Some Observations from an Elastic–Plastic Finite Element Study of a Mixed-mode Crack Initiation Problem

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ABSTRACT

Direction of initial crack extension in an elastic-plastic situation under mixed-mode loading has been predicted through incremental finite element analysis from the crack-tip stress-strain field using the criteria of maximum tangential stress (MTS), maximum tangential strain (MTSN) and maximum tangential principal stress (MTPS) of the linear elastic fracture mechanics(LEFM). The prediction of direction of crack extension as per these criteria remain unchanged at the various stages of loading. The plastic zone at the crack-tip develops symmetrically about a line whose inclination with the crack is close to the initial crack extension angle.

KEYWORDS

Direction of initial crack extension, maximum tangential stress criterion, maximum tangential strain criterion, maximum tangential principal stress criterion, elastic-plastic fracture, and mixed-mode crack extension.

INTRODUCTION

Direction of initial crack extension θ_c and critical load P_c at which crack initiates are the two important unknown in any mixed-mode fracture study. If the deformation is linear elastic, θ_c can be predicted using the criteria of maximum tangential stress (MTS) (Erdogan and Sih, 1963), strain energy density (SED) (Sih, 1973), maximum tangential strain (MTSN) (Wu, 1974) and maximum tangential principal stress (MTPS) (Maiti and Smith, 1983). Most metallic structures give rise to plastic deformation around the crack-tip upon loading and the situation falls more within the scope of elastic-plastic fracture mechanics (EPFM). In EPFM, the condition for the onset of crack extension is, many a times, defined in terms of the crack opening displacement (COD) and J-integral. These parameters have been mostly used to deal with mode I loadings. Very few studies concern elastic-plastic problems under mixed-mode loadings. Otsuka et al. (1957) performed a mixed-mode fracture test on a cracked tensile strip specimen of carbon steel and measured COD and stretched zone width (SZW) of an angled crack. They observed a correlation between the SZW and COD and proposed a COD based criterion for mixed-mode fracture. Application of J to mixed-mode fracture of ductile materials has been attempted by many authors (Cotterell et al., 1982, Ahmad et al., 1983, Kishimoto et al., 1980, Sakata et al., 1985). Cotterell et al. (1982) performed a mixed-mode plane stress ductile fracture test on staggered deep edge notch tension specimens of low alloy steel and demonstrated the applicability of J. Ahmad et al. (1983) carried out an investigation of crack initiation under both static and dynamic loading in three point-bend specimens with a slant crack and pointed out the usefulness of $\hat{J}$ approach (Kishimoto et al., 1980) for the purpose. They however indicate that more experiments are required to arrive at any firm conclusion. Sakata et al. (1985) have done experiments and finite element analysis for an angled crack problem in a compact tension specimen. They found $\hat{J}$ useful for the prediction of the fracture initiation load. All these studies deal with the prediction of P_c and no importance has been attached to the prediction of θ_c. This paper is intended to examine whether the direction of crack extension θ_c can be predicted using the criteria of MTS, MTSN and MTPS, which has so far been used within the framework of linear elastic fracture mechanics (LEFM). The investigation is based on an elastic-plastic finite element analysis. Some related observations on the crack-tip stress-strain field, plastic zone development and crack edge profile are also presented.

ELASTIC-PLASTIC FINITE ELEMENT ANALYSIS

The finite element analysis was done using a package based on an algorithm of Yamada et al. (1968). This algorithm is based on the tangent stiffness method. It uses small and varying increments of load just sufficient to yield a new Gauss point.

The specimen chosen for the analysis and other associated details are given in Fig.1 (inset). This configuration and material have been used by Sakata et al. (1985). Mises' yield criterion and plane stress condition are assumed. The stress-strain relation of the material is idealised by a Ramberg-Osgood relation.

8-noded rectangular elements are used to discretize the whole body except for the crack-tip zone, where quarter-point crack-tip elements are used. There are in total 200 elements and 680 nodes. The smallest element size at the crack-tip is 2.8 % of crack length 'a'.

