Some Guidelines on the Engineering Applications of Elastic–Plastic Fracture Mechanics

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ABSTRACT

Elastic-plastic J-integral fracture mechanics analyses are performed rather routinely. It is first shown that the accuracy of the results depends significantly on the accuracy with which the material true-stress true-strain curve is fit. Using exactly the same stress strain curve approximation, two independent finite element analyses of a circumferential flaw in a pipe are shown to yield similar results. Plastic zone size corrections to linear elastic methods are reasonably accurate up to near the engineering yield stress but become unacceptably non-conservative above this value. J analyses using the handbook procedure developed under the sponsorship of EPRI are shown to produce accurate results as the elastic plastic regime is entered if the Ramberg-Osgood fit of the stress strain curve is good in that region. Further into the elastic-plastic region acceptable conservatism is shown if the fit of the stress-strain curve is reasonably good. A Ramberg-Osgood fit to the mid-strain range (2 to 5%) is shown to be unacceptably conservative. Some guideline for obtaining acceptable approximations of finite element analyses in the elastic plastic region are given.

KEYWORDS

Cracked pipes, elastic-plastic, finite element, fracture mechanics, J-integral, Ramberg-Osgood, stress-strain.

INTRODUCTION

Elastic-plastic fracture mechanics analyses are performed rather routinely to assess the fracture potential of structures. One of the most commonly used approaches involves the J-integral methodology. The analytical tool preferred is perhaps that of using finite element modelling. Such an approach however may be quite costly, and simplified less expensive methods are often applied instead.

Three fracture assessment methods - finite element, plastic-zone size correction and finite element interpolation - are addressed in this paper. Rather simple structures, large pipes with through-wall circumferential flaws subjected to axial bending moments, are considered. Analyses are presented and results are compared. Guidelines for analyses are suggested.

COMPARISON OF FINITE ELEMENT RESULTS

It is generally assumed that finite elements analyses yield the most accurate state-of-the-art solution for elastic-plastic fracture behavior and serve as benchmarks for evaluations of other procedures. As a first approach, finite element analyses were made of a 34 inch outside diameter (O.D.) pipe, 2.5 inches thick, using two widely accepted computer codes, MARC (Marcal, 1969) and ADINA (Bathe, 1978) (modified for J-integral calculations). In the analysis using MARC the path integral was used to obtain J for a given crack size and load whereas in the modified ADINA analysis the virtual crack extension method (Yang and Palusamy, 1983) was used. The pipe material was stainless steel. A generic lower bound true-stress true-strain curve (henceforth referred to as stress strain curve) typical of this material at 600°F was applied.

Two representations of the stress stain curve were taken as shown in Fig. 1. One representation was a five-point approximation of the curve and the other was a bilinear representation as seen. MARC and ADINA as modified were run using the five point approximation while only ADINA as modified was run using the bilinear representation. The results are shown in Fig. 2 as a function of the applied moment. The crack length subtended an angle of 27°.

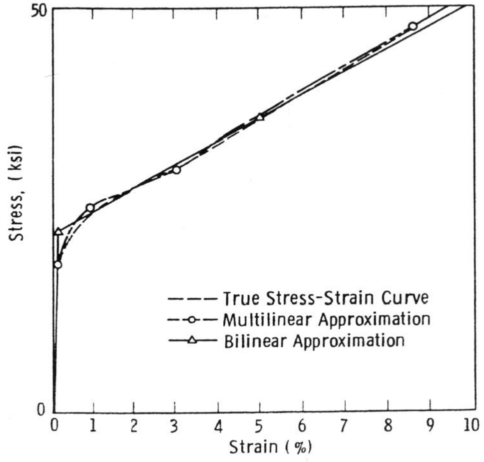

Fig. 1. Stress Strain Curve and Its Bilinear and Multilinear Approximations Used in Analyzing the 34 inch O.D. Pipe.

