problem. Note that as $n \to \infty$ the plastic strain singularity becomes logarithmic. The full field solution for this problem under small scale yielding conditions was determined numerically by Freund and Douglas (1983). The numerical results showed a plastic strain singularity much stronger than for the rate independent case, and it appeared from the numerical results that the domain of dominance of the asymptotic field within the crack tip plastic zone expanded with increasing crack tip speed. These observations are consistent with (11).

A Viscoplastic Crack Growth Model

A particularly interesting class of dynamic fracture problems are those concerned with crack growth in materials that may or may not experience rapid growth of a sharp cleavage crack, depending on the conditions of temperature, stress state and rate of loading. These materials may fracture by either a brittle or ductile mechanism on the microscale, and the focus of work in this area is on establishing conditions for one or the other mode to dominate. The phenomenon is most commonly observed in ferritic steels. Such materials show a dependence of flow stress on strain rate, and the strain rates experienced by a material particle in the path of an advancing crack are potentially enormous. Consequently, the mechanics of rapid growth of a sharp macroscopic crack in an elastic-viscoplastic material that exhibits a fairly strong variation of flow stress with strain rate has been of interest in recent years. The general features of the process as experienced by a material particle on or near the fracture path are straight forward. As the edge of a growing crack approaches, the stress magnitude tends to increase there due to the stress concentrating effect of the crack edge. The material responds by flowing at a rate related to the stress level in order to mitigate the influence of the crack edge. It appears that the essence of cleavage crack growth is the ability to elevate the stress to a critical level before plastic flow can accumulate to defeat the influence of the crack tip. In terms of the mechanical fields near the edge of an advancing crack, the rate of stress increase is determined by the elastic strain rate, while the rate of crack tip blunting is determined by the plastic strain rate. Thus, an equivalent observation is that the elastic strain rate near the crack edge must dominate the plastic strain rate for sustained cleavage. It is implicit in this approach that the material is intrinsically cleavable, and the question investigated in concerned with the way in which work can be supplied to the crack tip region.

The problem has been studied from this point of view by Freund and Hutchinson (1985). They adopted the constitutive description (9,10) with $n = 1$. This is indeed a situation for which the near tip elastic strain rate dominates the plastic strain rate. Through an approximate analysis, conditions necessary for a crack to run at high velocity in terms of constitutive properties of the material, the rate of crack growth, and the overall crack driving force were extracted under small yielding conditions.

Consider the crack gliding along through the elastic-viscoplastic material under plane strain conditions. At points far from the crack edge, the material remains elastic and the stress distribution is given in terms of the applied stress intensity factor K_I. Equivalently, the influence of the applied loading may be specified by the rate of mechanical energy flow into the crack tip region from remote points G, and these two measures are related by means of

$$G = \frac{1-\nu^2}{E} A(v) K_I^2 \tag{12}$$

where ν and E are the elastic constants of an isotropic solid and A is a universal function of the instantaneous crack speed v. The function has the properties that $A(0) = 1$, $A'(0) = 0$ and $A(v) \to \infty$ as $v \to c_r$. For points near the crack edge the potentially large stresses are relieved through plastic flow, and a permanently deformed but unloaded wake region is left behind the advancing plastic zone along the crack flanks. For material particles in the outer portion of the active plastic zone the rate of plastic straining is expected to be in the low or moderate strain rate range, whereas for particles close to the crack edge, the response is modelled by the constitutive law (9) with $n = 1$. Because of elastic rate dominance, the stress distribution within this region has the same spatial dependence as the remote field but with a stress intensity factor *different* from the remote stress intensity factor. The crack tip stress intensity factor, say K_{Itip}, is assumed to control the cleavage growth process. The influence of the remote loading is screened from the crack tip by the intervening plastic zone, and the main purpose of the analysis is to determine the relationship between the remote loading and the crack tip field. For present purposes, it is assumed that the crack grows as a cleavage crack with a fixed level of local energy release rate, say G^c_{tip}. The question then concerns the conditions under which enough energy can be supplied remotely to sustain the level of energy release rate G^c_{tip} at the crack tip.

