The Effects of Toughening Stresses on Liquid Impact Induced Fracture

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ABSTRACT

Toughening stresses are induced in glasses to improve their quasi-static fracture properties. The effects of these toughening stresses on the dynamic fracture properties of glasses are investigated in this paper. In particular an experimental study of liquid impact on thermally and chemically toughened glasses is presented. The results are discussed with reference to a theoretical simulation of a Rayleigh wave interacting with surface cracks.

KEYWORDS

Chemically toughened glass; dynamic fracture; liquid impact; Rayleigh waves; stress intensity factor; thermally toughened glass.

INTRODUCTION

Induced stresses are used to toughen glasses for use in components subjected to high applied stresses. Two distinct processes of toughening are used: thermal (physical) and chemical toughening. Thermally toughened glass (TTG) is manufactured by heating the glass to just below its softening point, followed by a rapid quench. This results in a parabolic stress distribution in the glass, with the surface in compression and the interior being held in tension. The stress distribution is modelled by

$$\sigma(z) = -\sigma_c\left(1 - \frac{6z}{d} + \frac{6z^2}{d^2}\right) \tag{1}$$

where σ_c is the magnitude of the surface compressive stress; z is the depth into the material and d is the thickness of the glass (see, for example, Lawn and Marshall, 1977). Solution of equation 1 shows that the compressive depth is ~21% of the sample thickness.

Chemically toughened glass (CTG) is prepared by placing the glass in a salt bath for a prescribed length of time, during which ion exchange takes place. The mismatch in ionic size results again in a surface compression layer. The layer is very thin; of the order of a few tens of microns. The stress distribution in this layer is modelled as varying linearly with depth such that

$$\sigma(z) = -\sigma_c\left(1 - \frac{z}{\delta}\right) \qquad 0 \le z \le \delta \tag{2a}$$

and

$$\sigma(z) = -\sigma_c\left(1 - \frac{d}{\delta} + \frac{z}{\delta}\right) \qquad d-\delta \le z \le d \tag{2b}$$

where σ_c is the magnitude of the surface compressive stress, δ is the layer thickness and d is the thickness of the glass. The interior tensile stress layer is modelled by

$$\sigma(z) = \frac{\sigma_c \delta}{d} \qquad \delta \le z \le d-\delta \tag{2c}$$

(see, for example, Lawn and Marshall, 1977).

In both cases there are improvements in the quasi-static strength of the material. In the case of the thermally toughened glass studied in the present work the improvement was ~100% and for the chemically toughened glass ~400%.

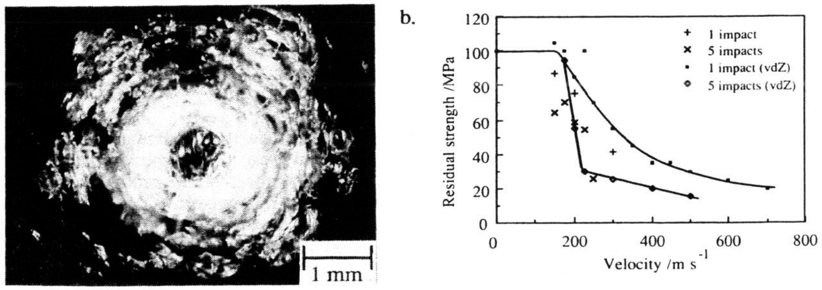


Fig. 1 a. Liquid impact damage on soda-lime glass
b. Residual strength curve for soda-lime glass (after van der Zwaag and Field (1983) plus data from current work)

LIQUID IMPACT

The use of toughened glasses as aircraft window materials means that they may be subjected to both liquid and solid impact. Damage resulting from liquid impact on brittle materials is a stress wave dominated phenomenon as the high 'water hammer pressures' generated during the initial compressible phases of the impact are of a short duration (typically ~0.01 - 0.1 µs). In particular surface cracks are extended by interaction with the Rayleigh surface waves generated by the impact resulting a circumferential crack pattern (see fig. 1a) (Bowden and Field, 1964).