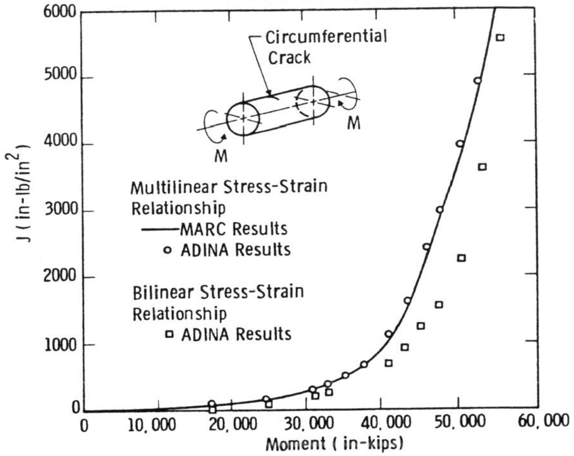

Fig. 2. Comparison of the Finite Element J Results for the 34 Inch O.D. Pipe Using MARC and ADINA.

There are two significant observations from Fig. 2. First, using the same stress strain curve almost identical results were obtained using two different finite element approaches. Secondly, the results using the bilinear representation compare very unfavorably in a non-conservative manner with the other results in the range of J from 500 in-lb/in^2 to 5000 in-lb/in^2 which spans the usual range of interest for stainless steel.

PLASTIC ZONE SIZE CORRECTION METHOD

One of the earlier approaches of addressing elastic-plastic behavior was to incorporate a plastic zone size correction into a linear elastic solution. For circumferential flaw in pipes one such linear elastic solution for applied bending moments is

$$K = \sigma_b \sqrt{\pi a}\, F(\alpha) \qquad (1)$$

where $F(\alpha)$ is the stress intensity calibration factor for bending, a is the half angle length, α is the half-crack angle, and σ_b is the remote fiber stress due to pure bending (Tada, 1983). Values for $F(\alpha)$ are given by Tada (1983). The effect of yielding near the crack tip can be addressed by incorporating the plastic zone size correction developed by Irwin (1960), into equation (1) by replacing a by the effective half-crack length, a_{eff}, defined by

$$a_{eff} = a + K^2/2\pi\sigma_y^{\,2} \qquad (2)$$

where the term added to a is the plastic zone size correction and σ_y is the engineering yield stress. Various other formulation of the plastic zone size for specific applications abound but all have the same character. The corrected K is found by using equation (1) to obtain the linear elastic K, correcting the half-crack length by equation (2) and then using equation (1) again to obtain the final K. J is found by the relation

$$J = K^2/E \tag{3}$$

where E is Young's modulus.

This method was used to analyze the problem solved by the finite element method in the previous section. A comparison of solutions is given in Fig. 3. It is noted that the comparison is very favorable up to near where the bending stress is equal to the yield stress. At this point the plastic zone size is slightly over one-half the half crack length. The plastic zone size is usually required to be small, theoretically, in applications; however, from a calculational standpoint, accuracy is seen to be maintained for quite a large size.

Results similar to those in Fig. 3 have been found for tensile loads and tensile and bending loads combined. The specific definition of the plastic zone size can impact the results somewhat but in general accuracy and conservatism, one gross yielding occurs, are lost. Calculationally, acceptable accuracy up to yield for the problem type under consideration is obtained. Restrictions on plastic zone size, such as for part through flaws, could further reduce the applicability of this method.

A FINITE ELEMENT INTERPOLATION METHOD

As noted previously, finite element elastic-plastic J-integral solutions are considered state-of-the-art and as such tend to serve as benchmarks for validation of other approaches. A rather unique and popular approach is one developed under the auspices of the Electric Power Research Institute (EPRI) and is presented in handbook form by Kumar et al (1981). This method will be referred to as the handbook procedure.

