Influence of the Material Inhomogeneity of Overmatched Welded Joints on the Crack Driving Force

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ABSTRACT

The effects on the J-integral of material inhomogeneity and weld width of the overmatched welded joints with a longitudinal crack or flaw have been studied. The overmatched weld is idealized to be a parallelly cracked and sandwiched hard layer, the yield stress of which is higher than that of the base metal. Numerical solutions were obtained by using the elastoplastic finite element method under the plane strain condition. The results show that the values of the J-integral are remarkably affected by the material inhomogeneity and the weld width especially when the nominal stress or nominal strain is comparatively high, which, therefore, should be taken into account when evaluating the crack driving force for an overmatched joint.

KEYWORDS

Welded joint; inhomogeneity; FEM analysis; crack; J-integral.

INTRODUCTION

The effect of the material inhomogeneity of welded joints on fracture toughness has already attracted attention of researchers. Satoh and Toyoda (1983, 1986) have contributed a lot in this field. But the effect of the inhomogeneity of welded joints on the crack driving force has not been paid enough attention to. At present, the inhomogeneity has not been taken into account either in the COD design curves used in flaw evaluation of welded structures, or in EPRI (Kumar *et al.*, 1981) method which is actually for homogeneous materials. It has been made clear by Ma *et al.* (1987a) that, if ignoring the effect of the inhomogeneity, the error in estimation of the crack driving force for undermatched welded joints would be unallowable sometimes. Only with correct estimation of the crack driving force on the basis of analysing the effect of the material inhomogeneity, can it be possible to properly assess the safety of welded structures. This would, in turn, provide a theoretical basis for designing reasonable welded structures, selecting correct welding materials, and determining proper welding

procedures.

THE MODEL AND NUMERICAL EVALUATION OF THE *J*-INTEGRAL

The idealized model of an overmatched welded joint with a longitudinal crack is shown in Fig.1. The weld is idealized to be a sandwiched hard layer with width h. The crack with length $2a$ is parallel to the boundaries between the weld and base metal. The distance between the crack and the boundary nearer to the crack is h_1. The ratio h_1/h represents the degree of crack eccentricity. When $h_1/h=0.5$, the crack lies on the weld center line; $0<h_1/h<0.5$, the crack lies apart from the center line; $h_1/h=0$, the crack lies on one of the boundaries (the case of a local lack of fusion between weld and base metal or a flaw between transition zone and base metal in a welded joint of different metals). The ratio of yield stress of weld to that of base metal is 1.37; the ratio of Young's moduli is 1.05, Fig.2.

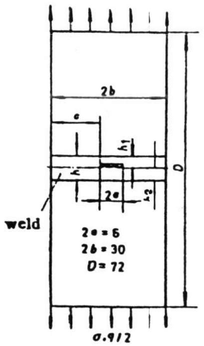

Fig.1. Idealized model of overmatched welded joints with a crack.

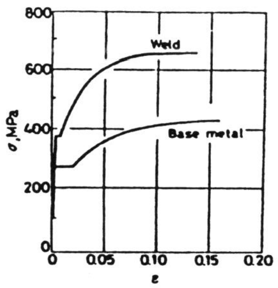

Fig.2. $\sigma-\epsilon$ curves of weld and base metal.

The *J*-integral for two dimensional crack problems defined by Rice (1968) is

$$J=\int_{\Gamma}\left(Wn_1-n_i\sigma_{ij}u_{j,1}\right)ds \tag{1}$$

where Γ is an arbitrary anti-clockwise contour surrounding the crack tip, W the strain energy density, n the direction cosine of the outward normal, σ_{ij} the stress tensor, u_{ij} the displacement tensor, and ds the arc element. The crack is parallel to the x axis. For homogeneous materials, plasticity deformation theory being applied, the *J*-integral defined by Eqn.(1) is path independent under unloading condition. Ma *et al.* (1987b) have proved that for a inhomogeneous body which is composed of two or more kinds of elastoplastic materials, and which contains a crack parallel to the boundaries between the materials, the *J*-integral is also path independent. Evidently the idealized model shown in Fig.1 can be classified into that type of inhomogeneous bodies.

