Elastic–Plastic Stress Analysis of Semi-elliptical Cracks in a Tubular Welded T-joint

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ABSTRACT

The effect of semi-elliptical cracks on the structural integrity of a tubular welded joint has been considered by evaluating stress intensity factors by a finite element method which uses three dimensional brick elements and a method involving elastic line springs used in conjunction with shell elements. The brick element model has been used to determine J under elastic-plastic conditions for two different strain hardening rates.

KEYWORDS

ELASTIC-PLASTIC; SEMI-ELLIPTICAL CRACK; TUBULAR WELDED JOINT

INTRODUCTION

The integrity of offshore oil structures is critically dependent on the behaviour of tubular welded joints which are subject to fatigue loading due to the action of the environment. Although design is usually based on an S-N approach, periodic inspection of the structures frequently reveals the presence of semi-elliptical cracks at sites of stress concentration near the chord-brace intersection. The development of such cracks under fatigue loading has been the object of experimental studies, by Dover and co-workers(1982) and Ritchie *et al.*(1987). To complement the experimental studies numerical analysis are performed by several methods, including, weight functions(Niu *et al.*,1986) ; line-springs(Rice *et al.*, 1972) used in conjunction with finite element shell analysis; and virtual crack extension (Parks,1974) used in conjunction with brick elements.

Although the development of the cracks by fatigue can be largely understood by reference to linear elastic fracture mechanics, it is necessary to determine the conditions under which cracks propagate under the overloads which arise in storm conditions. In order to ensure structural integrity under these conditions it is necessary to understand the elastic-plastic behaviour of tubular welded joints containing semi-elliptical defects.

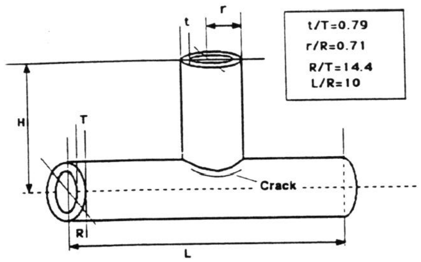
FINITE ELEMENT MODELS AND NUMERICAL METHOD

In the present work a tubular welded T-joint, the geometry of which is shown in Figure (1) has been examined. The tubular members were modelled with eight noded isoparametric doubly-curved shell elements while the critical region of the chord-brace intersection was initially modelled with twenty noded hybrid brick elements provided by the finite element code ABAQUS (1982). Compatability between the shell and brick elements was maintained by the use of transition elements with 18, 15 and 12 nodes produced by degenerating 20 noded hybrid brick elements using appropriate multi-point constraints.

Mesh generation was accomplished using commercial codes, the results of which were subsequently optimized for frontal solution , by renumbering the elements following the procedure given by Sloan and Randolph (1983). The models were subject to a uniform axial force on the brace while the ends of the chord were built in. Symmetry of the configuration, allows the structure to be represented by one symmetric quarter as shown in Fig.(2). The models involved approximately 12000 degrees of freedom. The problems were analysed on a Cyber 205 computer and required about 320 seconds cpu time,for an elastic solution. The same problem was also modelled using only eight noded doubly curved shell elements, while the semi-elliptical crack was represented by the linear elastic line spring concept of Rice and Levy(1972) as implemented in the ABAQUS(1982) finite element code. This formulation involved 4000 degrees of freedom and 56 cpu seconds. Further details of the elastic calculations and a comparison of the two calculation methods are given by Du and Hancock(1988).

In the present work attention is largely focussed on elastic-plastic behaviour using the formulation involving twenty noded hybrid brick elements although current work involves the use of non-linear line-springs. Full plasticity of the uncracked ligament was typically achieved in 50 increments each of which used about 4 iterations using a Newton method to obtain equilibrium with a Jacobian formed

from the elastic-plastic tangent stiffness and requiring a total cpu time of the order of 40,000 seconds.



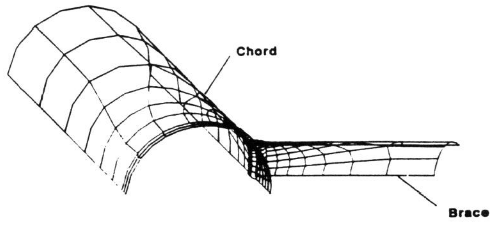


Fig1.) The geometry of the tubular welded joint.

Fig 2.) A finite element mesh using shell and brick elements.