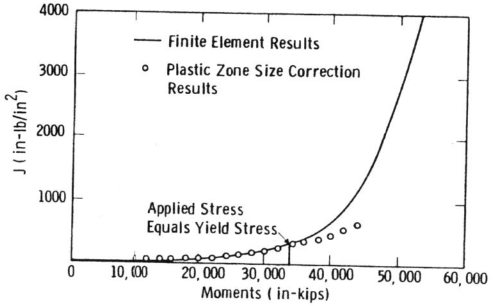

Fig. 3. Comparison of J Results Using Plastic Zone Size Corrections and the Finite Element Results for the 34 Inch O.D. Pipe.

The handbook procedure is in fact an interpolation scheme based on finite element results and would be expected to yield very accurate results if only interpolation were involved. However to provide a handbook methodology some assumption must be made about the stress strain curve. In the handbook procedure a Ramberg-Osgood representation (Ramberg and Osgood, 1943) of the stress-strain relationship is assumed of the form

$$\frac{\varepsilon}{\varepsilon_o} = \sigma/\sigma_o + \alpha\,(\sigma/\sigma_o)^N \tag{4}$$

where σ and ε denote stress and strain, respectively. α and N are the Ramberg-Osgood coefficients. Subscript o indicates the stress and strain at yield. Rigorously speaking, the yield stress (i.e., the 0.2 percent offset yield) has little relevance in finite-element elastic-plastic analyses. The yield stress for purposes of this discussion is taken as the proportional limit defined as the point of deviation from linearity of the stress strain curve.

Equation (4) represents the stress-strain behavior of the material of interest here (i.e., stainless steel) only to an approximation. As seen later, the method of approximating the stress-strain behavior can have an impact on J-integral results every bit as significant as changing flaw size, loading or geometry. Frequently, the key to performing an accurate or reasonably accurate analysis is the use of the proper α and N. This is especially true when the nominal stress is in the elastic-plastic regime.

Thus the problem is not with the accuracy of the handbook procedure itself but with the Ramberg-Osgood coefficients. The applicability of the Ramberg-Osgood relationship to the stress-strain behavior of stainless steel has been evaluated by Landes and McCabe (1986) and it is concluded that a reasonably good fit cannot be obtained over the full strain range. This is especially true if the restriction on N discussed later is enforced.

As a rationale for approaching the problem of determining the Ramberg-Osgood coefficients, there are certain facts which are helpful. First and foremost, it is generally known that when the nominal stress produces plastic flow the J value increases very dramatically. Thus an a priori requirement to obtaining fairly accurate results as the elastic-plastic regime is entered is to adequately fit the stress-strain relationship of interest as strain hardening occurs. It would seem reasonable, that to maintain accuracy as more plasticity occurs, the stress-strain curve of interest must continue to be reasonably described by the Ramberg-Osgood fit. It is also reasonable to judge that if the stress-strain relationship is not reasonably well fit as early plasticity occurs then the J-integral results may not be accurate in that stress region and will be questionable at higher plastic regions until such a high strain is obtained for which earlier effects are minimized.

The impact of the stress strain curve on the J results are readily seen in Fig. 2. Thus the approximation of the stress strain curve is of great concern even in finite element analyses.

Once having an acceptable stress strain curve available for an analysis, the handbook procedure requires an additional step - that of determining adequate values of the Ramberg-Osgood coefficients. In general, the handbook procedure (Kumar et al., 1981) limits the value of N in equation (4) to 7 or less for the configuration under consideration.

To illustrate the method for representing the stress strain curve for applying the handbook procedure, a 16 inch O.D. pipe, 1.6 inches thick was chosen from several examples available. The material is stainless steel and the representative lower bound stress strain curve is given in Fig. 4. Finite element analyses were run using ADINA as modified for two circumferential flaw sizes, one having an angle of 60°, the other, 26°. The finite element analyses used a ten point approximation of the stress-strain curve which well defined the early stages of strain hardening as well as the latter stages. A point consisting of a stress and the corresponding strain will be called a <u>couple</u>. The proportional limit was one of the ten input couples. The ten couples are plotted in Fig. 5 and compared with the stress-strain curve of Fig. 4. The stress strain curve was also fit with the Ramberg-Osgood equation using the same nine strain-hardening couples as used in the finite element analysis and four additional couples. The proportional limit was taken as σ_o in equation (4). The couples were so selected as to obtain a value of N less than 7. It is to be noted that in Fig. 5 the stress level of such a fit couple is plotted along with the strain obtained by the fit. The Ramberg-Osgood curve is also given in Fig. 5. The Ramberg-Osgood curve starts out slightly below the stress-strain curve, crosses between 19000 and 19500 psi, ranges somewhat above it to near 32000 psi then drops significantly below it as the stress increases. It is worth noting that at 40,000 psi the Ramberg-Osgood strain exceeds that of the stress strain curve by a factor of about 2. N was near 5.5 and α was near 0.6.

