

# HRR Field of a Moving Crack, an Experimental Analysis

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## ABSTRACT

An improved moiré interferometry was used to record simultaneously both the vertical and horizontal displacements associated with stable crack growth in uniaxially and biaxially loaded 2024-0 and 2024-T3 aluminum, single edge notched specimens. For stable crack growth up to 5 mm, the vertical displacement field showed the dominance of the HRR field but the HRR field was detected in the horizontal displacement only at the initial stage of loading. The far and near field J-integrals were path independent and yielded the correct crack tip displacements only at the initial stage of loading in the horizontal direction and at the terminal stage of loading in the vertical displacement field. The preliminary results indicate that J-characterization of a moving crack, including that of rapid propagation, may not be valid for these ductile materials for this specimen configuration.

## KEYWORDS

HRR field; moiré interferometry; biaxial; J-integral; J-resistance; ductile fracture; J-dominance region; aluminum

## INTRODUCTION

For the past two decades, an enormous amount of research and development efforts have been expended in justifying and quantifying the J-integral values of ductile engineering materials. From the view point of material testing, the J-integral offers an experimental convenience over other proposed ductile fracture criteria, such as crack opening displacement (COD) and crack tip opening displacement (CTOD). This advantage is manifested by the use of far field J-value, which under certain conditions represents the crack tip J-value through Rice's path independence proof (Rice, 1968). In contrast, the COD and the CTOD pose untold experimental challenges. The asymptotic crack tip solution, which justifies Rice's J-integral, for power hardening materials was later presented by Hutchinson (1968) and Rice and Rosengren (1968) and is commonly referred to as the HRR field.



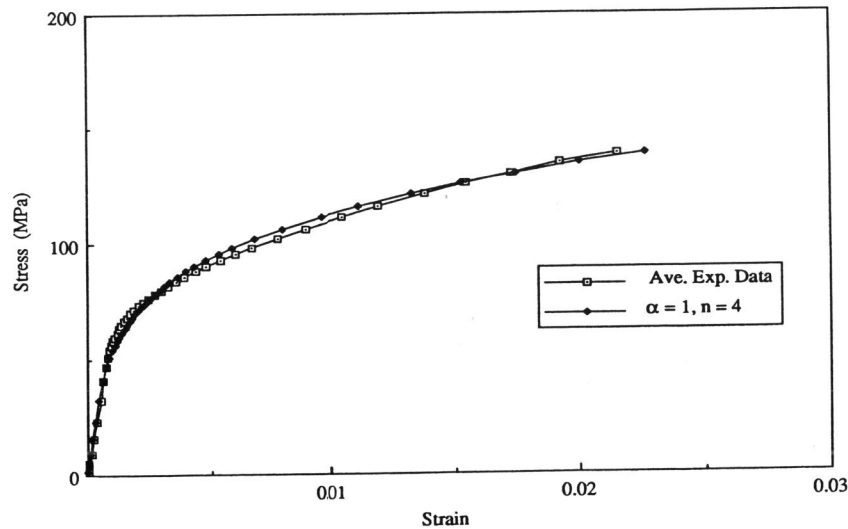


Fig. 2a. Uniaxial Stress-Strain Curve for 2024-0.

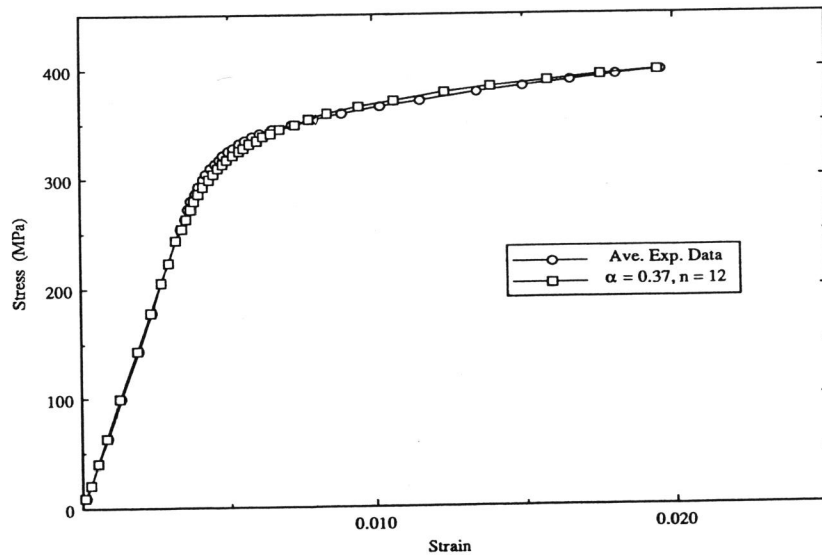


Fig. 2b. Uniaxial Stress-Strain Curve for 2024-T3.

