Numerical Analysis of the Initiation of Ductile Fractures

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ABSTRACT

Experimental data for the initiation of ductile fractures afe numerically analysed. The relationship between a new macroform parameter V_{gc} and microform ratio R_c / R_o of critical void growth rate is investigated. It is shown that V_{gc} can be conveniently used for the selection of materials.

KEYWORDS

Initiation of ductile fracture, Critical void growth rate, Macroform and microform parameters.

INTRODUCTION

The importance of crack initiation in ductile fracture has been recognised in many engineering processes. Thus, for example, the avoidance of cracking is essential for successful manufacturing operations, such as forging or extruding. Similarly, an easy crack initiation is particularly desirable in the cutting or machining of a variety of materials. It is therefore not surprising that the process of crack nucleation, together with subsequent propagation after large plastic flow, has been investigated in detail.

Ductile fracture is the result of an extensive amount of deformation. The microprocessors involved are generally considered to occur in three specific stages, namely the nucleation, the growth and the final coalescence of microvoids.

Systematic observations and measurements of the void development were the subject of earlier work (Zheng and Radon, 1983a,b) and the crack initiation process was investigated on a low alloy steel BS 4360-50D (Zheng and Radon, 1984). It was noted that the voids nucleated around the inclusions or second phase particles. At the same time, the crack-tip flanks showed wide opening and the actual tip was blunting excessively. In the final stage of this process, the blunted crack linked with the neighbouring void by rupturing the ligament.

THEORETICAL BACKGROUND

While the material is subjected to a large deformation, the effective stress may not vary significantly, but the effective strain can change within wide limits. Therefore, it is reasonable to describe the ductile fracture in terms of plastic strain and to express the critical condition of the fracture as:

$$\varepsilon_p = \varepsilon_f \tag{1}$$

where ε_p is the effective plastic strain under applied loading, and ε_f is the critical effective plastic strain at failure of the material. Equation (1) is the basic form of the ductile fracture criterion.

In general, equation (1) does not express an independent single-parameter criterion, because the effective fracture strain of the material, ε_f, is related to the stress states.

In conditions of quasi-static and proportional loading, the following assumptions may be accepted:

(1) The relationship of the effective stress, $\bar{\sigma}$, and the effective strain, $\bar{\varepsilon}$, for a given material is unique under different stress conditions.

(2) The position corresponding to the fracture point on the $\bar{\sigma}$-$\bar{\varepsilon}$ curve depends only on the stress state during the loading process.

(3) The influence of the ratio J_3'/J_2' on the position of the fracture point could be ignored when compared with the ratio of J_1/J_2'. Here, J_1 is the first invariant of the stress tensor; J_2' and J_3' are, respectively, the second and third invariants of the stress deviator.

Therefore:

$$\varepsilon_f = F_1\left(\frac{J_1}{J_2'}\right) = F_2\left(\frac{\sigma_m}{\bar{\sigma}}\right) = F_2(R_\sigma) \tag{2}$$

where F_1 or F_2 is a function of J_1/J_2' or $\sigma_m/\bar{\sigma}$, respectively, which can be derived experimentally or theoretically. R_σ is the triaxiality of stress which equals the mean stress, σ_m, divided by the effective stress, $\bar{\sigma}$.

The criterion in the form of equation (1) may be rewritten as follows:

$$\varepsilon(R_\sigma) = \varepsilon_f(R_\sigma) \tag{3}$$

where $\varepsilon(R_\sigma)$ is the effective strain of the material under the applied loading and the value of the stress triaxiality, R_σ, while $\varepsilon_f(R_\sigma)$ is the effective fracture strain of the material under the same value of R_σ.

It is obvious that the form of criterion, as presented in equation (3) or equation (1), is not very convenient for practical use. As a fracture criterion, it is usually expected that it works effectively under different stress states or triaxiality.

Equation (2) can be rewritten in the form:

$$\varepsilon_f\, F(R_\sigma) = \text{const.} \tag{4}$$

where $F(R_\sigma) = 1/F_2(R_\sigma)$. The new material constant in equation (4) will be expressed by the symbol V_{gc}, namely

$$V_{gc} = \varepsilon_f \cdot F(R_\sigma) \tag{5}$$

Equation (5) serves not only as a reformulation of equation (2) but also introduces a new ductile fracture characteristic of the material, V_{gc}, the macroform parameter of the critical void growth rate, whose microscopic physical meaning is as follows:

The experimental results obtained from the tests on five different types of steel in Zheng and Radon (1983a) confirmed that V_{gc} is a material constant insensitive to stress triaxiality. In general, the lower the ductility of the material, the smaller the value of V_{gc}.