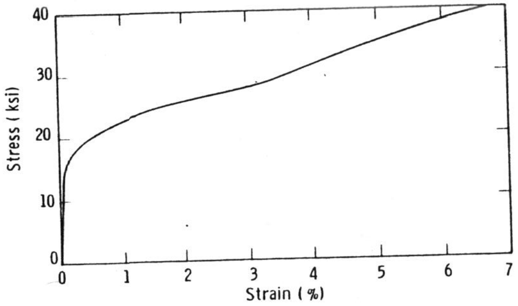

Fig. 4. Stress Strain Curve Used in Analyzing the 16 Inch O.D. Pipe.

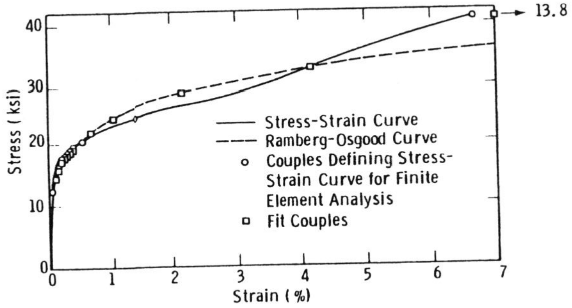

Fig. 5. Finite Element Approximation and Ramberg-Osgood Approximation of the Stress Strain Curve Used in Analyzing the 16 Inch OD Pipe

Interestingly, if a set of data describing a stress strain curve is fit by equation (4), N is independent of the selected yield stress. However, if σ_o and σ_1 are two different selected values of yield stress, the corresponding α's are related by

$$\alpha_1 = \alpha_o(\sigma_1/\sigma_o)^{N-1} \tag{5}$$

While generally, the proportional limit is recommended in this paper for use as the yield stress in fitting a stress strain curve, using the usually more available engineering yield stress does not impact the J result too adversely. For the handbook analyses considered in this paper using the engineering yield stress (about 60 percent greater than the proportional limit) as σ_o produced J values around 10 percent lower than that obtained using the proportional limit as σ_o in the stress range between the proportional limit and the engineering yield stress. As the applied stress increases the difference becomes considerably less approaching 1 percent for J values of 5000 in-lb/in^2 using a modest flaw size and a stress approaching 150 percent of the engineering yield stress.

Both finite element and handbook J results were obtained for axial moment loadings using the appropriate fit to the stress strain curve as shown in Fig. 5. The results are compared for the larger flaw size in Fig. 6 and for

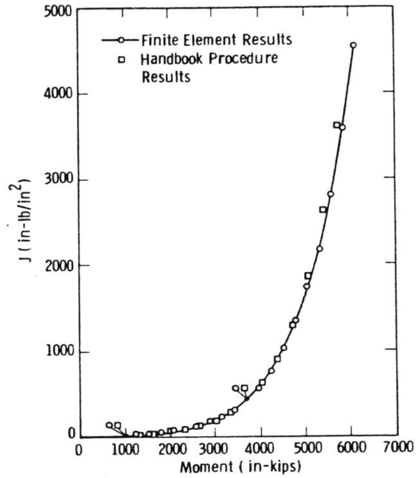

Fig. 6. Comparison of Handbook Procedure J Results with Those of the Finite Element Analysis for the 60 Degree Circumferential Flaw in the 16 Inch O.D. Pipe