Liquid impact is conveniently studied in the laboratory using the single jet impact technique (Bowden and Brunton, 1961; Field, van der Zwaag and Townsend, 1983). It is possible to relate the damage caused by particular sized jets to that produced by an 'equivalent' (usually 2mm diameter) spherical drop (Field *et al*, 1983; Hand, 1987). Quantitative assessment of any damage can be achieved by measuring the post-impact (residual) strengths using the hydraulic pressure test technique (Matthewson and Field, 1980). A residual strength curve for soda-lime glass is shown in fig. 1b. It can be seen that there exists a threshold velocity below which the stresses induced by impact are insufficient to extend the pre-existing surface defects. Once the threshold velocity is reached these defects are extended by interaction with the Rayleigh wave pulse.

The extent to which any flaw grows is dependent on both its initial size and its position relative to the impact site. This results in a bimodal transition region: in some of the samples no pre-existing flaws are significantly extended, whilst in others, with more serious flaws in the high stress region, they are. At high impact velocities many more of the flaws are extended and thus

the strength of all the samples is reduced. The width of the bimodal transition region of the residual strength curve is reduced for multiple impact. Surface flaws are able to grow on each loading cycle (assuming the threshold velocity has been exceeded) and thus the probability that a flaw close to the impact site will be extended significantly is increased. The probability for a reduction in strength of a sample is therefore increased and hence there is a reduction in the width of the bimodal region (see fig. 1b).

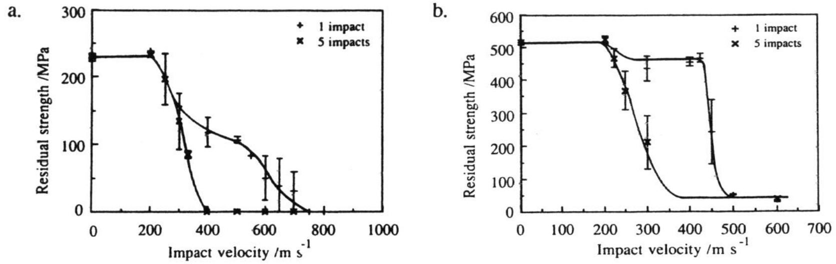


Fig. 2. Residual strength curve for a) TTG, b) CTG.

LIQUID IMPACT ON TOUGHENED GLASSES

The residual strength curve for thermally toughened glass is shown in fig. 2a. The threshold velocity has been raised to 200m s^{-1}, as compared to 150m s^{-1} for soda-lime glass. This is somewhat lower than the improvements in quasi-static strength would suggest.

At impact velocities greater than 400m s^{-1} for multiple and approximately 700m s^{-1} for single impact respectively, the sample failed catastrophically with a multiply bifurcated crack. It was observed that at intermediate velocities less than five impacts but more than one was required for catastrophic failure of the sample.

The residual strength curve for a chemically toughened glass is shown in fig. 2b. The difference in behaviour between single and multiple impact is again seen. The threshold velocity has again been raised to ca. 200m s^{-1}.

The reduction in strength due to single impact above the threshold velocity is substantially less than that seen for thermally toughened glass. At high impact velocities (either single or multiple impact) catastrophic failure of the chemically toughened glass does not occur. Cracks do penetrate right through the sample, and the residual strength is low, but catastrophic failure is avoided due to the low tensile stresses in the centre of these samples; using the standard CTG stress distribution (equations 2a-c) the tensile stresses in the centre glass studied is of the order of 6MPa. In the case of the thermally glass studied the equivalent stress reaches 55MPa.

SIMULATION OF LIQUID IMPACT ON TOUGHENED GLASSES

A simulation of the interaction of the Rayleigh wave with a pre-existing flaw distribution in the presence of toughening stresses was undertaken to relate this data to previous liquid impact results. As the toughening stresses vary with depth it is necessary to consider the variation of the Rayleigh stress wave with depth. The theoretical Rayleigh wave stress distribution, which for a Poisson's ratio of 0.25 is given by

$$\sigma_r(z) = \sigma_0(e^{-0.8475kz} - 0.5773e^{-0.3933kz}) \tag{3}$$

where k is the wavenumber of the wave (Kolsky, 1953), is therefore used in this analysis. This is unlike previous simulations of liquid impact which have either neglected (van der Zwaag and Field, 1983) or used approximate forms of this stress distribution (Evans *el al*, 1980).