In practical situations, when the applied factor, V_{appl}, in equation (4):

$$V_{appl} = \varepsilon_p \cdot F(R_\sigma) \tag{6}$$

increases to the critical value of the material, V_{gc}, namely

$$V_{appl} = V_{gc} \tag{7}$$

the material will fracture. Thus, equation (7) represents a new criterion of ductile fracture, i.e. the criterion of critical void growth rate.

As discussed earlier in Zheng and Radon (1983a,1984) and Zheng, Zhou and Liu (in press), the onset of void coalescence in a material corresponds to the failure initiation and thus to a critical value of the void growth rate. Based upon the experimental statistical results and the extended Rice-Tracey model, the coalescence condition can be described using the macroscopic parameters of equation (7) as follows:

$$V_{appl} = V_g = \varepsilon_p \exp\left(\frac{1.5\,\sigma_m}{\bar{\sigma}}\right) = V_{gc} \tag{8}$$

where V_g depends on the instantaneous void growth rate, ε_p is the effective elastic strain, σ_m is the mean stress; $\bar{\sigma}$ is the von Mises effective stress, and $\sigma_m/\bar{\sigma}$ is the parameter of triaxial stress state under loading conditions.

The amount of deformation in the very small region in front of the crack tip, for the cracked body of the elastic-plastic material, is very much larger than in the neighbouring regions. Therefore, the process of the nucleation of the void and subsequent growth of voids is most active close to the crack tip. It is reasonable to deduce that, in this region, the void begins to link with the crack tip and the fracture initiates as soon as the V_{gc} criterion, equation (8), is fulfilled.

The large voids, described in Zheng and Radon (1984) as primary, nucleated around inclusions of 10 μ diameter. Their coalescence with other large voids and their linking with the crack tip had been observed on a similar grade of low alloy steel. This internal necking of the ligaments between cavities is a well known process, documented on all the steels investigated in the present programme. However, it should be pointed out that the ligaments of the tested steels did not show any shear bands at lower strains (ε_p = 0.22), nor at very high strains of the order of

$\varepsilon_p \approx 0.95$ - 0.97. At these high strains, a large number of a different type of void, described as 'secondary voids', formed around submicron particles. Localised shear bands are not considered in this present work.

MATERIALS AND SPECIMENS

The main chemical composition of steel DE36 is shown in Table 1. The mechanical properties of this steel at room temperature are shown in Table 2. The dimensions of a three-point bend specimen were 20 × 24 × 120 mm. The notch was 0.08 mm wide and 10 mm deep.

The load-displacement (P-V) curve of cylindrical tensile specimens and the stress-strain (σ-ε) curve, modified by Bridgman's formula, are shown in Figures 1 and 2, respectively.

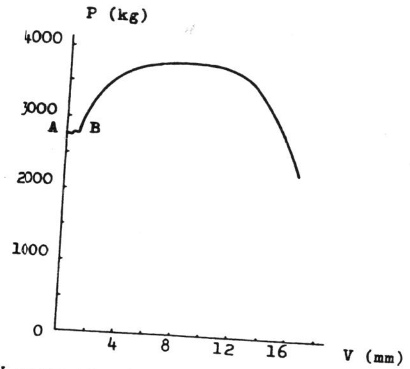

Fig. 1 The P-V curve of cylindrical tensile specimen

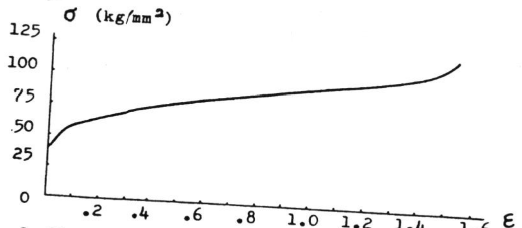

Fig. 2 The σ - ε curve

NUMERICAL COMPUTATION

The large elastic-plastic deformation finite element method adopted in this paper was carried out incrementally using the updated Lagrangian formulation (McMeeking and Rice, 1975). In order to improve the computational precision, to avoid drift and to reduce the computer time, the following methods and techniques were employed: (1) the equilibrium correction (Owen and Hinton, 1980); (2) the improved Euler method (Bajpai, Mustoe and Walker, 1978); (3) the combined initial/tangential stiffness method (Owen and Hinton, 1980); (4) the improved frontal method in which the stiffness matrix was partly updated. Note that in a certain deformation range, due to the variations of the most elastic elements, stiffness factors were very small (Lotsberg, 1978).