The cracks were located at the site of maximum stress concentration which under axial loading is situated at the toe of the weld adjacent to the saddle point as shown schematically in Fig (1). The cracks are formed on the chord side of the chord-brace intersection and two crack geometries have been analysed .These consist of semi-elliptical cracks with a maximum depth to thickness ratio a/T of 0.6 and 0.9, and a surface length 2c/T=4, where T is the thickness of the chord. The three dimensional elements were arranged as a focused mesh with three rings of elements concentric with the crack tip . In the inner ring of elements the mid-side nodes were located at the quarter point positions : a procedure which allows the elements to adopt the correct form of displacement function for the elastic singularity, as discussed by Barsoum(1976) and Henshell and Shaw(1975) . The stress intensity factors were determined by evaluating the J integral around three crack tip contours using virtual crack extension (Parks, 1974) . Although J was largely path independent , the values obtained entirely from the second contour using the outer corner nodes of the second ring of elements were preferred. Experience with this contour shown to produce the most reliable data,for reasons that are "generally considered to be more fortuitous than fundamental".

The crack front was represented by four elements sets with boundaries orthogonal to the crack front thus providing nine sites at which J could be determined. Due to the curvature of the crack front, the final ring of elements at the intersection with the chord surface was somewhat distorted and the contour using the midside nodes of these elements has been consequently ignored.

As three dimensional elastic-plastic problems are demanding, both in terms of computer storage and cpu time,the number of rings of element sets concentric with the crack tip was necessarily limited to three. To assess the accuracy of the solutions preliminary bench-marking calculations were undertaken using a plane strain edge cracked bar with an (a/W) ratio of 0.5 and the same crack tip element configuration and material response as the tubular welded joint. J was determined by the virtual crack extension method of Parks(1970) as implemented in ABAQUS and the results compared with formulae proposed by Kumar and co-workers(1981) for tension. The results agree to within 0.3% for the elastic problem, while at an applied load equal to twice the non-hardening limit load, Po, the results agreed to within 4.5%. The results for the tubular joint problem in which the crack tip elements have a similar configuration, may be expected to have a similar accuracy which is considered to be acceptable for the current problem.

In all cases the uni-axial material behaviour for the purpose of the analysis was modelled by a Ramberg-Osgood relationship of the form:

$$\varepsilon/\varepsilon_0 = \sigma/\sigma_0+\alpha(\sigma/\sigma_0)^n \qquad (1)$$

Poisson's ratio was set at 0.3, α at 3/7, while the ratio of the yield stress σ_0 to the elastic modulus E was 0.001. The finite element analyses were based on small strain theory and employed the Prandtl-Reuss flow rule (J_2 plasticity). The incremental form of this relationship is,

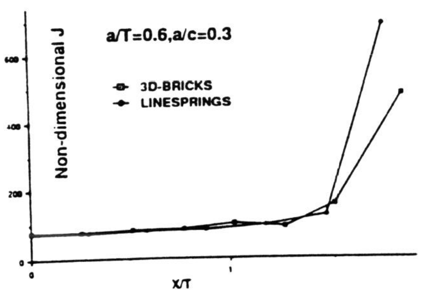
$$\delta\varepsilon_{ij}/\varepsilon_0=(1+\upsilon)d\sigma_{ij}/\sigma_0-\upsilon d\sigma_{kk}/\sigma_0\delta_{ij}+3/2n\alpha(\sigma_e/\sigma_0)^{n-2}s_{ij}/\sigma_0 d\sigma_e/\sigma_0 \qquad (2)$$

where s_{ij} are the stress deviators and σ_e is the equivalent stress.

RESULTS

The results of the elastic calculations are given in Figures (3) and (4) which show the non-dimensionalised J around the crack front for the two crack geometries (a/T=0.6 and a/T = 0.9) using the remote stress in the brace σ. The crack front position is defined by the distance X from the plane symmetry normalized by the chord

thickness T, the deepest point of the crack is thus located at the origin(X=0). The results of the line spring calculations are compared with the solutions using twenty noded bricks and agree to within 2.5 and 3.5% at the deepest point. The solutions even agree reasonably well near the ends of the crack where the physical basis of the line-spring method is less secure.



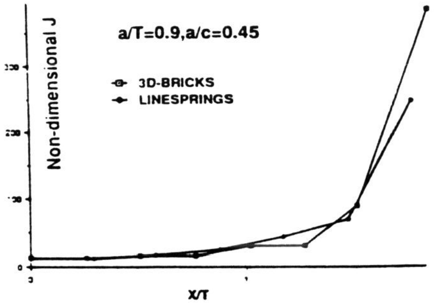


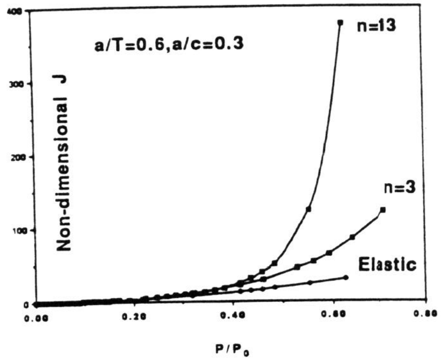
Fig 3.) Fig4.) A comparison of the non-dimensionalised $J=EJ/\sigma^2(1-\upsilon^2)a$ around the crack front from the line -spring and brick element calculations, where σ is the remote applied stress. X/T is defined as the normalized distance from the plane symmetry.