In order to make use of directly the stress, strain, displacement, etc. obtained by the finite element analysis for further numerical calculation of the *J*-integral, the variables in Eqn.(1) are replaced, and Eqn.(1) is

rewritten by using $n_1ds=dx_2$, $n_2ds=-dx_1$, $\sigma_{ij}=\sigma_{ji}$, $du_i=u_{i,j}dx_j$, , etc.:

$$J=\int_{\Gamma}\{[W-(\sigma_x\epsilon_x+\sigma_y\epsilon_y+\tau_{xy}\gamma_{xy})]dy+\tau_{xy}du+\sigma_y dv\} \tag{2}$$

In the finite element analysis of J, if the contour is specified as a list of node numbers or Gaussian integration point numbers in sequence $\{N_1, N_2, \ldots N_j, \ldots, N_{n-1}, N_n\}$ as shown in Fig.3, we have a formula based on Eqn.(2) for evaluating the *J*-integral numerically

$$\begin{aligned}J=\frac{1}{2}\sum_{i=1}^{n-1}\{&[(W_i+W_{i+1})-(\sigma_{xi}\epsilon_{xi}+\sigma_{yi}\epsilon_{yi}+\tau_{xyi}\gamma_{xyi})\\&-(\sigma_{xi+1}\epsilon_{xi+1}+\sigma_{yi+1}\epsilon_{yi+1}+\tau_{xyi+1}\gamma_{xyi+1})](y_{i+1}-y_i)\\&+(\tau_{xyi}+\tau_{xyi+1})(u_{i+1}-u_i)+(\sigma_{yi}+\sigma_{yi+1})(v_{i+1}-v_i)\}\end{aligned} \tag{3}$$

where W_i, σ_{xi}, ϵ_{xi}, ..., y_i, and u_i and v_i are the strain density, stress, strain, ..., ordinate and displacements corresponding to node or Gaussian integration sampling point N_i on the contour Γ.

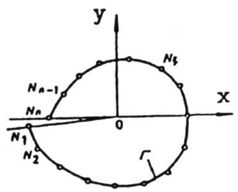

Fig.3. Schematic showing specification of *J*-integral contour.

In the present study, 9 contours labelled Γ_1, Γ_2, ..., Γ_9, with length and shape differing from one another, have been chosen, Fig.4. The reasons for the contour choice are to investigate the path independence of the *J*-integral using incremental plasticity theory, and to obtain more precise results by taking the mean value of the *J*-integrals evaluated along the 9 contours.

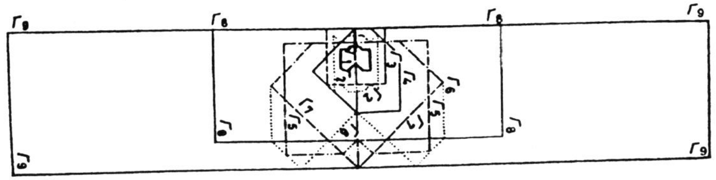

Fig.4. Contours for evaluation of J-integral.

In addition to Eqn.(3), the following approximate formula for center cracked-specimen recommended by Oji *et al.* (1978) for evaluating the *J*-integral is also used

$$J = J_e + \frac{2}{cB}\left(\int_0^q Pdq - \frac{1}{2}Pq\right) \qquad (4)$$

where J_e is the elastic part of J, i.e.,the crack extension force $G = K_I^2(1-\nu^2)/E$, c the ligament width, B the plate thickness, $P=\sigma bB$ is half the applied load, q the loading line displacement. Using two formulas to calculate the J-integral concerning one problem enables the obtained results to be checked by each other. And because Eqn.(4) is derived from the energy release rate expression $J=-1/B \cdot \partial U/\partial a$, by investigating the applicability of Eqn.(4) to overmatched welded joints through comparing the J-integral value evaluated from Eqn.(4) with that from Eqn.(3), the applicability of the formula $J=-1/B \cdot \partial U/\partial a$ to the inhomogeneous body can be investigated.

Elastoplastic analysis is carried out under the plane strain condition using the incremental self modifying chord stiffness method advanced by Ma *et al.* (1987c), which is much more precise than the widely used incremental methods. By making use of the geometrical symmetry of the model, a finite element mesh is drawn on one half of the model. The mesh contains 890 triangular elements and 494 nodes. The maximum side of the triangles around the crack tip is less than $a/360$ in length. When $h_1/h=0.5$, the model is both longitudinally and vertically symmetrical, therefore only a quarter of the model is taken to draw a finite element mesh which is composed of 427 triangular elements and 252 nodes. The source programme HCBAP is developed by Ma (1985) for the analysis of crack problems concerning inhomogeneous bodies. The reliability of the programme has already been demonstrated in full account elsewhere (Ma, 1985). For a material whose stress and strain relation obeys Ramberg-Osgood law

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^N$$

the J-integral values for a center cracked plate($a/b=1/5$, $\sigma_0=1097$ MPa, $\epsilon_0=0.05485$, $\alpha=0.365$, and $N=9.66$) computed by HCBAP almost coincide with those by EPRI method, with relative error within 3 percent at each nominal stress level, which is shown in Fig.5.