RESULTS AND DISCUSSION

Fig. 1 shows the variation of load per unit thickness (P/B) with deflection δ. δ is indicated in Fig. 1 (inset). The load-deflection plot is compared with the experimental and finite element based results of Sakata et al. (1985). The comparison is very good and it indicates the accuracy of our finite element analysis. Fig. 2 shows development of plastic zone at different stages. It is seen that the plastic zone developed is symmetrical about the line which is lying at an angle with the crack line. J against COD is plotted in Fig. 3 for four different contours. Initially there is not much difference in the J-δ plots but as the plastic zone develops J becomes slightly path dependent. Crack edge profile at different stages of loading are presented in Fig.4. The crack edge profile is asymmetric and upper surface moves more than the lower surface, which is in agreement with the observations of Sakata et al. (1985). Figs. 5, 6 and 7 gives the variations of tangential stress σ_θ, total equivalent stress, tangential strain ϵ_θ and shear stress $\tau_{r\theta}$ about the crack-tip at the three different stages of loading, 1st,60th and 100th. These plots show that the point of occurrence of the maximum or minimum of a parameter is constant for all the stages. Again the angle Φ made by the line of symmetry of the plastic zone is very nearly the same as α,the angle of crack initiation, as per the criteria of MTS, MTPS and MTSN. Therefore α can probably be predicted from a plot of the plastic zone, or a plot of σ_θ (or ϵ_θ, or $\tau_{r\theta}$) against θ at any step of loading. Even α can be predicted from the first load step, which is fully elastic. Unfortunately, no experimental result is available on θ_c, because of which it has not been possible to check the accuracy of our prediction.

CONCLUSION

(1) Direction of initial crack extension θ_c for an elastic-plastic mixed-mode situation can be predicted from an elastic analysis.

(2) The criteria used in linear elastic fracture mechanics for prediction of θ_c, e.g. the criteria of MTS, MTSN and MTPS, gives almost the same value of θ_c in case of a mixed-mode elastic-plastic problem considered here.

(3) Under mixed-mode loading plastic zone develops at the crack-tip symmetrically about a line whose inclination with the crack is found to be equal to the crack initiation angle.

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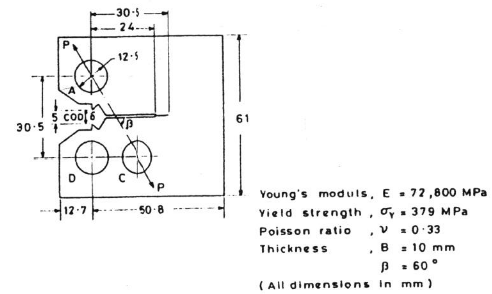
REFERENCES

Ahmad, J., C.R. Barnes and M.F. Kanninen (1983). An elastoplastic finite-element investigation of crack initiation under mixed-mode static and dynamic loading. ASTM-STP 803, Vol-I, I-214—I-239.

Cotterell, B., E. Lee and Y.W. Mai (1982). Mixed mode plane stress ductile fracture. Int. J. Fract., 20, 243-250.
Erdogan, F. and G.C. Sih (1963). On the crack extension in plates under plane loading and transverse shear. Trans. ASME J. Basic Engng., 85, 519-527.
Kishimoto, K., S. Aoki and M. Sakata (1980). On the path independent integral-J. Eng. Fract. Mech., 13, 841-850.
Maiti, S.K. and R.A. Smith (1983). Theoretical and experimental studies on the extension of cracks subjected to concentrated loading near their faces to compare the criteria for mixed mode brittle fracture. J. Mech. Phys. Solids, 31, 389-403.
Otsuka, A., T. Miyata, S. Nishimura, K. Kashiwagi and N. Kasai (1957). J. Soc. Naval Architects Japan, 136, 24-57 (in Japanese).
Sakata, M., S. Aoki and K. Kishimoto (1985). Mixed-mode elastic-plastic fracture of 2024-T351 aluminium alloy. In V.S. Goel (Ed.), The Mechanism of Fracture, Proc. of Int. Conf. and Exposition on Fatigue, Corrosion Cracking, Fracture Mechanics and Failure Analysis, Utah, U.S.A.
Sih, G.C. (1973). Introductory chapter: A special theory of crack propagation, In G.C. Sih (Ed.) Methods of Analysis and Solutions to Crack Problems, Mechanics of Fracture, Vol. I., Noordhoff International Publishing, Leyden, XXI-XIV.
Wu, H.C. (1974). Dual failure criterion for plane concrete. J. Engng. Mech. Div. ASCE, 100, 1167-1181.
Yamada, Y., N. Yoshimura and T. Sakuri (1968). Plastic stress-strain matrix and its application for the solution of elastic-plastic problems by the finite element method, Int. J. Mech. Sci., 10, 343-354.



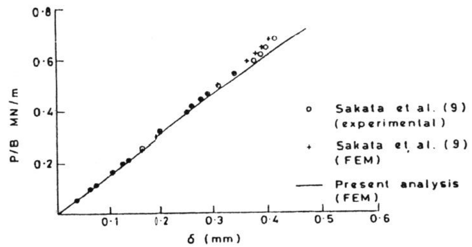


Figure 1 Comparison of P/B vs δ from computed and experimental results.