the smaller flaw size in Fig. 7. Excellent agreement is noted in Fig. 6 up to a moment of about 5000 in-kips which corresponds to a stress of around 21 ksi. It is noted in Fig. 5, that the Ramberg-Osgood curve agrees well with the stress strain curve up to this level. The point of deviation of the results in Fig. 6 is somewhat surprising since the Ramberg-Osgood fit intersects and falls above the actual stress strain curve near the 21 ksi level. This does not detract from the excellent to conservative comparison, however, and indicates the somewhat compensating effect of underestimating and overestimating, within limits, the stress strain curve of interest.

The comparison of results for the smaller flaw is excellent over the whole loading regime as noted in Fig. 7. In particular the small flaw analysis appears to give a somewhat better comparison than the large flaw analysis.

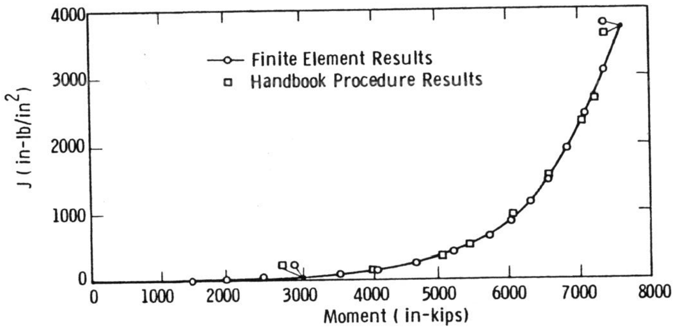

Fig. 7. Comparison of Handbook Procedure J Results with Those of the Finite Element Analysis for the 26 Degree Circumferential Flaw in the 16 Inch O.D. Pipe

An analysis of the 16 in O.D. cylinder using the handbook procedure was made based on a Ramberg-Osgood fit of the stress strain curve of Fig. 4 in the mid strain range (2 to 5 percent). The fit compared with the stress strain curve is given in Fig. 8. The fit curve falls well below the stress strain curve of interest in the early strain hardening regime, noting for instance that for a stress of 20 ksi, the Ramberg-Osgood strain is over a factor of two greater than that of the stress strain curve. Results from the handbook procedure for the larger flaw are compared with the corresponding finite element results in Fig. 9. The handbook procedure results would ordinarily be considered as far to conservative for engineering applications. This is true even for a loading stress level in the 2 to 5 percent strain range since the stress level for the finite element results presented goes up to around 26 ksi (i.e. slightly over 2 percent strain).
The behavior presented in this section is typical of that for other loading and flaw configurations.

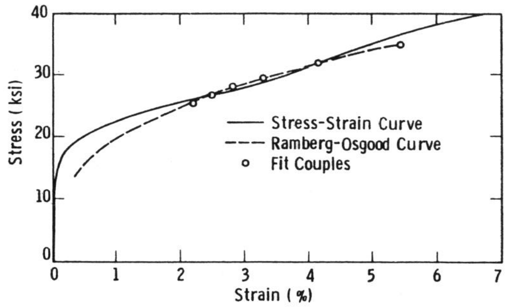

Fig. 8. Comparison of the Mid-Range Strain (2 to 5%) Fit Ramberg-Osgood Curve with the Stress Strain Curve Used in the Finite Element Analysis of the 16 Inch O.D. Pipe

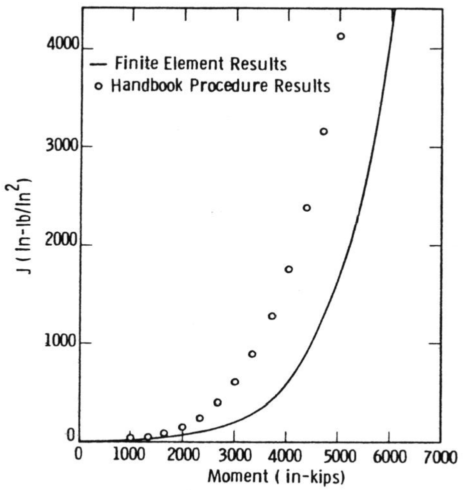

Fig. 9. Comparison of J Results Using the Handbook Procedure having a Mid-Range Strain Fit Ramberg-Osgood Curve With Those of the Finite Element Analysis for the 60° Circumferential Flaw in the 16 Inch O.D. Pipe

