Near-tip Fields of Plane-strain Crack Growing in Compressible Elastic Perfectly Plastic Material

K. C. HWANG and X. F. LUO
Department of Engineering Mechanics, Tsinghua University, Beijing, PRC

ABSTRACT

For plane-strain crack growing in compressible elastic perfectly plastic material, the near-tip field has 5-sector structure. There were intense discussions about the difference of solutions. In this paper a correct formulation is given and the solutions for stresses, flow factor and plastic strains are given for all the near-tip fields.

KEYWORDS

Crack growth; compressible material; near-tip fields; elastic perfectly plastic medium; plane strain crack.

INTRODUCTION

The solution with 4-sector structure given independently by Slepyan, Gao and Rice for near-tip fields of plane-strain crack growing steadily in incompressible ($\nu=\frac{1}{2}$) elastic perfectly plastic material is now widely accepted. But there were discussions if compressibility of material is considered ($\nu<\frac{1}{2}$). Rice et al.(1980) extended the 4-sector solution to the case of compressible material. Gao(1981) pointed out that the yield condition is violated in the unloading sector of the 4-sector solution, and gave a 5-sector solution(Fig.1) with strain singularity ahead of the tip. Drugan et al.(1982) obtained another 5-sector solution without strain singularity ahead of the tip. In recent works, the present authors noticed that in the solution given by Drugan et al.(1982) the flow factor λ occurs to be negative and hence the plastic flow law is violated in part of the "non-singular plastic sector" C (after the terminology by Rice(1982)). With the formulation in terms of Airy's stress function, the present paper gives the correct 5-sector solution. The limiting process of the degeneration of the present solution to the solution for incompressible material as $\nu\to\frac{1}{2}$ is studied by Hwang and Luo(1988). The results obtained coincide with those obtained by Luo and Hwang(1988) within the framework developed by Rice(1982) and refined in the spirit of this paper.

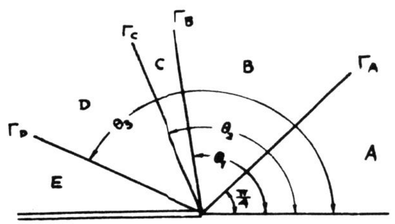

Fig.1. The 5-sector construction of near-tip field

BASIC EQUATIONS (Gao and Hwang(1981a))

Denote by r, θ the polar coordinates centered at crack tip. Take the leading term of Airy's stress function $\phi(r,\theta,\nu)$ in the form

$$\phi(r,\theta,\nu) = r^2 F(\theta,\nu)$$

where ν is Poisson's ratio. Then we have

$$\sigma_r = F'' + 2F \qquad \sigma_\theta = 2F \qquad \sigma_{r\theta} = -F'$$

where a prime is used to denote derivative with respect θ. Yield condition can be written as

$$\frac{1}{4}F''^2 + F'^2 + \frac{1}{3}\epsilon^2\varsigma^2 = \tau_0^2 \tag{1}$$

where

$$\epsilon = \frac{1}{2} - \nu \qquad \varsigma = -1.5 S_{33}/\epsilon$$

Constitutive law for perfect plasticity is as follows

$$\dot{\varepsilon}_{ij} = \frac{1+\nu}{E}\dot{\sigma}_{ij} - \frac{\nu}{E}\delta_{ij}\dot{\sigma}_{kk} + \lambda S_{ij} \qquad (i,j,k = 1,2,3) \tag{2}$$

For quasi-static steady crack growth, we can assume speed of crack growth $\dot{a}$=1, and then have for any quantity ( )

$$(\dot{\ }) = -(\cos\theta\frac{\partial}{\partial r} - \sin\theta\frac{1}{r}\frac{\partial}{\partial\theta})(\ ) \tag{3}$$

Then (2) can be rewritten in the asymptotic form (r→ 0)

$$\dot{\varepsilon}_{ij} = \frac{1}{r}\sin\theta(\frac{1+\nu}{E}\sigma'_{ij} - \frac{\nu}{E}\delta_{ij}\sigma'_{kk}) + \lambda S_{ij} \qquad (i,j,k = 1,2,3) \tag{4}$$

In plane-strain condition $\dot{\varepsilon}_{33}$=0, and from (4) it follows that

$$\varsigma' + \frac{2}{3}\frac{1}{\sin\theta}Er\lambda\varsigma = F''' + 4F' \tag{5}$$