The matter of relating the applied G to G^c_{tip} was pursued by enforcing an overall energy rate balance. The balance may be cast into the form

$$G^c_{tip} = G - \frac{1}{v}\int_A \sigma_{ij}\dot{\epsilon}_{ij}\, dA - \int_{-h}^{h} U_e^*\, dy \tag{13}$$

where A is the area of the active plastic zone in the plane of deformation, h is the thickness of the plastic wake far behind the crack tip, and U_e^* is the residual elastic strain energy density trapped in the remote wake. This relation simply states that the energy being released from the body at the crack tip is the energy flowing into the crack tip region reduced by the energy dissipated through plastic flow in the plastic zone, and further reduced by the energy trapped in the wake due to incompatible plastic strains. The expression is exact.

Through several approximations, the complete energy balance (13) was reduced by Fre-

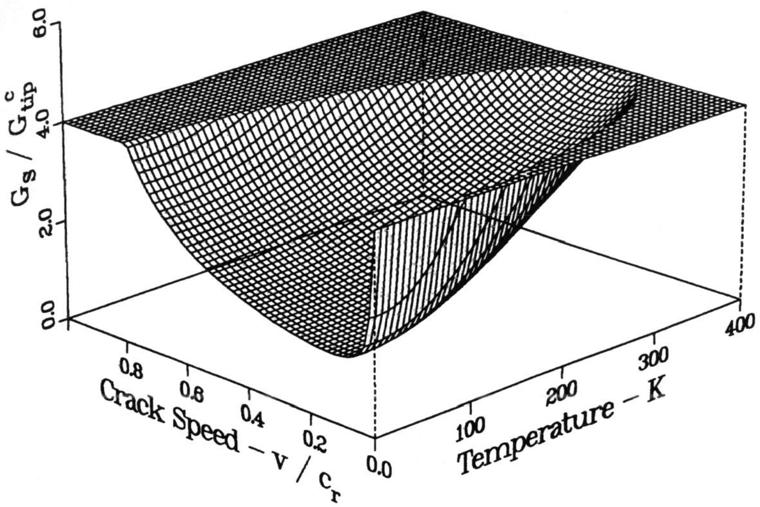

Fig. 4. A surface representing conditions on applied crack tip driving force G_S, crack tip speed v and temperature that correspond to steady crack propagation, as predicted by (14). The plateau is an artifice arising from truncation of the surface at a suitable level.

und and Hutchinson (1985) to the simple form

$$G/G^c_{tip} = 1 + D(m)P_c \tag{14}$$

where the dimensionless parameter P_c is $\dot{\gamma}_o\sqrt{\mu\rho}\,G^c_{tip}(1 + 2\dot{\gamma}_t\mu/\dot{\gamma}_o\tau_t)/3\tau_t^3$ and $D(m)$ is a dimensionless function of crack tip speed $m = v/c_r$ and ρ is the material mass density. P_c is a monotonically increasing function of temperature for steels with values in the range from about zero to twelve as temperature varies from $0\,K$ to about $400\,K$; see Fig. 5. The function $D(m)$ is asymptotically unbounded as $m \to 0$ and $m \to 1$, and it has a minimum of order unity at an intermediate crack tip speed. The applied crack tip driving force, say G_S, is related to the crack tip energy release rate by $G_S = G/(1 - v/c_r)$ for a semi-infinite crack in an otherwise unbounded body, and this relationship is adopted here as an approximation. A graph of G_S/G^c_{tip} is shown in Fig. 4 in the form of a surface over the crack speed—temperature plane.

The graph in Fig. 4 gives the locus of combinations G_S, v, T for which steady state propagation of a sharp crack can be sustained. The implication is that if a cleavage crack can be initiated for a combination G_S, v, T that is *above* the surface, then the crack will accelerate to a state on the stable branch of the surface (i.e, the side with increasing G_S at fixed temperature). If the driving force diminishes as the crack advances, or if the local material temperature increases as the crack advances, then the state combination will move toward the minimum point on the surface at the local temperature. If the

