Effect of Material Structure on the Non-linear Fracture Parameters of Aggregative Concrete

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ABSTRACT

Comparisons of experimental data on the effect of material structure on the composite fracture properties of concrete with a recently developed theoretical model is presented. The meso-scale theoretical model of tension-softening behavior is based on the initiation and spreading of a dominant crack from the largest aggregate particle, subsequently intersecting other aggregates in its path, followed by complete material separation of the eventual crack plane by frictional pull-out of the aggregates. The limited but favorable comparisons between data and the theoretical predictions suggest the usefulness for further investigation into this preliminary model.

KEYWORDS

Model, tension-softening, fracture energy, concrete, aggregate volume fraction, aggregate size.

INTRODUCTION

It has become increasingly clear that the post-peak tensile behavior of concrete plays an important role in controlling the deformation behavior of concrete structural members. Recent computational mechanics studies of concrete structures (e.g. Hillerborg et al, 1976, Ingraffea and Gerstle, 1985) have used such post-peak behavior, often termed the tension-softening curve, as a constitutive relation for describing the fracture process zone, in a concrete structure loaded to failure. The tension-softening curve may be regarded as a non-linear fracture 'parameter', and the area under this curve has been shown through a J-integral analysis (Rice, 1968) to be equal to the fracture energy of concrete. Use of the tension-softening curve in cohesive crack models has already led to explanations of size-effect of concrete structural members and possible concrete structure design-code revisions (Hillerborg, 1986, 1988).

Several pieces of recent work (Mihashi, 1988, Wittmann et al, 1988, Cornelissen et al, 1986) have focused on experimental studies of the effect of varying the internal structure, specifically the aggregate size, volume fraction, and water/cement ratio, on the fracture properties of concrete. In this paper, a simple theoretical model on the meso-scale (Huang and Li, 1988) is briefly reviewed. The model predictions in relation to experimental results are discussed. In spite of the highly idealized nature of the theoretical model, most of the experimental results to date can be explained and interpreted in the context of this theoretical model.

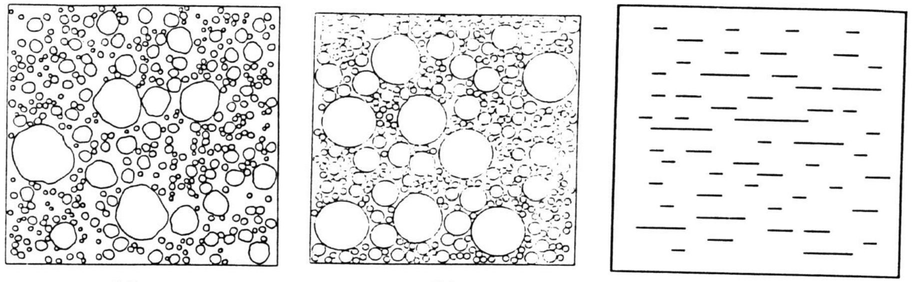
A MODEL FOR TENSILE STRENGTH

Concrete is modelled as a solid with 'meso' cracks of random location. These cracks arise because of interfacial weaknesses between aggregates and the cement matrix. In concrete, the aggregate size is usually graded, and the Fuller distribution is a commonly adopted one. In the present model, the initial size of the cracks at peak load is assumed to be equal to the aggregate size, and the mesocrack size distribution is therefore also described by the Fuller curve (Figure 1). In fact prior to peak load, stable propagation of the interfacial cracks around the aggregates could occur, leading to pre-peak inelastic deformation (Huang and Li, 1988). The present model assumes that the tensile strength coincides with the branching of the largest interfacial crack into the cement matrix. Propagation of this dominant crack then leads to formation of the eventual fracture plane. Thus

$$f_t = \alpha \frac{K_{IC}^{eff}}{\sqrt{\pi R_{max}}} \quad (1)$$

in which α is a correction factor introduced to account for the interaction of the dominant crack with neighboring interfacial cracks at peak load, to be discussed later. The effective toughness K_{Ic}^{eff} is the cement matrix toughness K_{Ic}^{m} modified by the factors f_1 and f_2

$$K_{Ic}^{eff} = K_{Ic}^{m} f_1(V_f) f_2(V_f) \quad (2)$$



(a) (b) (c)

Fig 1 Simulated structure of concrete with (a) aggregates having random geometry, (b) aggregate size distribution following Fuller curve, (c) crack distribution at tensile strength.

f_1 and f_2 have been introduced to account for the interaction of the propagating dominant crack with intersecting aggregates and to account for the shielding effect on the dominant crack due to the presence of other presumably arrested interfacial cracks.