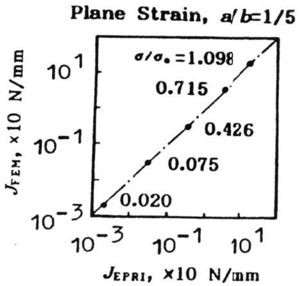

Fig.5. J values computed by FEM programme HCBAP and those estimated by EPRI method.

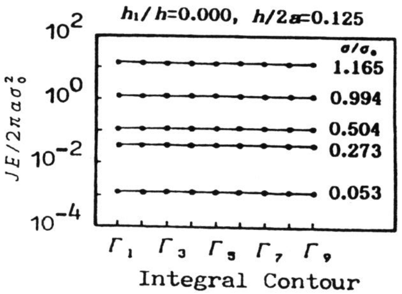

Fig.6. Path independence of J for overmatched welded joints, $a/b=1/5$, plane strain.

RESULTS OF THE NUMERICAL ANALYSIS

For the idealized model shown in Fig.1, the relative error of the computed J-integral value integrated along any one of the 9 contours at each stress level to the mean of the computed J-integral values along the 9 contours shown in Fig.4 is within ±3 percent. Fig.6 shows the computed results for the model with $h/2a=0.125$ and $h_1/h=0$, where E and σ_0 are Young's modulus and yield stress of the base metal respectively. Thus the path independence of the J-integral has been confirmed by the numerical analysis based on the incremental plasticity theory.

Before the base metal starts general yielding (at this stage, the deformation state of the whole model is basically the same as the assumption under which Eqn.(4) was derived from $J=-1/B \cdot \partial U/\partial a$), the J-integral value (labelled J_u) computed from Eqn.(4) is very close to the J-integral value (labelled J) from Eqn.(3) derived from the definition of J-integral, which is shown in Fig.7, showing that Eqn.(4) is still valid before general yielding and that the energy release rate expression of the J-integral still holds for inhomogeneous overmatched welded joints. The fact that coincident results have been obtained by different methods demonstrates further the reliability of the obtained results.

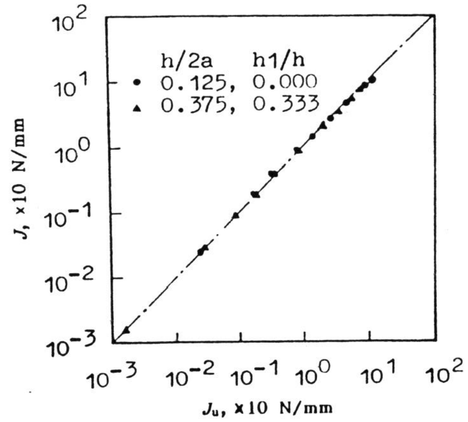

Fig.7. Comparison of J_u values evaluated from Eqn.(4) with J values evaluated from Eqn.(3), $a/b=1/5$, plane strain.

Fig.8 and 9 show the J curves against nominal stress(σ) and nominal strain(q/D) respectively. It can be seen that though the yield stress of the base metal is only 37 percent less than that of the weld, the effects of the inhomogeneity and the relative weld width($h/2a$) on the J-integral value are remarkable, especially when σ or q/D is relatively high. In comparison with the J-integral value corresponding to the homogeneous body with $h/2a\to\infty$, the different degrees of yielding in the base metal on two sides of the weld leads to different degrees of rising in the J-integral value. When σ is constant, the larger the weld width, the lower the J-integral value. When

q/D remains unchanged, with the presence of overmatched weld with different $h/2a$, the J-integral value will decrease to some extent in comparison with that corresponding to the homogeneous body with $h/2a$=0, and the larger the $h/2a$, the lower the J-integral value. Sometimes, the change in $h/2a$ can cause a ten fold change in the J-integral value. The trends of the variation of the J-integral values with the weld width coincide with the COD experimental results (Ma, 1985; Tian and Ma, 1987) as shown in Fig.10.