DISCUSSION AND CONCLUSIONS

Finite element analyses of through-wall circumferential flaws in a pipe are benchmarked one against the other. Two different approaches were used and excellent agreement was found using the same stress-strain representation. The use of a different stress-strain representation yielded significantly different results which emphasizes the importance of the elastic-plastic stress strain relationship in analyses where gross section yielding occurs.

An evaluation of plastic-zone corrections to account for crack tip plasticity was shown to be reasonably acceptable for loadings up to near gross section yielding even though large plastic zone sizes are calculated. Restrictions on plastic zone sizes, based on theoretical considerations, could reduce the stress range of applicability appreciably.

Various analyses using the handbook approach are compared with finite element elastic-plastic fracture mechanics results. The major concern is the proper fitting of the stress strain curve of interest with the Ramberg-Osgood expression for stress and strain. To obtain good comparisons with the finite element results in the region of gross section yielding, a good fit of the Ramberg-Osgood equation in this region is required using σ_o of equation 4 as the proportional limit. Using σ_o as the engineering yield stress and the same data produced $\mathcal{J}$ values about 10% low in this region. For higher loads the percentage decreased significantly. Within the restrictions of the handbook procedure the complete stress strain diagram cannot be adequately fit for the material of interest in this paper (i.e., stainless steel). Very good comparisons of handbook procedure results with finite element results were found up to reasonably high loads when the stress strain curve was well fit by the Ramberg-Osgood expression as gross yielding occurred and reasonably fit to higher strains. With these restrictions, the handbook results were shown either to agree very well with the finite element results or to err on the side of conservatism.

Fitting the Ramberg-Osgood expression to the mid-strain range (2 percent to 5 percent strain) was seen to be potentially unacceptably conservative even at high load levels. Although some success has been obtained at fitting Ramberg-Osgood expressions to the stress strain curve for higher load situations (not discussed in this paper), it is judged that for those situations, where undue conservatism cannot be tolerated, finite element analyses (or equally accurate methods) may be the only recourse.

The results presented in this paper are representative of a larger analysis base and the conclusions and observations are based on the results presented, strengthened by additional exemplary and definitive results. The material of interest is stainless steel however.

Ductile tearing has not been addressed in this evaluation. If significant tearing is judged to occur then such tearing must be taken into account. A simple procedure for doing this if the J-R curve is known is discussed by Witt (1987).

SOME GUIDELINES

Based on the selected representative results presented in this paper the following guidelines for analyses of stainless steel components are judged to be warranted:

Elastic plastic finite element fracture mechanics analyses should depict accurately the stress strain curve of the material of interest.

Plastic zone size corrections to linear elastic analyses are reasonably accurate up to near the 0.2% offset yield stress, calculationally, but limitation on the zone size could further restrict applications.

Often the Ramberg-Osgood expression cannot be made to fit accurately the material stress strain curve with the restriction on N (e.g., $N \leq 7$ for the applications made in this paper) further mitigating the fit.

For the handbook procedure to accurately duplicate finite element results, the stress strain curve should accurately be fit by the Ramberg-Osgood expression as yielding occurs and preferably up to the stress of interest.

For many applications the handbook procedure produces accurate or reasonably conservative results if both a good Ramberg-Osgood fit is obtained in the initial yielding region and a reasonably good fit to higher stress levels representative of the loading condition is obtained.

Fitting the Ramberg-Osgood equation to the mid strain range (2 to 5%) is expected to produce results which may be restrictively conservative.

Finite element analyses (on other equally accurate methods) appear to be the only recourse if accurate results are required for very high stress levels.

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