To account for the deflection of the dominant crack due to aggregate interception, the model of Faber et al (1983) was applied. Extension of the deflected portion of the crack was considered to be governed by the requirement that the reduced stress intensity factor at the deflected crack tip reaches the matrix toughness. Since the dominant crack can intercept an aggregate at any position, the stress intensity factor averaged over all possible deflecting angles was calculated based on a probabilistic analysis. The result is

$$f_1 = \sqrt{1.0 + 0.87\, V_f} \quad (3)$$

where V_f is the volume fraction of aggregate. For V_f = 0.1, 0.3, 0.5, and 0.7, f_1 = 1.04, 1.12, 1.20 and 1.27.

The formation and propagation of a dominant crack implies a stress relaxation and possibly closure of interfacial cracks in the material remote from the dominant crack. However, in the zone of material ahead of the dominant crack, the high tensile stress there would keep the interfacial cracks open, resulting in a material with Young's modulus effectively smaller than that of the original uncracked material. By considering the path independence of the J-integral, Evans and Faber (1983) showed that the dominant crack tip stress intensity factor is lowered by the reduced local modulus, resulting in a toughening effect. Hence

$$f_2 = \sqrt{E_m(1-\nu)/E(1-\nu_m)} \quad (4)$$

where E_m is the Young's modulus for the material devoid of cracks, E is the modulus with interfacial cracks, and ν, ν_m are the corresponding Poisson's ratios. By using the self-consistent technique employed by Budiansky and O'Connell (1976) in treating the distributed cracked body, Huang and Li (1988) found

$$\frac{E}{E_m} = 1 - \frac{\pi^2}{16}(1-\nu^2)\, V_f \quad (5)$$

Substitution of (5) into (4), assuming $\nu \approx \nu_m$, results in the toughness increase:

$$f_2 = \sqrt{\frac{1}{1 - \frac{\pi^2}{16} V_f (1-\nu^2)}} \quad (6)$$

For ν = 0.2, and V_f = 0.1, 0.3, 0.5, and 0.7, f_2 = 1.03, 1.1, 1.19, and 1.30.

The stress intensity factor (SIF) of the dominant crack when it propagates into the matrix may be expected to be raised as a result of interaction with neighboring interfacial cracks. Effectively, the tensile strength is reduced by the factor α in (1).

The interaction problem is simplified as a dominant crack of length $2R_{max}$ interacting with all other cracks of average length $2R_{avg}$. For Fuller distribution, $R_{avg}=1/3R_{max}$. The stress field generated by the average length crack at the dominant crack face is evaluated, which gives an additional contribution to the SIF at the dominant crack tip. By adding this part of SIF to the one associated with the ambient tensile stress, the increase in SIF at the dominant crack tip is calculated, so that the factor α in (1) is given by

$$\alpha = \frac{1 - f(a,b)f(b,a)}{1 + f(a,b)} \qquad (7)$$

and

$$f(a,b) = \frac{1}{2a}\left(\sqrt{(r+a)^2 - b^2} - \sqrt{(r-a)^2 - b^2} - 2a\right)$$

where $a=R_{max}$, $b=R_{avg}$, $r=R_{max}+[\sqrt{(\pi/V_f)}-1]R_{avg}$.

A MODEL FOR POST-PEAK TENSION-SOFTENING BEHAVIOR

The post-peak behavior of concrete is assumed to have two main failure mechanisms: a crack-like deformation controlled mode and an aggregate frictional pull-out mechanism.

At the beginning of the post-peak stage, the progressive formation of the failure plane is modelled as the expansion of the dominant crack, and the crack opening displacement, δ, is equated to the average opening contribution of this dominant crack. The propagation of this crack within the matrix is governed by the equation:

$$\sigma\sqrt{\pi(L + R_{max})} = K_{IC}^{eff} \qquad (8)$$

where σ is the ambient tensile stress, L is the length of crack in the cement matrix. Each time the dominant crack intercepts an aggregate, it may join with the interfacial crack at the aggregate interface, leading to a jump of dominant crack length by aggregate diameter. Including the jumping effect and relating average opening δ to the crack length L, the resulting tension-softening curve is obtained:

$$\delta = \frac{(K_{IC}^{eff})^2(1-\nu^2)}{E(1-V_f)\, f_t}\frac{1}{(\frac{\sigma}{f_t})}\left[1 - (\frac{\sigma}{f_t})^3\right] \qquad (9)$$