To simplify the calculations it is assumed that all the cracks are through edge cracks (fig. 3) (surface crack growth is thus neglected); that they are all orientated perpendicularly to the stress wave and that they are extended in this direction only. Experimental photographic studies (see, for example, Field 1966; van der Zwaag, 1981) show that the real cracks are propagated obliquely to the surface during liquid impact.

The dynamic stress intensity factor

To model the interaction of a Rayleigh surface wave with a crack it is necessary to calculate the dynamic stress intensity factor at the crack tip. Freund (1972, 1973) has analysed the case of a plane wave incident on a semi-infinite crack. The dynamic stress intensity factor has the following general form

$$K_I^d(t,c,v) = k(v)\,\kappa_d(t)\,K_s(c) \tag{4}$$

where $\kappa_d(t)$ is the time-dependent part of the stress intensity factor due to the shape of the stress pulse; $K_s(c)$ is the quasi-static factor for a crack of length c and k(v) is a modifying factor to account for the crack velocity. In this work the following function for k(v) (Freund, 1972; calculated for $\nu = 0.25$) is used:

$$k(v) = \frac{1}{\left(1 + \frac{v}{(C_R - v)}\right)\left(1 - 0.531\frac{v}{C_R}\right)^{1/2}} \tag{5}$$

The current crack velocity (initially set equal to zero) is evaluated from the dependence of the crack velocity on the dynamic stress intensity factor:

$$v = v_{max}\left(1 - \left(\frac{K_{Ic}}{K_I^d}\right)^2\right) \tag{6}$$

where v_{max} is the maximum crack velocity (Kerkhoff and Richter, 1969), assumed to be 1580m s^{-1} for soda-lime glass (Field, 1971).

For a time-dependent stress profile, $\sigma(t)$, the pulse may be considered as an incremental sum such that $\delta\sigma(t) = \sigma'\delta t$. The time-dependent stress intensity factor may then be evaluated using

$$\kappa_d(t) = \frac{2}{\pi}\int_0^t \sigma'(s)(t - s)^{1/2}ds \tag{7}$$

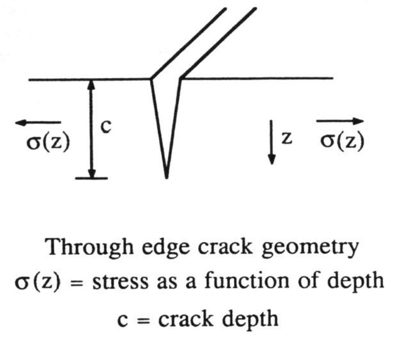
(Freund, 1973).

This analysis is based on a semi-infinite crack. For a finite crack, length c, the result must be modified to account for stress wave reflection from the free surface (straight forward application of equation 7 would otherwise suggest that, for a finite length crack, $\kappa_d \rightarrow \infty$ as $t \rightarrow \infty$). In this case the time dependent stress intensity factor can be expressed in the following form

$$K_d(t) = f\left(\frac{vt}{c}\right)K_s(c) \tag{8}$$

where v is the crack velocity and $K_s(c)$ is the quasi-static stress intensity factor (Evans, 1979). Sih (1973) has evaluated f(vt/c) numerically. The general form for a step pulse is that K(t) increases initially as $t^{1/2}$ reaching a maximum value 1.25 times greater than the quasi-static value (Thau and Lu, 1971, calculate the maximum to be 1.3 times the quasi-static value). It then decays in an oscillatory fashion towards the quasi-static value as the time tends to infinity. This behaviour has been approximated in the present study by a stress intensity factor of the form

3.



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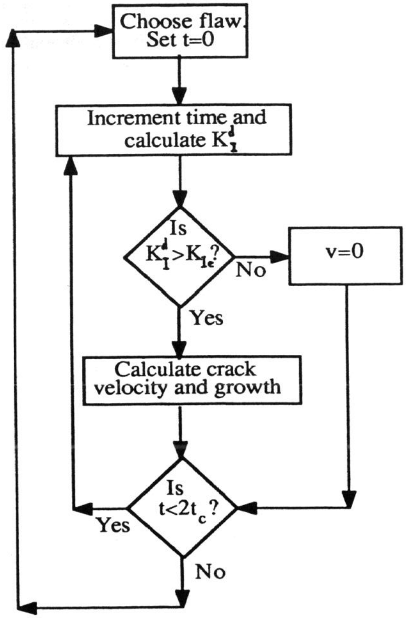