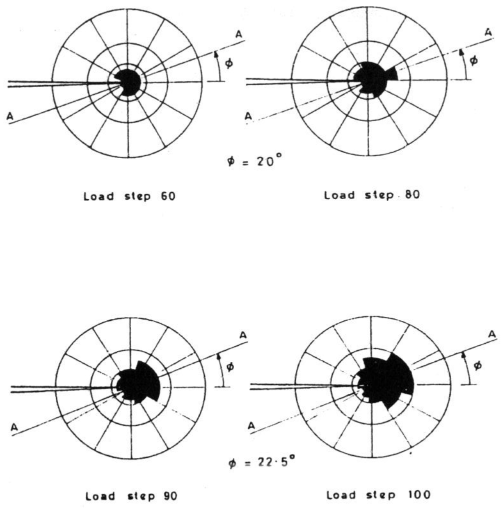


Figure 2 Development of plastic zone at different stages of loading

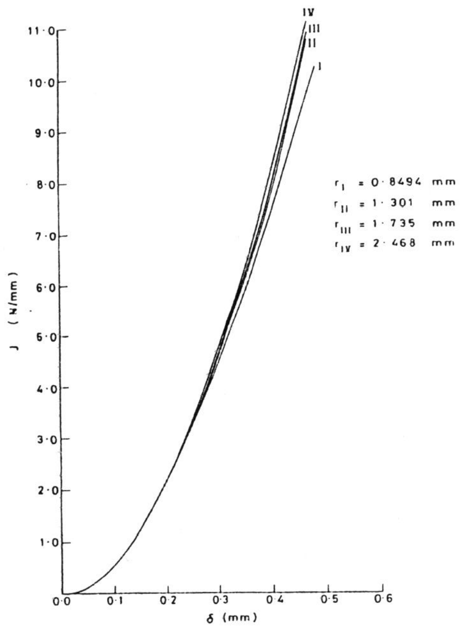


Figure 3 J vs. δ for four different circular contours.

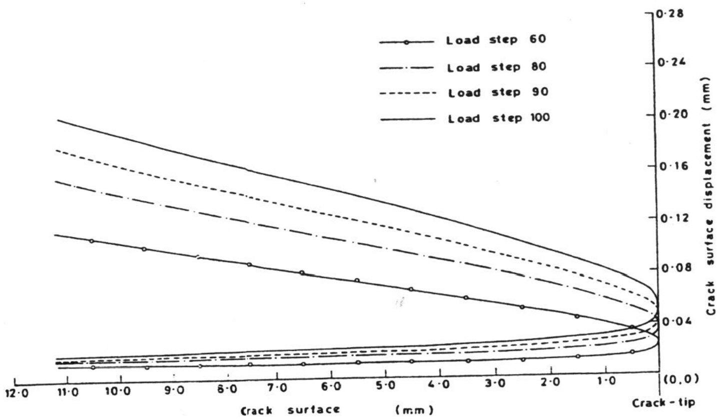


Figure 4 Crack edge profile at different stages of loading

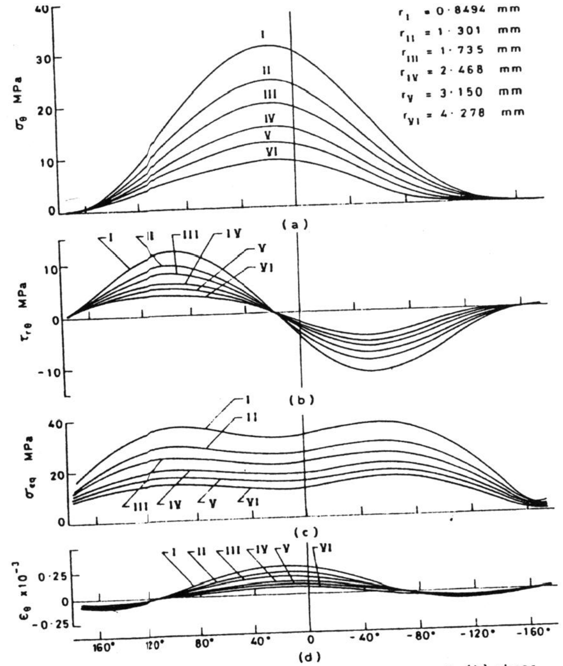


Figure 5 Variations of (a) tangential stress σ_θ vs θ, (b) shear stress $\tau_{r\theta}$ vs θ, (c) total equivalent stress $\sigma_{eq.}$ vs θ and (d) tangential strain ϵ_θ vs θ on circles of six different radii around the crack tip in the first load step.