In the final stage of an uniaxial tension failure, a macroscopic fracture plane forms through the cross-section of the specimen. The coarse aggregate particles are subsequently extracted from either face of the fracture plane. In the model, a material parameter, η, is introduced, which defines the roughness of the aggregate surface. We envision that the aggregate can be completely pulled out when the roughness on the aggregate surface is being sheared off. By

geometrical analysis, the critical opening δ_c is related to η and the maximum aggregate size:

$$\delta_c = \sqrt{2R_{max}\,\eta} \qquad (10)$$

Assuming that the cement matrix is very brittle so that the load is carried by the frictional force at the interface of the aggregate at the pull-out stage, and considering the equilibrium condition, the relation between stress σ and crack opening δ is obtained:

$$\frac{\sigma}{f_t} = \frac{3\,V_f}{2R_{max}}\,\delta_c\,\frac{\tau^*}{f_t}\,(1 - \frac{\delta}{\delta_c}) \qquad (11)$$

where τ^* is the interfacial shear strength. The combination of the crack-like deformation controlled stage and the frictional pull-out stage gives the complete tension-softening curve. The fracture energy G_c may be obtained by integrating the area under the σ–δ curve.

COMPARISONS OF MODEL PREDICTIONS WITH EXPERIMENTAL DATA

Based on the model presented above, it is possible to predict the tension-softening curve of concrete. Figures 2a and 2b shows two such predictions using model parameters consistent with the actual values of the particular concretes from which experimental data have been obtained. The only uncertain parameter is the matrix toughness, which was not reported as part of the experiments, and a typical range of K_{Ic}^m=0.25 to 0.35 MPa $\sqrt{m}$ was used. The two concretes having different values of Young's modulus E produce different predicted tension-softening curves. The comparisons between the model predicted and experimentally deduced tension-softening curves appear quite satisfactory.

The model predicts that concrete strength will moderately increase with aggregate volume fraction when R_{max} is fixed. This is shown in Fig 3 together with experimental data from Stock et al (1979). The present model is not appropriate for predicting the strength of plain cement (at $V_f = 0$). At high volume fraction (V_f>0.78), the interaction analysis predicts α --> 0 due to cracks intersecting each other, so that the basic assumption of a dominant crack propagating into a matrix breaks down.

There have been several studies of the fracture energy of concrete with different maximum aggregate size $D_{max} = 2\,R_{max}$. Figure 4 shows a collection of such data (Hillerborg, 1984, Mihashi, 1988) and a trend of increasing G_c with D_{max} indicated by the linear regression line. It should be noted that D_{max} and V_f are not independent quantities for concrete, but are related by design rules to maintain workability in the fresh state. The model predicted trends are also presented, for three values of matrix toughness.

The value of critical opening δ_c was examined as a function of maximum aggregate size D_{max} by Mihashi (1988) experimentally. His data is plotted in Fig 5 normalized against the δ_c value for the concrete with D_{max} = 32 mm. The model prediction is also shown. In the normalized form, the unknown roughness parameter η is not needed in the calculation.

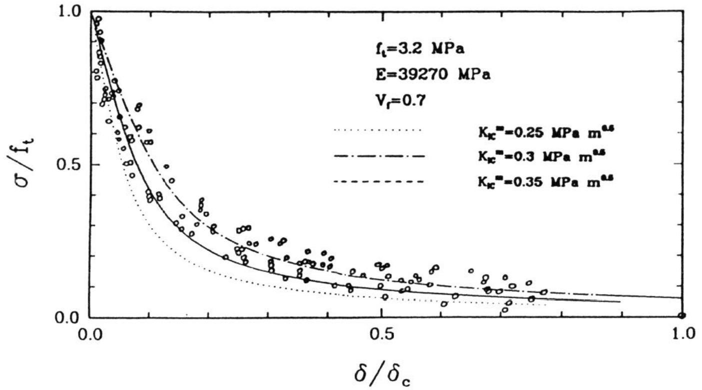


Fig 2a Comparison of predicted tension softening curves with data from Cornelissen (1986).

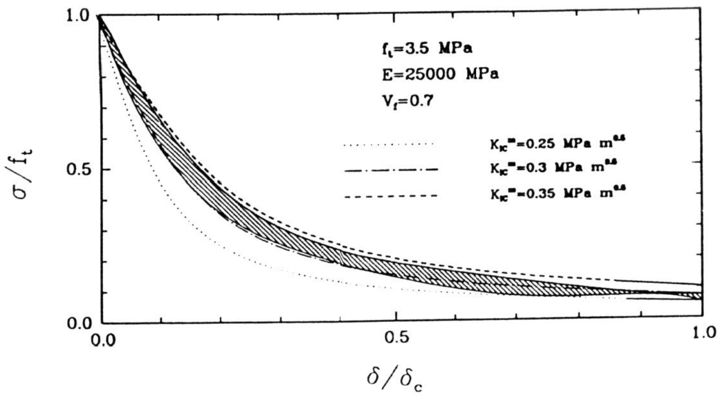


Fig 2b Comparison of predicted tension softening curve with data from Petersson (1981).