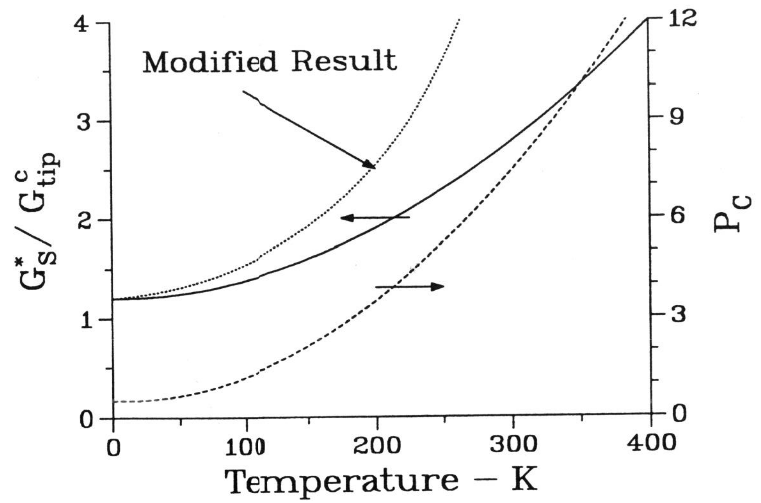

Fig. 5. The minimum driving force G^*_S needed to drive the crack dynamically as a function of temperature. The solid line results from Fig. 4, and the dashed line from a modified result due to Mataga et al (1987).

driving force is further decreased, or if the temperature is further increased, then growth of a sharp cleavage crack cannot be sustained according to the model. The implication is that the crack will arrest abruptly from a fairly large speed, and a plastic zone will then grow from the arrested crack.

Of special significance is the observation that, at any given temperature, the variation of required driving force with crack speed has an absolute minimum, say G^*_S/G^c_{tip}. This implies that, according to this model, it is impossible to sustain cleavage crack growth at that temperature with a driving force below this minimum. Thus, this minimum as a function of temperature may be interpreted as the variation of the so-called arrest toughness for the material with material temperature. This minimum is plotted against temperature for the case of mild steel in Fig. 5.

Further crack growth beyond the first arrest is possible if either a ductile growth criterion can be met or if cleavage can be reinitiated through strain hardening in the evolving plastic zone. The details of the model have been refined through full numerical solution of the problem (Freund et al, 1986), but the essential features have not changed with more precise analysis. A modification of the basic model was proposed by Mataga et al (1987) that provides a description that is in better agreement with full field numerical solutions than the model outlined above. In the original development, it was assumed that the plastic dissipation was completely controlled by the near tip stress intensity factor field, say K_{tip}. However, this stress intensity factor, which is asymptotically correct, must give way to the far field stress stress intensity factor K with increasing distance from the crack edge. It was observed by Mataga et al (1987) that the estimate of plastic dissipation

was improved significantly, at least in comparison with finite element simulations, if the plastic dissipation was estimated on the basis of a 'mean' stress intensity factor $\sqrt{K_{tip}K}$. The graph of arrest toughness versus temperature for parameters corresponding to mild steel are also shown in Fig. 5.

Experimental Observations

Important experiments on crack propagation and arrest in steel specimens are currently being carried out by deWit and Fields (1987). Their specimens are enormous single edge notched plates loaded in tension. The growing crack thus experiences an increasing driving force as it advances through the plate. A temperature gradient is also established in the specimen so that the crack grows from the cold side of the specimen toward the warm side. Based on the presumption that the material becomes tougher as the temperature is increased, the crack also experiences increasing resistance as it advances through the plate. The specimen material is A533B pressure vessel steel, which is both very ductile and strain rate sensitive. In the experiments, the fracture initiates as a cleavage fracture and propagates at high speed through the specimen into material of increasing toughness. The crack then arrests abruptly in material whose temperature is *above* the nil ductility temperature for the material based on Charpy tests. A large plastic zone grows from the arrested crack edge, and cleavage crack growth is occasionally reinitiated. The essential features of the experiment appear to be consistent with the model of high strain rate crack growth outlined in above, and this model appears to provide a conceptual framework for interpretation of the phenomenon. An analysis of rapid crack growth in a rate dependent plastic solid has also been carried out by Brickstad (1983) in order to interpret some experiments on rapid crack growth in a high strength steel.