The compatibility and rate of compatibility equations are, respectively,

$$\frac{1-\nu^2}{E}\Delta\Delta\phi + \frac{\partial^2\varepsilon^p_{11}}{\partial x_2^2} + \frac{\partial^2\varepsilon^p_{22}}{\partial x_1^2} - 2\frac{\partial^2\varepsilon^p_{12}}{\partial x_1\partial x_2} = 0 \tag{6}$$

$$\frac{1-\nu^2}{E}\Delta\Delta\frac{\partial\phi}{\partial x_1} - \frac{1}{2}(\nabla_1\nabla^1 - \nabla_2\nabla^2)\phi\cdot(\nabla_1\nabla^1 - \nabla_2\nabla^2)\lambda - 2(\nabla_1\nabla^2\lambda)(\nabla_2\nabla^1\phi)$$
$$-\frac{1}{2}\lambda\Delta\Delta\phi - (\nabla_\alpha\lambda)(\nabla^\alpha\Delta\phi) - \frac{2}{3}\epsilon^2\Delta(\lambda\varsigma) = 0 \qquad (\alpha = 1,2) \tag{7}$$

The in-plane plastic strain components can be obtained by integration

$$\varepsilon^p_{11} = \frac{1}{2}\int_{x_1}^{x_A}\lambda(\sigma_{11} - \sigma_{22})dx_1 + \epsilon^2\frac{1}{E}(\sigma_{11} + \sigma_{22} - \varsigma)$$
$$\varepsilon^p_{22} = \frac{1}{2}\int_{x_1}^{x_A}\lambda(\sigma_{22} - \sigma_{11})dx_1 + \epsilon^2\frac{1}{E}(\sigma_{11} + \sigma_{22} - \varsigma)$$

$$\varepsilon^p_{12} = \int_{x_1}^{x_A}\lambda\sigma_{12}dx_1 \tag{8}$$

The bordering line Γ between the ahead plastic loading sector and the rear elastic unloading sector (see Γ_c in Fig.1) is called " the unloading boundary ". We restate the theorem for the unloading boundary for the case of elastic perfectly plastic medium under plane strain (Gao and Hwang, 1981a, 1983a) as follows:

Theorem: If on an unloading boundary Γ we have at the side of the plastic sector (denoted by Γ(p))

$$\epsilon\varsigma \neq 0 \qquad \text{or} \qquad \sigma_{rr} - \sigma_{\theta\theta} \neq 0 \qquad \text{at } \Gamma(p) \tag{9}$$

then

$$\lambda|_{\Gamma(p)} = 0 \tag{10}$$

It should be noted that the conclusion (10) is, indeed, nontrivial. Actually, for the case of incompressible material, (10) is not true at the unloading boundary Γ between the plastic centered fan sector and the elastic unloading sector, and we have a discontinuity jump of λ across Γ. The reason is that both equalities in (9) then hold. It is not intuitively evident why λ should be continuous across Γ for the case of compressible material. This explains why the condition (10) is liable to be disregarded. We refer the mathematical proof to Gao and Hwang(1981a,1983a).

Besides, symmetry about the crack line requires

$$F' = 0 \qquad \text{as} \qquad \theta = 0 \tag{11}$$

and traction-free conditions of crack surface require

$$F = F' = 0 \qquad \text{as} \qquad \theta = \pi \tag{12}$$

DISCONTINUITY CONDITIONS

Gao and Hwang(1981b,1983a,b) proved that the rate of compatibility equation (7) can be rewritten as

$$A_{1111}\frac{\partial^4\dot{\phi}}{\partial x_1^4} + A_{1112}\frac{\partial^4\dot{\phi}}{\partial x_1^3\partial x_2} + A_{1122}\frac{\partial^4\dot{\phi}}{\partial x_1^2\partial x_2^2} + \cdots = 0 \tag{13}$$

where

$$A_{1111} = \frac{16\epsilon^2}{9}\varsigma^2(1-\nu^2) + (\sigma_{22} - \sigma_{11} + \frac{4}{3}\epsilon^2\varsigma)^2$$

The characteristic equation of (13) is

$$A_{1111} = 0$$

which can be satisfied only when

$$\epsilon\varsigma = 0 \tag{14a}$$
$$\sigma_{11} = \sigma_{22} \tag{14b}$$

(14b) gives the orientation of the characteristic line. Gao and Hwang (1981b,1983a,b) also proved that only the shear strain may suffer discontinuity. Moreover, the line of discontinuity is certainly characteristic line, where both of (14) are satisfied.