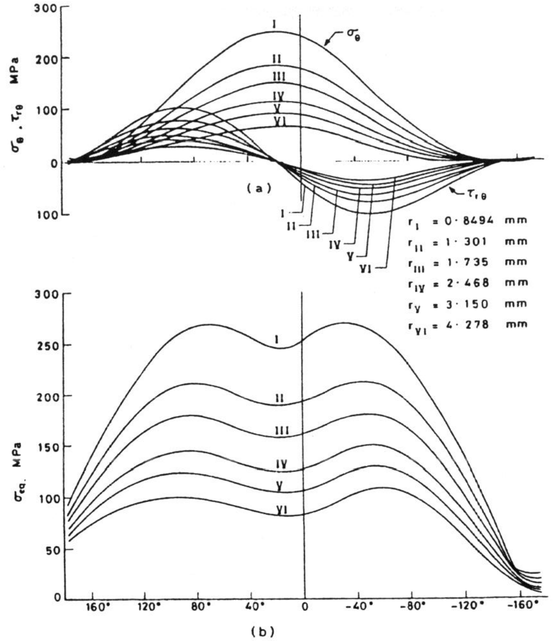


Figure 6 Variation of (a) tangential stress σ_θ and shear stress $\tau_{r\theta}$ vs θ, (b) total equivalent stress $\sigma_{eq.}$ vs θ on circle of six different radii around the crack tip in the 60 th load step.

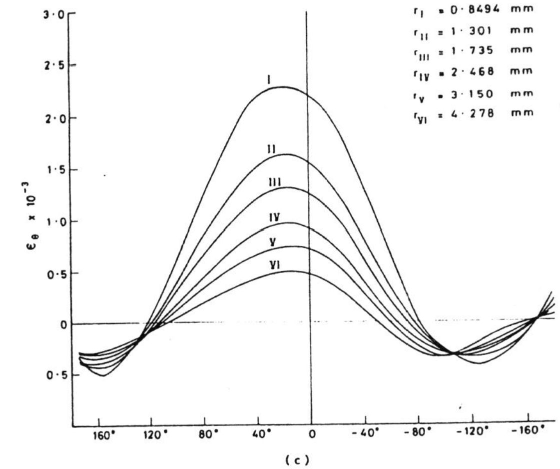


Figure 6 (Contd.) Variation of (c) tangential strain ϵ_θ vs θ on circle of six different radii around crack tip in the 60th load step.

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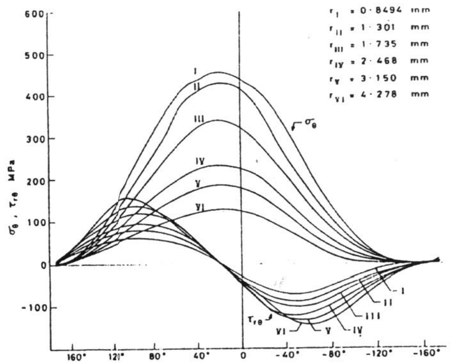
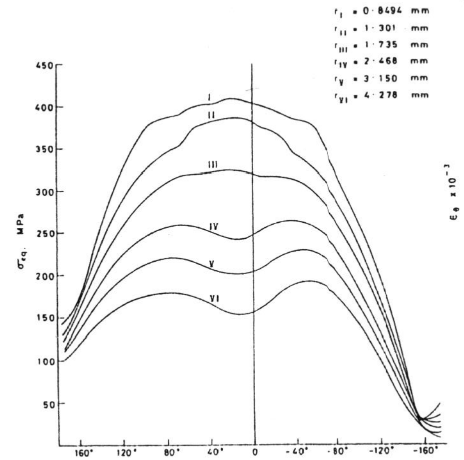


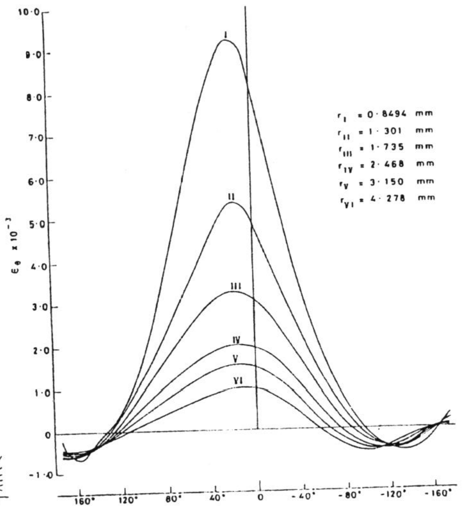
(a)

Figure 7 Variation of (a) tangential stress σ_θ and shear stress $\tau_{r\theta}$ vs. θ on circle of six different radii around the crack tip in the 100 th load step.



(b)

Figure 7 (Contd.) Variation of (b) total equivalent stress vs. θ on circle of six different radii around the crack tip in 100 th load step.



(c)

Figure 7 (Contd) Variation of (c) tangential strain ϵ_θ vs θ on circle of six different radii around the crack tip in the 100 th load step.