## RESULTS

The 2024-0 and 2024-T3 aluminum specimens were loaded uniaxially, i.e., at a biaxial ratio  $B = 0$ , and biaxially with the horizontal load being twice the vertical load or  $B = 2$ . Table 1 shows the test matrix of this limited study together with the maximum crack extensions achieved in each of the five tests. Figure 3 shows the load versus load-line displacement for a biaxially loaded ( $B = 2$ ) 2024-0 aluminum specimen.

Table 1. Test Matrix

Material Biaxial Load Ratio	2024-T3	2024-0
	Max. Crack Extension (mm)	
$B = 0$	3 MD031687	1.34 MD050288
$B = 1$	5.6 MD102687	—
$B = 2$	4 MD101987	2.2 MD050388

Figures 4a and 4b show typical moiré fringes, i.e., the vertical displacement,  $v$ , and the horizontal displacement,  $u$ , for the biaxially loaded 2024-0 aluminum specimen. Also shown are typical integration contours used for the J-integral determination. The J values obtained along these contours are shown in Table 2. Path independence, i.e., within a nine percent scatter in the J-values, is noted. Similar results were noted in the other four tests.

Figures 5 and 6 show typical log-log plots of the  $v$ - and  $u$ -displacement fields obtained from the moiré fringes. Also shown are the log-log plots of displacements versus radial distance  $r$  of the linear elastic (LEFM) and HRR fields at a crack tip polar angle of  $\theta = 45^\circ$ . The HRR field requires that this slope be  $1/(n + 1)$ , which is 0.2 for 2024-0 aluminum. These plots indicate that the  $v$ -field exhibited a nearly LEFM field which later changed to an HRR field as the terminal load was approached. The  $u$ -field, on the other hand, exhibited an HRR field in the initial loading stage, but was quickly replaced by a non-linear zone at load exceeding 2600 (N). A similar trend was noted in the other three tests.

Figure 7 shows plots of the measured  $v$ - and  $u$ -displacements at a point of  $r = 1.2$  mm (1-1/2 plate thickness) and  $\theta = 45^\circ$ , where HRR field was shown to extend the furthest (Chiang et al., 1986), with increasing applied load. Also shown are the corresponding  $u$ - and  $v$ -displacement fields for the LEFM and HRR fields at the same location. The displacements for the HRR field were calculated by using the average J-integral values obtained through contour integrations of the moiré data. The corresponding displacement field for the LEFM crack tip were obtained by the equivalent plane-stress stress intensity factor computed from these J-integral values. A total of 34 numbers of log-log plots of the  $u$ - and  $v$ -fields were evaluated to arrive at the conclusion that only the  $v$ -field exhibited the HRR field through the loading and stable crack growth process.

The results are summarized in Fig. 8 which shows the J-resistance curve of this 2024-0 specimen for  $B = 0$  and 2. Also shown are the approximate J-integral values, which were reported by Kang et al. (1988), for the same material but for a much smaller specimen. These

Table 2. J-integral Values Under Stable Crack Growth in 2024-0 Aluminum Specimen MD050388, B = 2.0.

Load in X (N)	Load in Y (N)	J (kPa-m)		$\Delta a$ (mm)
		Contour 1	Contour 2	
4066	2086	4.0	3.8	0.14
5489	2896	11.0	10.5	0.6
5591	3305	18.1	18.4	0.85
5845	3888	31.0	29.0	1.37
6076	3914	32.0	29.2	1.4
5720	4524	34.0	31.0	1.68
6810	4626	50.0	47.0	2.2