The reasoning of Gao and Hwang leading to (14) is correct. Its direct consequence for compressible material ($\epsilon \neq 0$) is

$$\begin{cases} \varsigma = 0 & (15a) \\ \sigma_{11} = \sigma_{22} & (15b) \end{cases}$$

(15a,b) are the results obtained later by Drugan and Rice(1984,eq.(4.16b)). Unfortunately, Gao and Hwang drew an excessive conclusion that the two conditions in (15) cannot be satisfied simutaneously along a curve Γ , and hence, the basic equation is elliptic. In fact, if $\lambda \sim O((\ln r)/r)$ in a zone (for instance, singular plastic sector in this paper), then from (5) we see that in this whole zone

$$\varsigma = 0$$

And discontinuity of plastic shear strain can exist across the characteristic line where (14b) is satisfied.

The contiguity conditions given by Gao and Hwang(1981a,1983a) for bordering line Γ between neighboring sectors are:

$$[\![F]\!]_\Gamma = 0 \quad (16)$$

$$[\![F']\!]_\Gamma = 0 \quad (17)$$

$$[\![F'']\!]_\Gamma = 0 \quad (18)$$

$$\frac{1-\nu^2}{E}\frac{1}{r}[\![F''']\!]_\Gamma + \frac{1}{2\sin\theta_\Gamma}[\![\lambda F'']\!]_\Gamma + \frac{2}{3}\epsilon^2\frac{1}{\sin\theta_\Gamma}[\![\lambda\varsigma]\!]_\Gamma - 2\frac{d}{dr}[\![\varepsilon^p_{r\theta}]\!]_\Gamma = 0 \quad (19)$$

$$\frac{1-\nu^2}{E}\frac{1}{r^2}[\![F^{(4)}]\!]_\Gamma + \frac{1-\nu^2}{E}\cot\theta_\Gamma\frac{1}{r^2}[\![F''']\!]_\Gamma + \frac{1}{2\sin\theta_\Gamma}\frac{1}{r}[\![\lambda' F'']\!]_\Gamma + \frac{1}{2\sin\theta_\Gamma}\frac{1}{r}[\![\lambda F''']\!]_\Gamma$$
$$+ \frac{2}{\sin\theta_\Gamma}\frac{1}{r}[\![\lambda F']\!]_\Gamma + \frac{2}{3}\epsilon^2\frac{1}{\sin\theta_\Gamma}\frac{1}{r}[\![\frac{\partial}{\partial\theta}(\lambda\varsigma)]\!]_\Gamma + \frac{2}{\sin\theta_\Gamma}F'\frac{d}{dr}[\![\lambda]\!]_\Gamma = 0 \quad (20)$$

where $[\![\ ]\!]_\Gamma$ is used to denote the jump across Γ . (16)-(19) follow from continuity of stresses and displacements, while (20) simply denotes the difference of compatibility equation (7) at both sides of Γ. It should be emphasized that in deriving (16)-(20) we have assumed a possible singularity of curvature of Γ at crack-tip which has the order less than $(\ln r)^{-1}/r$ as $r \to 0$. Besides, these contiguity conditions are correct since its derivation is not at all influenced by the above-mentioned excessive conclusion drawn by Gao and Hwang.

ASYMPTOTIC SOLUTION OF NEAR-TIP FIELDS

The 5-sector structure is shown in Fig.1, where A---constant stress sector, B---singular plastic sector, C and E---nonsingular plastic sectors, and D---elastic unloading sector. The solutions for various sectors are as follows.