Under elastic-plastic conditions attention is focused on the deepest point of the crack where the development of J with applied load is given in Figures(5) and (6) for (n=3) and (n=13) for both crack depths. The result are presented in a non-dimensional form in which J is non-dimensionalized with respect to the yield stress σ_0 and the maximum crack depth a. The relations between force and the load point displacement v are shown in Fig(7) and (8).

The results of the elastic-plastic calculations are expressed, following convention, as the sum of elastic and plastic terms. The elastic results may be written in the form

$$Je = \sigma_0\varepsilon_0 af^e(a/T)(P/P_0)^2 \qquad (3)$$

where P is the applied force on the brace,and P_0 is a reference load



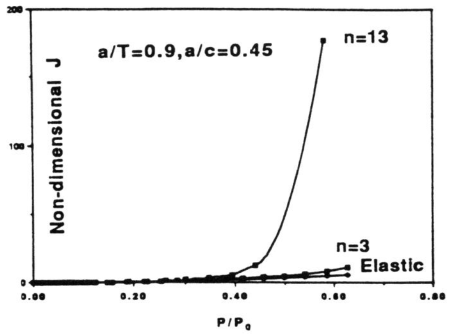
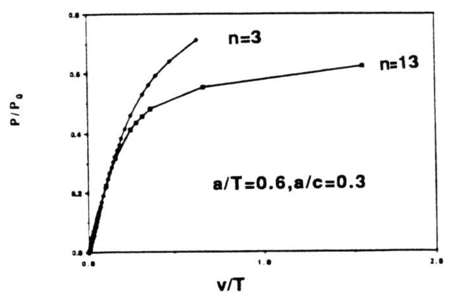


Fig.5) Fig.6) A comparision of the non-dimensionalised $J=EJ/\sigma_o^2(1-\upsilon^2)a$ at the deepest point



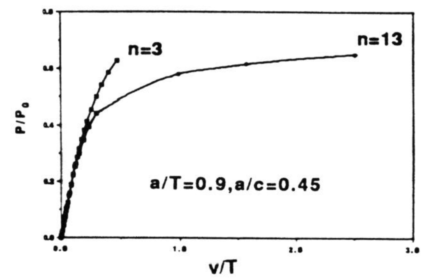


Fig.7) Fig.8) The relationship between the applied load and the load point displacement

which is usually taken to be the limit load for a perfectly plastic material. In the case of tubular welded joints it is convenient to take P_0 as the load to cause full plasticity of the brace remote from the joint i.e. $2\pi Rt\sigma_0$. J is written as the sum of elastic and plastic terms it is however usual to use Irwin's plastically adjusted crack length (a_e) to provide a smooth interpolation between the elastic and plastic terms

$$a_e = a + r_y * \varnothing \tag{4}$$

where

$$r_y = 1/6\pi^* \ (n-1)/(n+1)(K_I/\sigma_0)^2 \tag{5}$$

$$\varnothing = 1/(1+(P/P_0)^2) \tag{6}$$

and the total J is

$$J = J_{(a_e)} + J_p \tag{7}$$

The plastic component must adopt the dimensional form given by Goldman and Hutchinson (1975):

$$Jp = \alpha\sigma_0\varepsilon_0 a f^p(a/T,n)(P/P_0)^{n+1} \tag{8}$$

Calculations with strain hardening exponents n=3 and 13 show that the plastic component Jp is indeed proportional to P^{n+1} and the corresponding values of the geometric functions f^p and f^e are given in table (1)

Table 1. f^p and f^e as a function of a/T and n

	a/T=0.6	a/T=0.9
n=1	70.3	12.4
n=3	1080.1	138.3
n=13	3.8E6	8.86E5

Discussion

A feature of the stress intensity factors of semi-elliptical cracks in tubular welded joints is that the stress intensity factor does not increase markedly with crack length.(Dover *at al.*, 1982; Ritchie *et al.*, 1987, Huang and Hancock, 1988). In fact, in the present case at constant applied load the stress intensity factor decreases with crack length over the range (a/T>.0.6). Under elastic-plastic conditions, with a constant applied load, the tearing modulus (dJ/da)/E is also negative implying that an increasing load history is required to maintain elastic-plastic growth at the deepest point, for cracks of depth greater than (a/T=0.6). As the crack develops in the through thickness direction the loads to produce a given remote displacement are similar for both a/T=0.6 and 0.9 under elastic-plastic conditions, and this parallels the elastic observation(Huang and Hancock, 1988) that the stiffness of the joint is maintained until cracks penetrate the chord wall,when it susequently undergoes a rapid decay. As the joint stiffness is maintained until the chord is penetrated, constant remote loading and displacement conditions produce similar results, and the tearing modulus will be negative for both cases favouring stable crack growth in the through thickness direction.

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