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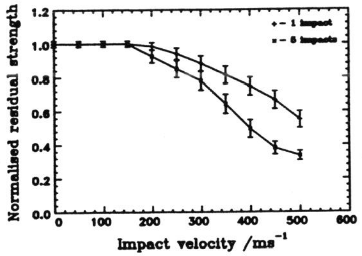

Fig. 3. Through edge crack geometry.
Fig. 4. Flow diagram of simulation program.
Fig. 5. Simulation of liquid impact on soda-lime glass.

$$K_d(t) = \left[1.25\int_0^{t-\tau}\sigma'(s)ds + \int_{t-\tau}^{t}\sigma'(s)(t - s)^{1/2}ds\right]K_s(c) \tag{9}$$

where $\tau = (1.25\pi)^2c/(4C_R)$. C_R is the Rayleigh wave velocity (the appropiate wave velocity for this problem). The short duration of the high pressure regime is such that the decay of the stress intensity factor towards the quasi-static value may be ignored.

The quasi-static stress intensity factor, $K_s(c)$ in equation 9 is calculated using the through edge crack analysis of Hartranft and Sih (1973) as a convenient approximation which increases in validity as the flaws are extended the growth along the surface during loading. In this case the quasi-static stress intensity factor is given by

$$K_s(c) = 2Y\left(\frac{c}{\pi}\right)^{1/2}\int_0^c \frac{\sigma(z)(1 + F(z/c))}{(c^2 - z^2)^{1/2}}dz \tag{10}$$

where $\sigma(z)$ is given by equation 3 and F(z/c) by

$$F\left(\frac{z}{c}\right) = \left(1 - \left(\frac{z}{c}\right)^2\right)\left[0.2945 - 0.3912\left(\frac{z}{c}\right)^2 + 0.7685\left(\frac{z}{c}\right)^4 - 0.9442\left(\frac{z}{c}\right)^6 + 0.5094\left(\frac{z}{c}\right)^8\right] \tag{11}$$

As the spatial decay of the Rayleigh wave is frequency dependent it is necessary to determine the wave frequency for a given velocity. Being a single pulse the Rayleigh surface wave does not have a single well-defined frequency. For the purpose of this simulation it is not feasible to Fourier analyse the pulse and use all the frequency components thus generated. Instead the pulse is assumed to have the frequency of the fundamental component. The wavenumber of this component for each impact velocity was calculated using $k = 2\pi C^2/(3C_R rv)$ where r is the drop radius, v is the velocity of impact and C is the shock wave velocity in water.

Flaw statistics

The strength of brittle materials is governed by a distribution in size of 'Griffith' type flaws. The flaw distribution used in this work is the probability density function for flaw dimensions developed by Jayatilaka and Trustrum (1977) which is given by

$$f(c) = \frac{c_0^{n-1}}{(n-2)!} c^{-n} e^{-c_0/c} \tag{12}$$

where c_0 is the mean flaw size, taken to be 10μm in this work. The parameter n is related to the Weibull parameter m (Rickerby, 1980). Jayatilaka and Trustrum (1977) estimated n to be 2.67±0.44 for glass. To obtain analytical results for the distribution it is necessary to have n integer. In this work therefore 3 is taken to be a reasonable value for n.

The calculation of crack growth

Initially a set of 40 flaws of random size subject to the chosen flaw distribution and in random positions were generated. The orientation of each flaw was taken as being normal to the radius vector from the centre of the distribution. The maximum flaw size was found and used to calculate the undamaged strength of the 'specimen' giving a worst case post impact strength.

The impact was simulated by a triangular pulse which has a maximum value, σ_{max}, given by $\sigma_{max} = \beta\rho CV$ where β is a constant, and ρCV is the 'water hammer' pressure. Swain and Hagan (1980) have shown that to a reasonable accuracy such a pulse approximates to the initial, tensile, part of the Rayleigh wave. The pressure distribution under a liquid impact is approximated by a flat punch pressure distribution and β is evaluated by using the proof given by Way (1940). The pulse moves radially out from the centre of the 'specimen' and its amplitude decreases as $r^{1/2}$ where r is the normalised radial coordinate. The duration of the pulse was taken to be invariant with distance, because the Rayleigh wave is non-dispersive. Each flaw was considered in turn for one to five impacts for a range of ten impact velocities; a flow diagram of the simulation is given in fig. 4.