Sector A

$$F = \frac{\tau_0}{2}(b + \cos 2\theta) \quad (21)$$

$$\lambda = O(1) \quad (22)$$

$$\sigma_{11} = \tau_0(b-1) \qquad \sigma_{22} = \tau_0(b+1) \qquad \sigma_{12} = 0 \quad (23)$$

From (1) and (8) we obtain, respectively,

$$\varsigma = 0 \quad (24)$$

$$\varepsilon^p_{\alpha\beta} = O(1) \qquad (\alpha, \beta = 1, 2) \quad (25)$$

Sector B

$$F = \frac{\tau_0}{2}\{b - 2(\theta - \frac{\pi}{4})\} \qquad \lambda = A\frac{1}{r}\ln r + O(\frac{1}{r}) \quad (26)$$

It follows from the yield condition (1)

$$\varsigma = 0 \quad (27)$$

At the bordering line $\Gamma_A(\theta=\pi/4)$, the first 3 contiguity conditions (16)-(18) are already satisfied. From the 4th and 5th contiguity conditions, we have

$$[\![\varepsilon^p_{r\theta}]\!]_{\Gamma_A} = \frac{1}{2}(3+4\epsilon)\frac{\tau_0}{E}\ln\frac{\bar{R}}{r} \quad (28)$$

$$\lambda = -\frac{\sqrt{2}}{E}(\frac{3}{4}+\epsilon)\frac{\ln r}{r} + O(\frac{1}{r}) \quad (29)$$

Plastic strain components in Sector B follow from (8)

$$\varepsilon^p_{11} = -\varepsilon^p_{22} = -\frac{2\sqrt{2}}{E}(\frac{3}{4}+\epsilon)\sin\theta\ln\frac{\bar{R}}{r} + O(1) \quad (30)$$

$$\varepsilon^p_{12} = \frac{\sqrt{2}}{E}(\frac{3}{4}+\epsilon)\{\cos\frac{\pi}{4}(\ln\frac{\tan(\theta/2)}{\tan(\pi/8)} + 2\cos\theta) - 1\}\ln\frac{\bar{R}}{r} + O(1)$$

Sector C

$$\lambda = O(\frac{1}{r}) \qquad \varsigma \neq 0 \quad (31)$$

Making a consistent choice of sign, we obtain from (1)

$$\varsigma = -\frac{\sqrt{3}}{\epsilon}(\tau_0^2 - \frac{1}{4}F''^2 - F'^2)^{1/2} \quad (32)$$

Then from (5) and (7), we have

$$\lambda = \frac{3}{2Er}\frac{\sin\theta}{\varsigma}(F''' + 4F')(1 + \frac{3}{4\epsilon^2\varsigma}F'') \quad (33)$$

$$\frac{1}{E}(\frac{3}{4} + \epsilon + \frac{3}{2\varsigma}F'' + \frac{9}{16\epsilon^2}\frac{F''^2}{\varsigma^2})(F''' + 4F') = C_1\cot\theta + C_2 \quad (34)$$

where C_1 , C_2 are integration constants. It can be proved that the plastic strain components suffer no jump across Γ_B . From the 4th and 5th contiguity conditions (19) and (20), it follows, respectively,

$$C_1 = -\frac{\sqrt{2}}{2}\frac{\tau_0}{E}(3+4\epsilon)\sin\theta_1 \quad (35)$$

$$C_2 = -\frac{\sqrt{2}}{2}\frac{\tau_0}{E}(3+4\epsilon)(\sqrt{2} - \cos\theta_1)$$

F, ς and λ in Sector C can be obtained by integrating (32), (33) and (34). Since ς =0 at Γ_B , integration of (34) should be started from an angle in the near neighborhood of θ_1 (say $\theta=\theta_1+0.1°$). Their starting values are taken from the power series expansion at θ_1 obtained from (32),(33) and (34):

$$F = \frac{\tau_0}{2}\{b - 2(\theta_1 - \frac{\pi}{4})\} - \tau_0(\theta-\theta_1) + \frac{(3+4\epsilon)^2}{240\epsilon^2\sin^2\theta_1}\tau_0(\theta-\theta_1)^5 + \cdots \quad (36)$$

$$\varsigma = -\frac{(3+4\epsilon)\tau_0}{4\sqrt{2}\epsilon^2\sin\theta_1}(\theta-\theta_1)^2 + \cdots$$

The integration of (34) for F is performed from $\theta_1+0.1°$ in increment of 0.0005°, and ς and λ can be calculated from (32) and (33). The angle θ_2 where λ =0 corresponds to the unloading bordering line Γ_C .