"Load-Load Line Disp." MD050388

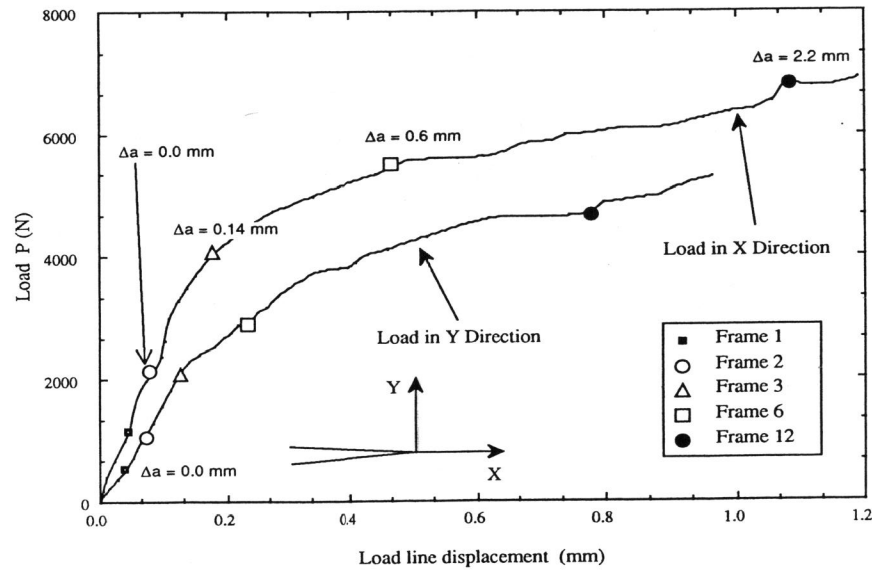


Fig. 3. Load versus Load-line Displacement in 2024-0 Aluminum Specimen, B = 2.

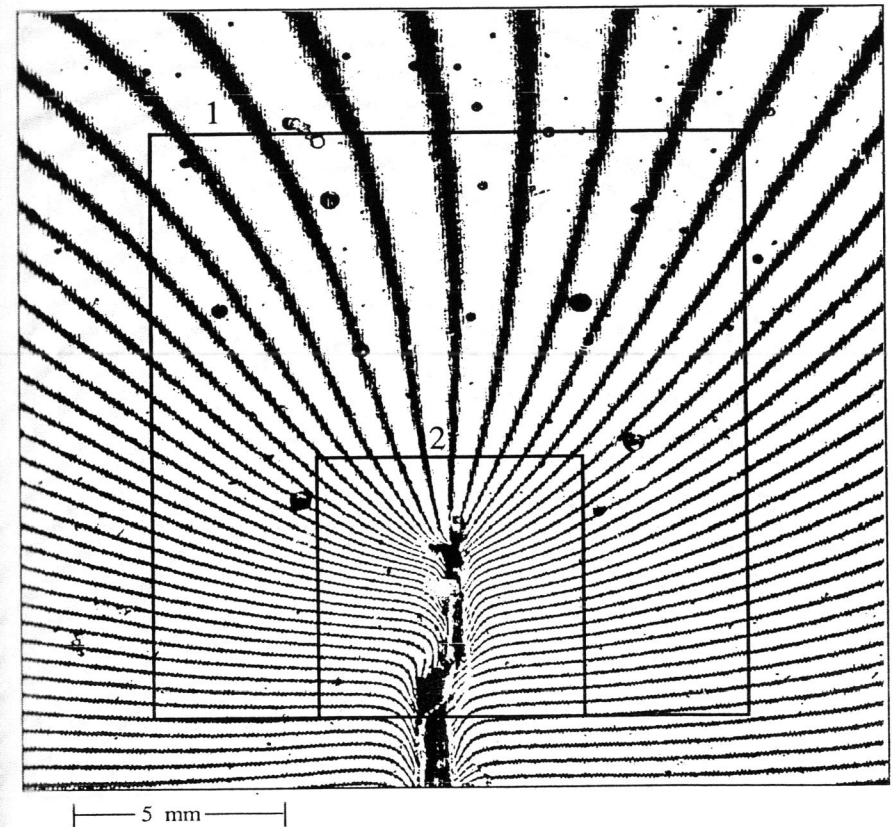


Fig. 4a. v-Displacement in 2024-0 Aluminum Specimen, MD050388-3, B = 2.0,  $\Delta a = 0.14$  (mm),  $F_x = 4066$  (N) and  $F_y = 2086$  (N).