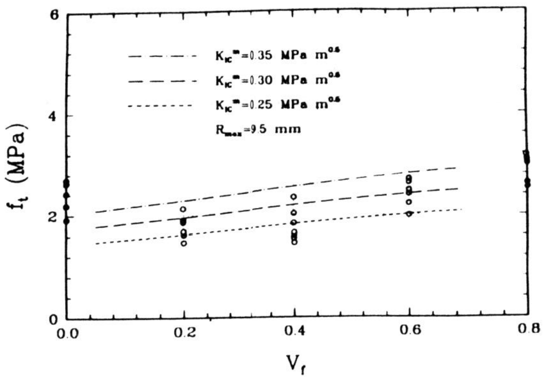


Fig 3 V_f-dependence of predicted tensile strength and data from Stock et al (1979).

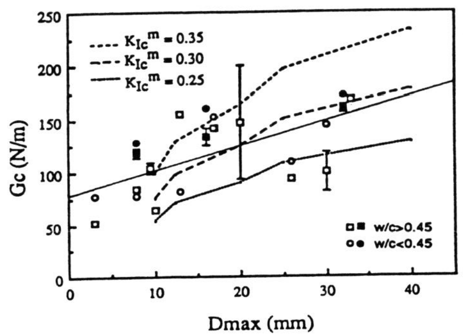


Fig 4 D_{max}-dependence of Gc. Solid symbol data from Mihashi (1988). Open symbol data from Hillerborg (1984).

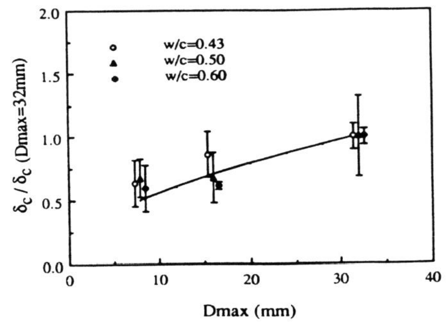


Fig 5 D_{max}-dependence of δ_c. Data from Mihashi (1988).

It may be expected that the cement matrix toughness will increase with decreasing w/c ratio, and with age. Such is indicated in an experimental test of notched 3-point bent cement beams (Figure 6, Zielinski, 1983) . However, the fracture toughness of pure cement and that of cement as a matrix in a concrete may not be the same, especially for the cement matrix in the neighborhood of an aggregate. For this reason, the information provided in Fig 6 can only be used in our model with the understanding that it represents an approximation of the actual cement matrix toughness.

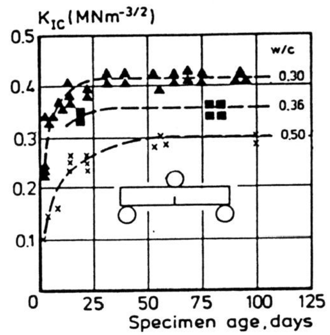
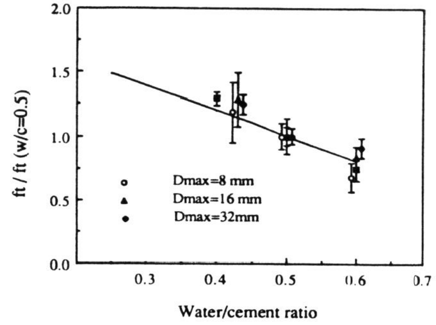


Fig 6 Fracture toughness of cement paste (Zielinski, 1983).

Wittmann et al (1987) studied the effect of w/c on the tensile strength of concrete with different aggregate size. Their data, in normalized form, is presented in Fig 7, together with model prediction based on (1), in which the matrix toughness K_{Ic}^m is regarded as a function of w/c, as extrapolated from the data in Fig 6. Wittmann et al (1987) also studied the effect of concrete fracture energy as a function of w/c ratio, and his normalized data is shown in Fig 8. Model prediction is overlaid on the same figure.



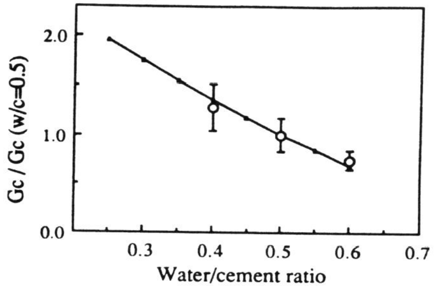


Fig 7 w/c-dependence of f_t. Open symbol data from Mihashi (1988), x-symbol data from Wittmann (1987).