Sector D

In the elastic unloading sector, λ=0, and

$$\varepsilon^p_{\alpha\beta} = \varepsilon^p_{\alpha\beta}(x_2) \qquad (\alpha, \beta = 1, 2)$$

Assume

AFR-1—N*

$$\frac{d^2\varepsilon_{11}^P}{dx_2^2} = \frac{D_0}{x_2^2}$$

Then the solution of equation (6) is

$$F = \frac{E}{1-\nu^2}\{D_1 + D_2\theta + D_3\cos 2\theta + D_4 \sin 2\theta + F^*\} \tag{37}$$

where

$$F^* = -\frac{D_0}{4}\{(\cos 2\theta - 1)\ln\sin\theta + (\theta + \tfrac{1}{2}\cot\theta)\sin 2\theta\} \tag{38}$$

The contiguity conditions (16)-(20) at Γ_c lead to

$$\begin{aligned} &D_1 + D_2\theta_2 + D_3\cos 2\theta_2 + D_4 \sin 2\theta_2 + F^*(\theta_2) = \frac{1-\nu^2}{E}F_c(\theta_2) \\ &D_2 - 2D_3\sin 2\theta_2 + 2D_4\cos 2\theta_2 + F^{*\prime}(\theta_2) = \frac{1-\nu^2}{E}F_c'(\theta_2) \\ &-4D_3\cos 2\theta_2 - 4D_4\sin 2\theta_2 + F^{*\prime\prime}(\theta_2) = \frac{1-\nu^2}{E}F_c''(\theta_2) \\ &D_2 = \tfrac{1}{4}C_2 \\ &D_0 = C_1 \end{aligned} \tag{39}$$

where F_c , F_c' , F_c'' denote values in Sector C. The above equations can be used for obtaining D_1 , D_2 , D_3 , D_4 and D_0 for given θ_1 and θ_2 . From (5) we get

$$\varsigma = F'' + 4F + G \tag{40}$$

where the constant G can be obtained from the continuity of ς across Γ_c . The plastic strain components in Sector D are

$$\begin{aligned} &\varepsilon_{11}^P = -\varepsilon_{22}^P = -\frac{\sqrt{2}}{2}(3+4\epsilon)\frac{1}{E}\sin\theta_1 \ln\frac{A\sin\theta_1}{x_2} + O(1) \\ &\varepsilon_{12}^P = \frac{\sqrt{2}}{4}(3+4\epsilon)\frac{1}{E}\left\{\cos\frac{\pi}{4}\left(\ln\frac{\tan(\theta_1/2)}{\tan(\pi/8)} + 2\cos\theta_1\right) - 1\right\}\ln\frac{A\sin\theta_1}{x_2} + O(1) \end{aligned} \tag{41}$$

Sector E

$$\lambda = O(\tfrac{1}{r}) \qquad \varsigma \neq 0 \tag{42}$$

The governing equations for ς , λ and **F** in Sector E remain to be (32), (33) and (34). It follows from the 4th and 5th contiguity conditions that the constants C_1 and C_2 for Sector E remain the same as for Sector C. The initial values necessary for starting the integration of (34) are offered by the first 3 contiguity conditions at Γ_D.

Boundedness of each term in (5) as $\theta \to \pi$ implies

$$\varsigma \to 0 \qquad \text{as} \quad \theta \to \pi \tag{43}$$

Hence the integration of (34) cannot be performed directly to the free crack surface ($\theta=\pi$). It is proved by Hwang and Luo(1988) that the condition $F'=0$ in (12) can be replaced by

$$\tilde{F}'^2 = -\tilde{F}'\tilde{F}''\eta + 2(\tilde{F}'^2 + \tfrac{1}{3}\epsilon^2\tilde{\varsigma}')\eta^2 \tag{44}$$

where "~" denotes the value taken at $\theta = \pi - \eta$ (say, $\eta = 0.1°$).

NUMERICAL RESULTS

For the asymptotic solution, we have to solve the differential equation (34) in Sectors C and E, and to determine constant b and angles θ_1 , θ_2 and θ_3 for Γ_B, Γ_C , Γ_D (Fig.1). It costitutes a one-parameter (i.e. θ_1) shooting problem, which can be solved by the following steps of iteration:

1. Assume a value of θ_1 ;
2. Determine C_1 and C_2 from (35);
3. Calculate from (36) the initial values necessary for starting the integration of (34) from $\theta=\theta_1+0.1°$ (take, for convenience, for the time being b=0);
4. Integrate (34), calculate pointwise values of λ , determine the angle θ_2 for Γ_c from the requirement (10);
5. Calculate from (39), (40) the constants D_1 , D_2 , D_3 , D_4 , D_0 and G for Sector D;
6. Determine from (1) the angle θ_3 for the reloading plastic boundary Γ_D ;
7. Calculate from the first 3 contiguity conditions the initial values at Γ_D necessary for starting the integration of (34);
8. Integrate (34) from Γ_D to $\theta=\pi-\eta$ (say, $\eta=0.1°$), and examine whether the condition (44) is satisfied. If it is not, then repeat the above set of steps with newly assumed value of θ_1 . The procedures are continued until (44) is satisfied within prescribed accuracy;
9. Determine the constant b from the condition $F(\pi)=0$ (see (12)).