The computational model represents one half of the specimen by reference to the symmetry. The computational mesh is shown in Figure 3; it consists of 132 elements and 451 nodes. The loading was carried out by applying displacement increments in the range of 0.002 to 0.015 mm at the loading point. The assumption of plane strain was accepted for the computation.

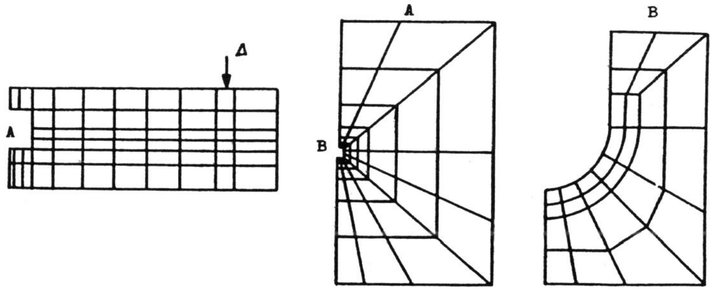

Fig.3. The computation mesh

Because there was an obvious yielding stage in the P-V curve (Figure 1), the whole σ-ε curve adopted in the computation was divided into three ranges: the elastic stage (O-A), the yielding stage (A-B), determined by Hermite's interpolating curve, and the hardening, modified by Bridgman's formula after the necking observed in tensile tests of cylindrical specimens.

RESULTS: THE INSTANTANEOUS VALUE OF THE PARAMETER V_g AND ITS APPLICATION FOR PREDICTING FRACTURE INITIATION

Computational results show that V_g in front of the crack tip is monotonically increasing with the increase in loading. From experimental measurements (Hua, 1986), the V_{gc} value of steel DE36 was equal to 3.4. According to the V_{gc} criterion, it was predicted that the fracture initiation in the specimen would occur at Δ = 2.2 mm. This was compared with the following experimental results:

(1) Using a series of interrupted TPB specimens with an incremental displacement of the loading point, Δ, the crack began to initiate at Δ = 2.2 mm. The real increment of the crack, a, measured in the middle thickness of the specimen was Δa = 0.05 mm.

(2) Using the standard microhardness method on a series of interrupted tensile tests, the relationship of the effective plastic strain, ε_p, versus the microhardness HV was obtained. The effective plastic strain, ε_p, at the crack tip was 1.4 and the strain distribution in front of the crack tip is shown in Figure 4. For materials where crack tip blunting occurs at a very early stage of straining, the value of the parameter $\sigma_m/\bar{\sigma}$ will be close to 0.577. It may be noted that, for plane strain conditions and Poisson's ratio of ν = 0.5, the $\sigma_m/\bar{\sigma}$ value at the blunt crack tip is $1/\sqrt{3}$. Using again equation (8), the parameter V_g will reach the value of 3.3 for the point of fracture initiation. Therefore, it may be concluded that the computational results applied in the V_{gc} criterion for the fracture initiation prediction are in good agreement with the experimental values.

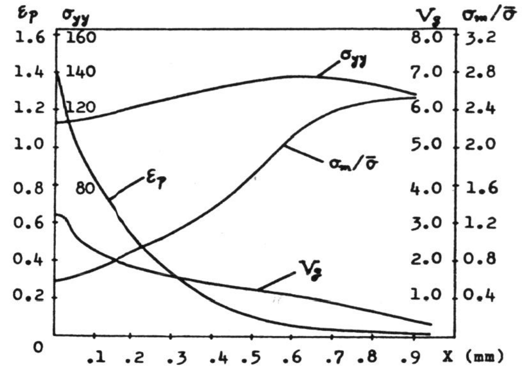

Fig. 4. The distribution of ε_p, $\sigma_m/\bar{\sigma}$, σ_{yy} and Vg in front of the crack tip before fracture initiation occurs

Table 1. Chemical composition (wt%)

C	Si	Mn	P	S	Cu	Al	Nb
0.16	0.37	1.60	0.014	0.004	0.04	0.05	0.04

Table 2. Mechanical properties

E (GPa)	ν	σ_y (MPa)	σ_u (MPa)	ψ (%)	δ_s (%)	K (MPa)	n
208	0.362	382	534	40.1	77.3	969	0.22

CONCLUSION

Fracture initiation in a cracked body can be predicted by computer simulation using the elastic-plastic large deformation finite element method combined with the V_{gc} criterion.

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