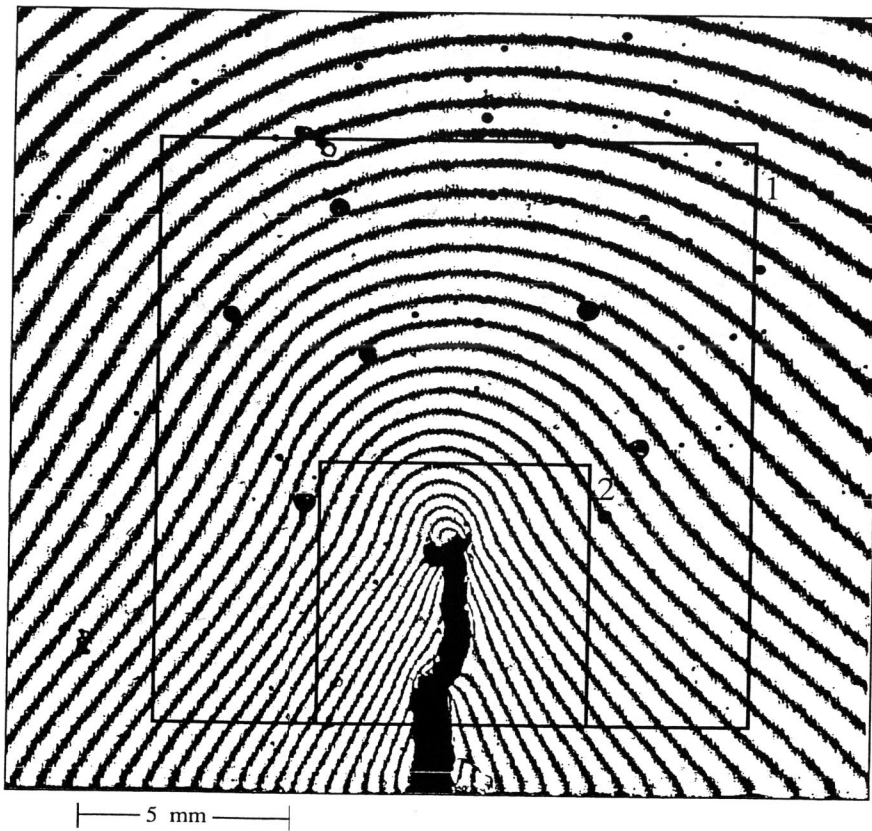


Fig. 4b. u-Displacement in 2024-0 Aluminum Specimen, MD050388-3,  $B \approx 2.0$ ,  $\Delta a = 0.14$  (mm),  $F_x = 4066$  (N) and  $F_y = 2086$  (N).

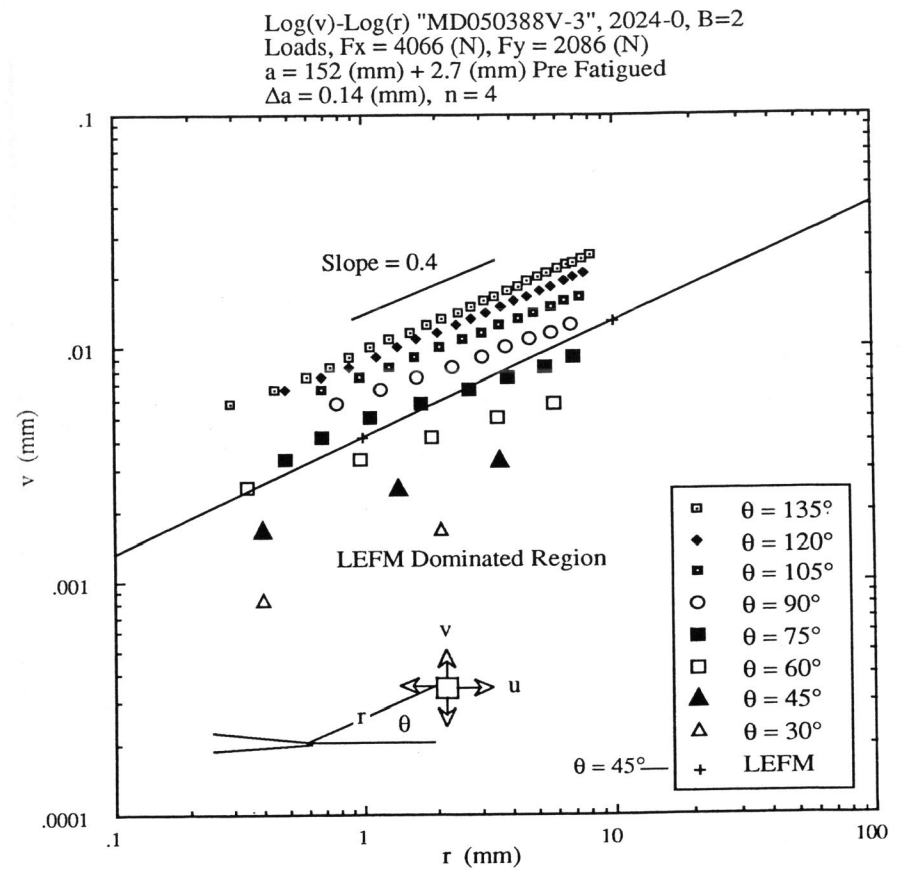


Fig. 5a. v-Displacement Versus  $r$  Relation for Various  $\theta$ , 2024-0 Aluminum MD050388-3  $B \approx 2.0$ ,  $\Delta a = 0.14$  mm,  $F_x = 4066$  (N) and  $F_y = 2086$  (N).