Results

The results for soda-lime glass are shown in fig. 5. Each data point represents the average residual strength of forty samples. The generated curves are of the same basic form as the experimental results for soda-lime glass and other brittle materials. The threshold is very similar to that seen experimentally, but as both β and the geometrical factor, Y (assumed to be equal to 1, see equation 10) are not accurately known exact agreement cannot be expected. The threshold region is not as sharp as that seen experimentally, although the basic features are otherwise the same.

The results for thermally toughened glass are given in fig. 6a. It is clear that similarly good agreement in the form of the curves is obtained. The threshold velocity has been increased by 50m s^{-1}. The single and multiple liquid impact results exhibit the different behaviour observed experimentally. As for the soda-lime glass, the threshold region is not as sharp as that seen experimentally. Zero residual strength (equivalent to catastrophic failure) is seen in some individual samples, but not in all 40 cases and thus the average residual strength curve does not go to zero.

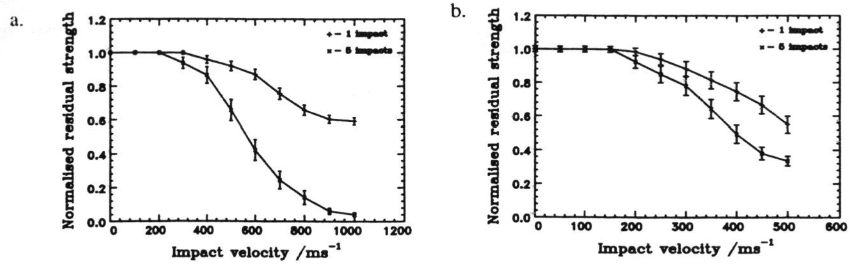

Fig. 6. Simulation of liquid impact on a) TTG, b) CTG.

The results for chemically toughened glass are not so good: the strength of the material is maintained for multiple as well as single impact (fig. 6b). The threshold velocity is again 200m s^{-1}. The poorer simulation of liquid impact on chemically toughened glass is thought to be a result of the simplicity of the assumed chemical toughening stress distribution (equations 2a-c).

DISCUSSION

The data presented here were generated using a 0.8mm diameter jet. The equivalent 'threshold' velocities (see above) for liquid impact by 2mm diameter drops are estimated to be 220m s^{-1} for soda-lime glass and 280m s^{-1} for the toughened glasses.

The small increase in the threshold velocity for damage is the same for both chemically and thermally toughened glasses which ties in well with the experimental results. The longer duration of the high pressure impact regime at higher impact velocities means that there is a decrease in the wavenumber of the Rayleigh wave This means that the attenuation of the Rayleigh wave with depth is reduced with increased impact velocity and thus cracks of a given depth are subjected to a greater impact stress than a simple calculation of the 'water hammer' pressure would suggest. Additionally cracks can be extended to a greater depth by the action of the stress pulse.

The difference between single and multiple impact curves may be explained by considering the growth of a surface crack subjected to a Rayleigh surface wave. Repeated impacts give the crack further opportunities to grow; hence under normal circumstances the threshold region is sharpened (see above). In the case of toughened glasses, however, further growth causes the crack to move from a region of compressive stress into a tensile one. This results in severe strength degradation as the crack tip loses the benefits of the compressive toughening stresses: the closure force at the crack tip becomes an opening force.

The plateau region for single impact at velocities greater than 200m s^{-1} is a result of the details of the stress profiles involved. The Rayleigh wave stress falls off exponentially with depth (Kolsky, 1953), whereas the compressive toughening stress falls off either linearly or quadratically depending on the toughening process. Thus a given crack will grow to a similar length for a range of impact velocities despite the increase in the maximum intensity of the stress pulse, as the toughening stresses become the dominant stress field with depth.

CONCLUSIONS

Toughening stresses modify not only the quasi-static behaviour but also the dynamic behaviour of toughened glasses. The benefits of the toughening stresses are of less benefit under liquid impact than the quasi-static results would suggest. This may be explained by a consideration of the interaction of a Rayleigh wave with a flaw distribution taking into account the presence of the toughening stresses.

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