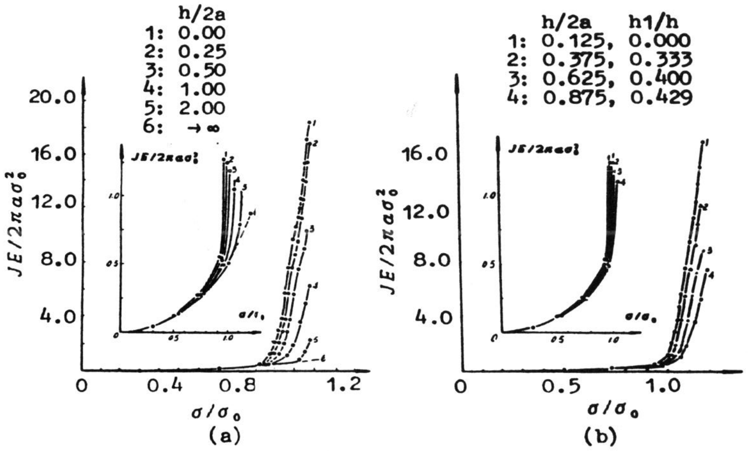

Fig.8. J-integral vs nominal stress. Plane strain, a/b=1/5. (a) h_1/h=0.5; (b) $0\le h_1/h<0.5$.

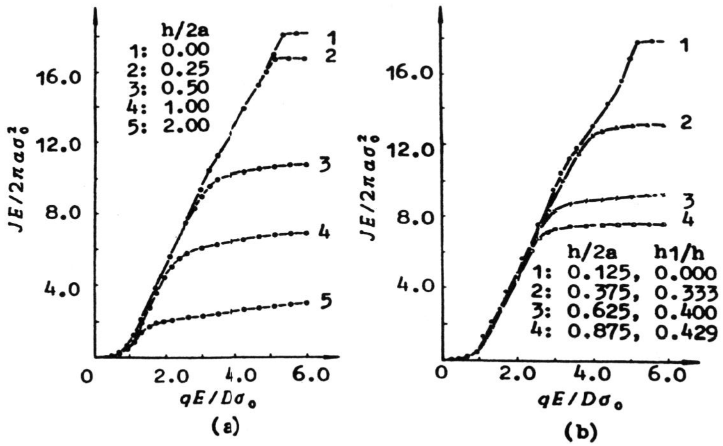

Fig.9. J-integral vs nominal strain. Plane strain, a/b=1/5. (a) h_1/h=0.5; (b) $0\le h_1/h<0.5$.

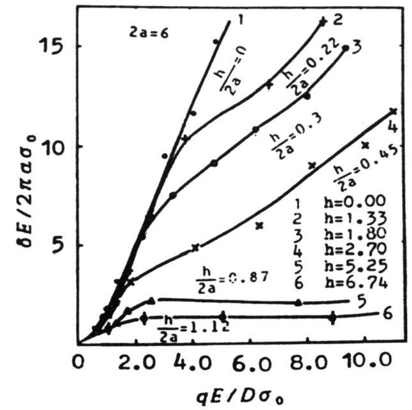

Fig.10. Experimental COD at crack center vs. nominal strain (Ma, 1985; Tian and Ma, 1987).

CONCLUSIONS

1. The path independence of the J-integral and the applicability of the energy release rate expression $J=-1/B\cdot\partial U/\partial a$ in overmatched welded joints have been confirmed by the results of the elastoplastic finite element analysis using incremental plasticity theory.

2. The material inhomogeneity and weld width of the overmatched welded joints with a crack have nonnegligible influence on the J-integral value, especially when the crack is in the high stress region or high strain region.

3. Even if the yield stress of the weld is only 37 percent higher than that of the base metal, the change in the weld width may sometimes cause a ten fold change in the J-integral value.

4. Under the same nominal stress or strain, with the presence of the overmatched weld, the J-integral value is lower than that corresponding to the homogeneous condition $h/2a$=0, and the larger $h/2a$, the lower the J-integral value.

5. The relation between J and nominal stress or strain is characterized by a group of curves which are dependent on the material inhomogeneity and the weld width. Therefore using only one curve instead of the group might lead to a great error.

6. When estimating the crack driving force for an overmatched welded joint, the influences of the material inhomogeneity and the weld width should be taken into account.

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