Fig 8 w/c-dependence of Gc. Data from Wittmann (1987).

CONCLUSIONS

The composite tensile strength, post-peak tension-softening behavior, fracture energy and critical separation of concrete are related to the aggregate volume fraction, maximum aggregate size, cement matrix toughness, and indirectly to the w/c ratio, through a theoretical model of a crack spreading from the interface of the largest aggregate into a heterogeneous matrix. Comparisons with experimental data on such fracture properties using concrete with variable material structures provide encouraging support, even though the model includes several critical simplifying assumptions which may require further verification. As an example, the present model assumes that the interface between aggregate and matrix is the weak link, and the dominant crack always propagate from the largest aggregate. An experimental program is now underway to examine the validity of such assumptions.

The present model may be useful in describing the fracture behavior of other brittle composites containing second phase particles with weak interfaces.

REFERENCES

Budiansky, B. and R.J. O'Connell (1976), Elastic moduli of a cracked solid. *Int. J. Solids Structures,* 12, 81-97.

Cornelissen, H.A.W., D.A. Hordijk, and H.W. Reinhardt (1986). Experiments and theory for the application of fracture mechanics to normal and lightweight concrete. In: *Fracture Toughness and Fracture Energy of Concrete*, (F.H. Wittmann, ed), pp. 565-575.

Evans, A.G., and Faber, K.T. (1983). On the crack growth resistance of microcracking brittle materials. In: *Fracture in Ceramic Materials* (Evans, ed.) pp.109-136.

Faber, K.T., A.G. Evans, and M.D. Drory (1983). A statistical analysis of crack deflection as a toughening mechanism in ceramic materials. In: *Fracture Mechanics of Ceramics,* 6, (Bradt, ed.), pp.77-91.

Hillerborg, A., M. Modeer, and P-E. Petersson (1976). Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cement and Concrete Research,* 6, 773-782.

Hillerborg, A. (1984). Additional concrete fracture energy tests performed by 6 laboratories according to a draft RILEM recommendation. Report to RILEM TC50-FMC Div of Building Materials, Lund Institute of Technology, Sweden.

Hillerborg, A. (1988). Fracture mechanics and the concrete code. To appear in: *Application of Fracture Mechanics to Cementitious Materials and Concrete Structures* (Li and Bazant, ed.) American Concrete Institute.

Huang, J. and V.C. Li (1988). A Meso-mechanical model of the tensile behavior of concrete, Part I: modelling of pre-peak stress-strain relation. Submitted to *Composites*.

Huang, J. and V.C. Li (1988). A Meso-mechanical model of the tensile behavior of concrete, Part II: modelling of post-peak tension-softening behavior. Submitted to *Composites*.

Ingraffea, A.R. and W. H. Gerstle (1985). Nonlinear fracture models for discrete crack propagation. In: *Application of Fracture Mechanics to Cementitious Composites* (Shah, ed.) 247-286.

Mihashi, H., N. Nomura and M. Izumi (1988). Influence of internal structure on strain softening and fracture energy of concrete. Proceedings of the International Workshop on *Fracture Toughness and Fracture Energy - Test Methods for Concrete and Rock.* Sendai, Japan.

Petersson, P.E. (1981). Crack growth and development of fracture zones in plain concrete and similar materials. Report TVBM-1006 Div. of Building Materials, Lund Institute of Technology, Sweden.

Rice, J.R. (1968). Mathematical analysis in the mechanics of fracture. In: *Fracture: An Advanced Treatise P* (Liebowitz, ed.), Vol. 2pp.191-311, Academic Press.

Stock, A.F.D., J. Hannant, and R.I.T. Williams (1979). The effect of aggregate concentration upon the strength and modulus of elasticity of concrete. *Mag Concr Res* 31, 225-234.

Wittmann, F.H., P.E. Roelfstra, H. Mihashi, Y. Huang, X. Zhang and N. Nomura (1987). Influence of age of loading, water-cement ratio and rate of loading on the fracture energy of concrete, *Materials and Structures*, 20, 103-110.

Wittmann, F.H., K. Rokugo, E. Bruhwiler, H. Mihashi, and Ph. Simonin (1988). Fracture energy and strain softening of concrete as determined by means of compact tension specimens. *Materials and Structures*, 21, 21-32.

Zielinski, A.J. (1983). Behavior of concrete at high rates of tensile loading: A theoretical and experimental approach. Report 5-83-5 Stevin Lab, Delft U. of Techn.