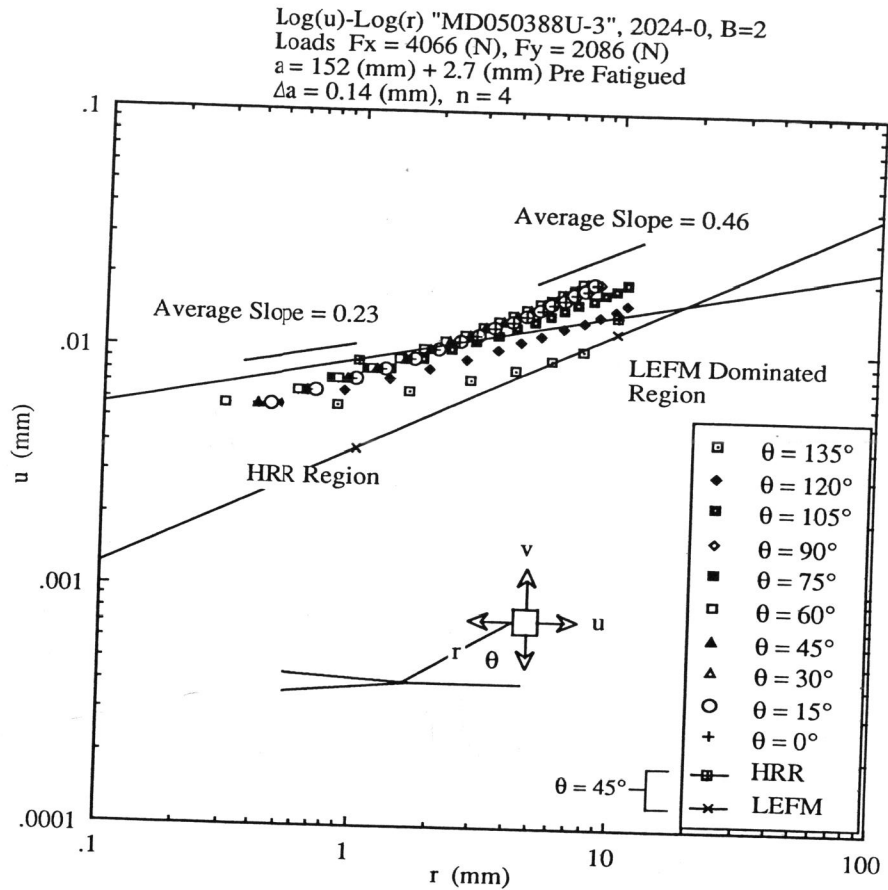


Fig. 5b. u-Displacement Versus r Relation for Various  $\theta$ , 2024-0  
 Aluminum MD050388-3 B  $\approx 2.0$ ,  $\Delta a = 0.14$  mm,  $F_x = 4066$  (N) and  $F_y = 2086$  (N).