In the solution by Drugan et al.(1982) the flow factor λ turns out to be negative, hence the plastic flow law is violated in a portion of Sector C. The reason for this is that Drugan et al.(1982) disregarded the condition (10) implied by the " unloading boundary theorem ", replaced it by a superfluous condition (43), which is actually a consequence of the equation (33) (or (5)) itself, and converted the problem to a two-parameter (i.e. θ_1 and θ_2) shooting problem. Besides, the Gao's solution(1981) is not acceptable, since it does not degenerate to the corresponding solution for incompressible materials as $\nu \to \frac{1}{2}$ because of the existence of strain singularity ahead of the crack-tip.

The full near-tip fields for stresses, strains and flow factor can be obtained. Here only angles θ_1 , θ_2 and θ_3 are compared in Table 1. the present results shown in Table 1 coincide exactly with the value obtained by Luo and Hwang(1988) within the framework developed by Rice(1982) and refined by the present authors.

Table 1. Comparison of θ_1 , θ_2 and θ_3 between the results of present paper and those by Drugan et al. (1982)

	θ_1	θ_2	θ_3
Present results	110.09°	118.20°	160.42°
Drugan et al.(1982)	110.26°	123.13°	160.38°

REFERENCES

Drugan, W.J. and J.R. Rice(1984). Restrictions on quasi-statically moving surface of strong discontinuity in elastic-plastic solids. In: Mechanics of Material behavior,The Doniel C. Drucker Anniversary Volume(G.J. Dvorak and R.T. Shield, ed.),pp.59-73. Elsevier Science Publishers.

Drugan, W.J., J.R. Rice and T.-L. Sham(1982). Asymptotic analysis of growing plane strain tensile crack in elastic-ideally-plastic solids. J. Mech. Phys. Solids, 30, 447-473.

Gao, Y.C.(1981). The influence of compressibility on the elastic-plastic field of a growing crack. Proceedings of ASTM Second International Symposium on Elastic-Plastic Fracture Mechanics, Philadelghia, 6-10, October,1981.

Gao, Y.C. and K.C. Hwang(1981a). Elastic-plastic fields in steady crack growth. In: Three-Dimensional Constitutive Relations and Ductile Fracture (S. Nemat-Nasser, ed.),pp.417-473. North-Holland Pub. Co..

Gao, Y.C. and K.C. Hwang(1981b). The plane strain problem for perfectly elastic-plastic medium. Acta Mechanica Sinica, Special Issue(1981),pp.111-120.

Gao, Y.C. and K.C. Hwang(1983a). On the formulation of plane strain problem for elastic perfectly-plastic medium. Int. J. of Engng. Sci.,21,765-780.

Gao, Y.C. and K.C. Hwang(1983b). The discontinuity in quasi-static plastic fields. Proc. of ICF Int. Symp. on Fracture Mech.(Beijing,1983),pp.24-30.

Hwang, K.C. and X.F. Luo(1988). Near-tip fields for cracks growing steadily in elastic-perfectly-plastic compressible material, paper presented on the IUTAM Symposium on Recent Advances in Nonlinear Fracture Mechanics, Caltech,Pasadena,CA,U.S.A.,March 14-16,1988.

Luo, X.F. and K.C. Hwang(1988). Correct formulation and solution of near-tip field problem for crack growing in compressible elastic-plastic materials, to be published in Scientia Sinica,Series A.

Rice, J.R., W.J. Drugan and T.-L. Sham(1980). Elastic-plastic analysis of growing crack. Fracture Mechanics,ASTM STP 700,pp.189-219.

Rice, J.R.(1982). Elastic-plastic crack growth. In: Mechanics of Solids, Rodney Hill 60th Anniversary Volume(H.G. Hopkins and M.J. Sewell, ed.), pp.539-562. Pergamon Press, Oxford.