A new experiment for studying dynamic fracture processes that occur due to loading pulses of extremely short duration has been developed by Ravichandran and Clifton (1986). A thin disk of a high strength steel containing a mid-plane prefatigued edge crack that has been propagated halfway across the diameter is impacted by a thin flyer plate of the same material. A compressive pulse propagates through the specimen and reflects from the rear surface as a step tensile pulse of duration of about 1 μs. This plane wave loads the crack and causes propagation of the fracture. The motion of the rear surface of the specimen is monitored during the event by means of a laser interferometric technique. In effect, this situation is that of plane strain deformation of a semi-infinite crack in an unbounded body subjected to plane wave loading, at least for a microsecond or two. For 4340 steel with R_C = 52 tested at a temperature of $-100\,C$, the cracks grew predominantly as cleavage cracks. Based on optical measurements of the surface motion of the specimen and comparison with detailed elastic-viscoplastic calculations, it appeared that the cracks grew more nearly at constant velocity crack than with a fixed level of energy release rate or stress intensity factor. This observation is similar to that made by Ravi-Chandar and Knauss (1984) who studied crack growth in the brittle polymer Homalite-100. In both cases, this observation was made in situations where the load was suddenly applied and the load level was very intense compared to the minimum load necessary to induce fracture in the same situation. The results suggest that the one parameter characterization of the crack tip conditions may not be adequate to describe fracture response under such conditions, and that damage evolution in advance of the main crack is important in the process.

CONCLUDING REMARKS

The results described in the preceding sections reflect some progress toward discovery of the role played played by crack tip plastic fields in establishing conditions for rapid advance of a crack in an elastic-plastic material. Understanding of this issue is far from complete, and a few of the open questions that could be profitably pursued are identified in this concluding section. For example, much of the modeling that has resulted in a detailed description of crack tip elastic-plastic fields has been based on the assumption that the fields are steady as seen by a crack tip observer. This approach overlooks all transient aspects of the process. The picture of the way in which a crack tip plastic zone develops in a cracked, rate sensitive structural material under the action of stress wave loading is not clear, but the question is important in the sense that these fields determine whether or not the crack will advance. The same issue appears to be at the heart of the cleavage initiation process in steels, but on a microstructural scale. Here, the sudden cracking of carbides or other brittle phases due to incompatible plastic strains provides a nucleation mechanism, and the question is whether or not these dynamic microcracks penetrate into the adjacent ferrite as sharp cracks. The answer seems to hinge on the way in which plastic strains develop near the carbide-ferrite interface due to the appearance of the microcracks in the brittle phases.

The transients of the arrest of a cleavage crack in a structural material are also unclear at this point. A running crack appears to arrest because conditions for the continuous reinitiation of cleavage cannot be maintained (Irwin, 1987). In terms of the model discussed in section 4, arrest occurs because conditions for elastic rate dominance of the local field cannot be maintained. However, the model does not provide information on the process thereafter. It appears from the experiments reported by deWit and Fields (1987) that arrest is quite abrupt, that a large plastic zone grows from the crack edge following arrest of the cleavage crack, that the crack may grow subsequently in a ductile mode, and that cleavage may be reinitiated at a later stage. It is not clear if the cleavage reinitiation is due to a rate effect or to a combination of strain hardening and constraint in the interior portions of the specimen.

Modeling of plasticity effects in dynamic crack growth has been restricted to two dimensional systems, for the most part. It is likely that a number of three dimensional effects are of sufficient importance to warrant further investigation. For example, crack propagation studies are often carried out with plate specimens. For such specimens, the transition from plane stress conditions in regions far from the crack tip compared to plate thickness to plane strain or generalized plane strain conditions near the crack edge is not clear. Yang and Freund (1985) suggest that plane stress conditions prevail only for points beyond about one-half the plate thickness from the crack edge for elastic deformations. Out-of-plane inertia is of potential importance in these three dimensional fields, but this effect has not been investigated to date. Furthermore, the role of ductile shear lips at the free surfaces or of ductile ligaments left behind a cleavage crack as it advances through a structural metal are not clear at this time. In a study of fracture initiation in dynamically loaded specimens of a ductile material by Nakamura et al (1986), it was shown that these three dimensional effects are potentially very significant.