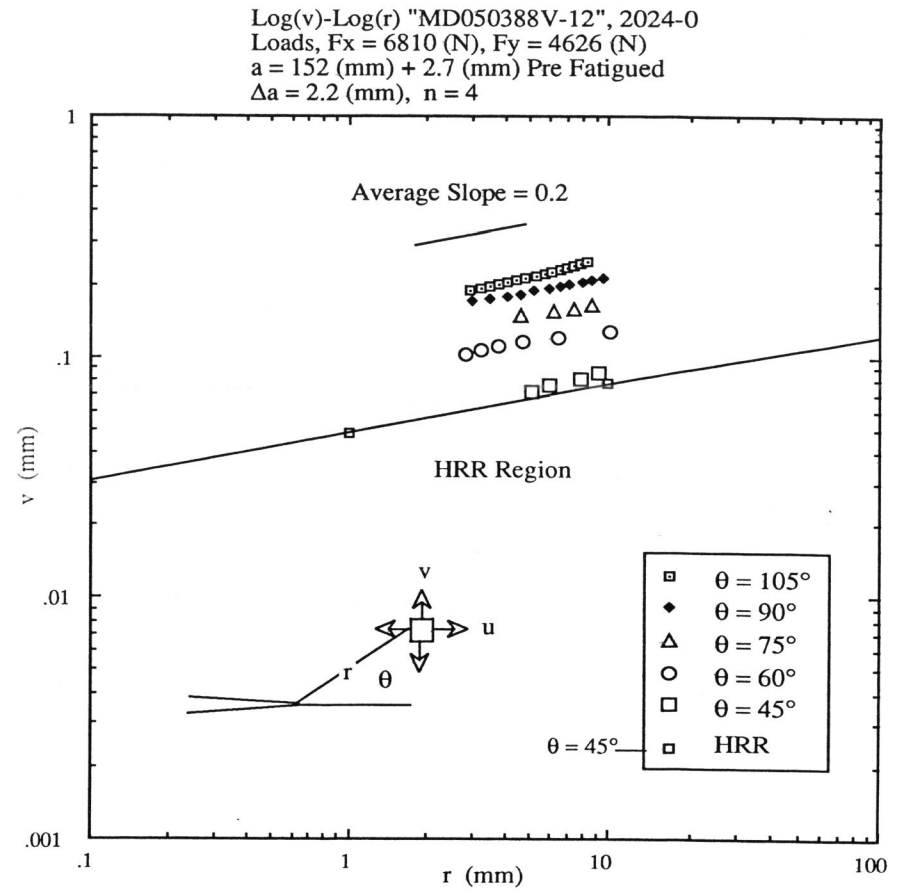


Fig. 6a. v-Displacement Versus r Relation for Various  $\theta$ , 2024-0  
 Aluminum MD050388-12 B  $\approx 2.0$ ,  $\Delta a = 2.2$  mm,  $F_x = 6810$  (N) and  $F_y = 4624$  (N).

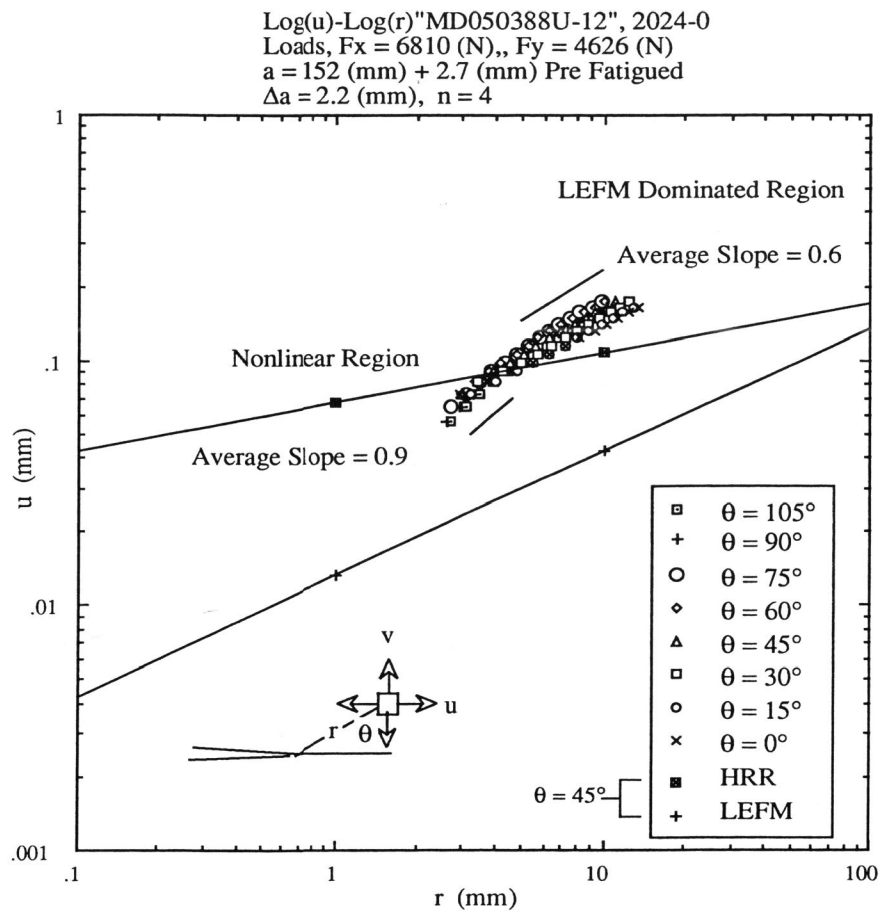


Fig. 6b. u-Displacement Versus r Relation for Various  $\theta$ , 2024-0  
 Aluminum MD050388-12 B  $\approx 2.0$ ,  $\Delta a = 2.2$  mm,  $F_x = 6810$  (N) and  $F_y = 4624$  (N).