Finally, it is noted that the analytical models discussed above have been developed for the study of rapid crack growth in a rate-dependent elastic-plastic material under

conditions than permit crack advance in a cleavage mode, and separately for rapid crack advance in an elastic- plastic material when the crack advances by means of a local ductile mechanism. However, models suitable for study of rapid crack growth that permit either mode of crack advance, with the operative mode being determined by which of two competing fracture criteria prevails, have been elusive. A preliminary study of a very simple model of this type has been reported by Lee and Freund (1988). The process of dynamic tensile crack growth in a material was analyzed under small scale yielding conditions with the crack tip plastic zone modeled as a strip yield zone extending ahead of the advancing crack tip. Following Glennie (1971) and others, rate dependence of plastic flow was taken into account by assuming that the cohesive stress in the yield zone depends linearly on the local rate of opening of the yield zone. The conditions under which a crack can advance steadily according to either of two criteria were then considered. A crack tip opening criterion was identified with a locally ductile mode, and a critical stress condition was identified with a cleavage mode. The analysis led to conditions among the applied stress intensity factor, the crack speed and the material viscosity that are necessary for sustained crack growth in either case, with the implication that the criterion that is easiest to satisfy will establish the mode by which the crack advances.

A representative result is shown in Fig. 6 in the form of a surface of applied stress intensity factor necessary to sustain growth according to either criterion over the plane of crack speed v normalized by the elastic shear wave speed c_s and a viscosity parameter β that characterizes the sensitivity of the cohesive zone stress to the opening rate. The dynamic stress intensity factor is normalized by the initiation toughness K_c which necessarily satisfies the crack tip opening criterion.

With reference to Fig. 6, the following crack growth behavior is represented. Suppose that a cracked body characterized by a particular value of β is loaded so that the crack begins to advance from speed $v = 0$, and that the applied stress intensity factor increases as the crack advances. The result implies that the crack will accelerate with the separation on a local scale occurring according to a ductile mechanism. In terms of the surface in Fig. 6, the state of K and v follows the surface along a path for which $\beta = constant$, starting from $K = K_c$ and $v = 0$. The crack accelerates until a speed corresponding to the position of the 'ridge' in the surface is attained. At this point, the mode of material separation converts to a stress controlled mode due to the rate induced elevation of flow stress. The only way for the applied stress intensity factor to further increase is for the crack speed to suddenly become very large, so that the state ends up on the rapidly rising portion of the surface associated with inertial effects (not shown in Fig. 6). Thereafter, if the applied stress intensity factor decreases, then the state falls to the minimum in the path for fixed β. Further decrease of the applied stress intensity factor implies crack arrest, or no further growth can be sustained according to either of the possible criteria. These general features are consistent with the observations made above concerning crack arrest in rate sensitive materials.

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Various aspects of analytical and experimental research on plasticity effects in dynamic

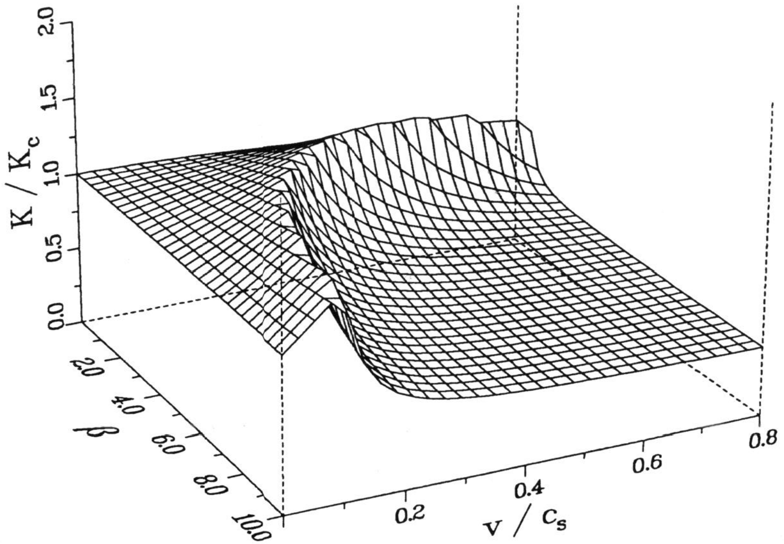

Fig. 6. Dependence of the applied dynamic stress intensity factor on crack speed v and viscosity parameter β implied by the condition that *either* a critical crack tip opening displacement growth criterion or a critical stress criterion can be satisfied by the material, with the choice determined by whichever can be satisfied by the lower level of K.

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