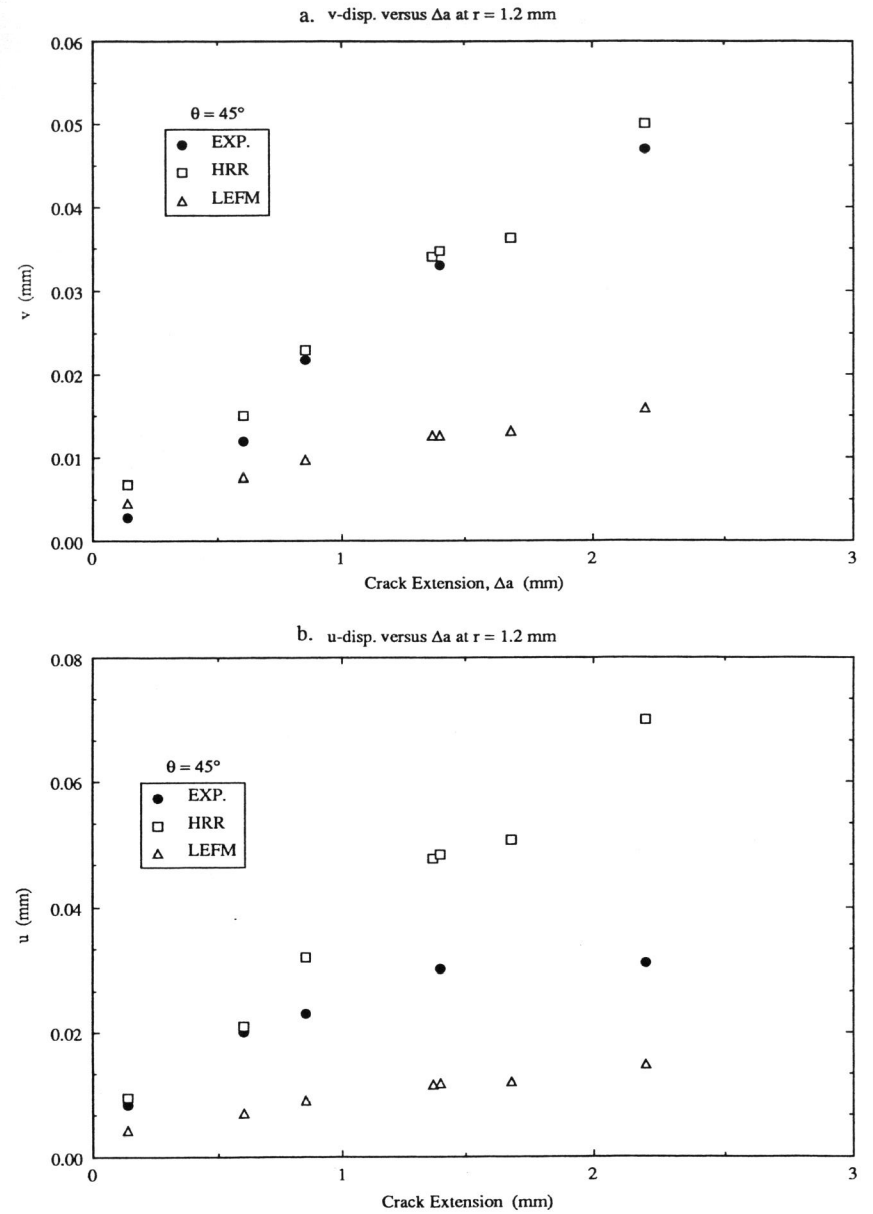


Fig. 7. v- and u-Displacements at a point of  $r = 1.2$  mm and  $\theta = 45^\circ$  versus Crack Extension

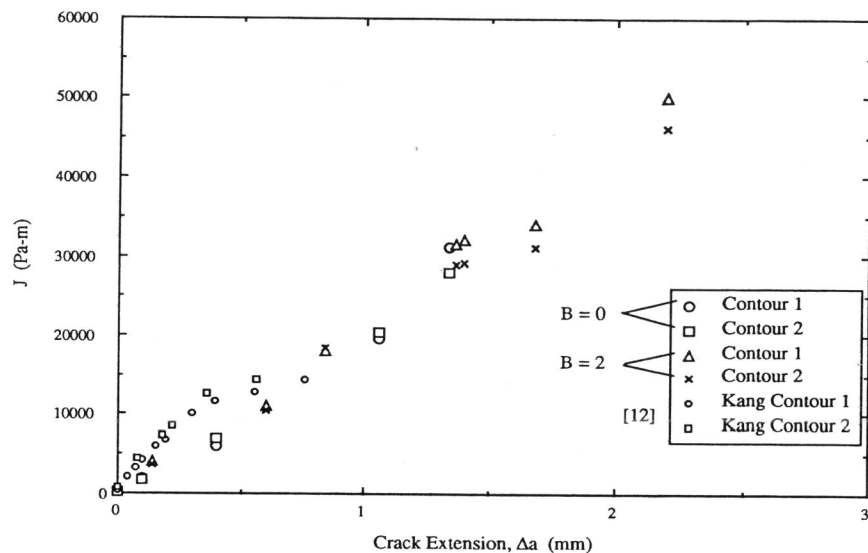


Fig. 8.  $J_k$  Curve of all 2024-0 Aluminum Tests.

J-resistance curves differ with others in that crack extension occurs at very low applied load and not due to blunting as reported by others (Paris et al, 1979). Fig. 9 shows the J-resistance values for 2024-T3 tests during stable crack growth. Also shown is the J-resistance curve obtained by de Koning (de Koning, 1978) for a 2024-T3 center cracked panel with a thickness of 2-mm and Ernst (1981) for 2024-T351 4T compact tension specimen. The major difference between the results of the two tests (Hutchinson, 1983) is the knee, which occurred at the lower loads.

#### DISCUSSION

The requirements for the J-dominance region for plane strain condition were discussed extensively in the articles by Hutchinson (1983) and Shih (1985). This present discussion is limited to the analysis of HRR region of a moving crack in plane stress condition. Figure 7 showed that under the same loading, only the v-field exhibited the expected progression from the LEFM to the HRR crack tip fields. At higher loading where the HRR singularity field should prevail, u-field does not show any HRR dominated region, but the v-field exhibited a prominent HRR field as shown in Fig. 7. These results indicate that under rapid crack propagation, the desired HRR field may not exist in ductile materials and thus the dynamically modified J cannot be used as a firm field parameter to characterize the ductile crack propagation.

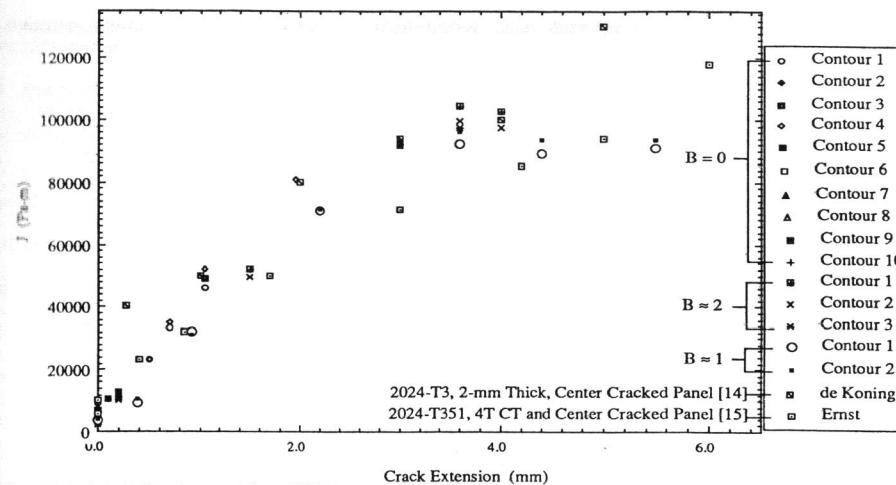


Fig. 9.  $J_R$  Curve for all 2024-T3 Aluminum Tests of this study, Ref. [14] and [15].

#### CONCLUSIONS

- 1) HRR field exists only during initial loading for u-field and extended up to 2 mm from the crack tip.
- 2) HRR field exist in the v-field through the stable crack growth process and extended up to 10 mm from the crack tip.
- 3) J is path independent for crack extension of 5.6 mm for 2024-T3 and 2.2 mm for 2024-0 aluminum alloys.
- 4)  $J_R$  curves are identical within the scatter of data for 2024-T3 and 2024-0 for both B = 0, B = 1 and B = 2.

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