

Dynamic Fracture Toughness Measurement Methods for Brittle and Ductile Materials

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ABSTRACT

After a brief review of several current test methods for measuring dynamic fracture toughness, two methods are described in greater detail, namely, stress wave loading of a circumferentially notched round bar (fatigued precracked) and the dynamically loaded three-point-bend bar. The time to fracture, measured from the instant of the arrival of the stress wave at the crack front, is about 20 to 25 μs for the notched round bar and ranges from 300 μs to milliseconds for the three-point-bend bar. Full field finite element simulation of both experiments have been carried out. The computational investigation showed that the measurements made in both experiments can be interpreted within the framework of fracture mechanics. These studies have also provided information on the test conditions and the specimen size requirements which ensure that the J -integral properly characterizes the near tip state at the onset of fracture. These aspects and procedures for the determination of the dynamic fracture toughness J_{IC} using the measured load-displacement record are discussed.

KEYWORDS

nonlinear fracture mechanics; dynamic fracture; fracture toughness; strain rate effects.

1. INTRODUCTION

Many loading conditions occur in practice which cannot be treated as quasi-static and in these cases, the inertia of the material as well as its rate sensitivity must be taken into account. A potential danger arises in the use of materials which exhibit high sensitivity to loading rate because this behavior can lead to a significant reduction in fracture toughness and thus to unexpected failures in service under dynamic loading.

Fracture initiation resulting from the application of dynamic loads has received considerable attention in recent years. Reviews of recent work have been presented, for instance, by Kalthoff (1985) and Shockey (1985). The latter describes techniques ranging from crack arrest, through Charpy, all the way to the plate-impact fracture test. At medium loading rates,

some of the early important contributions to this field of research are due to Krafft (1964) who tested mild and high strength steel specimens over a range of temperatures and loading rates. Rapid loading was achieved through the use of a hydraulic testing machine developed at the Naval Research Laboratory. Krafft was able to obtain a correlation between the plane strain fracture toughness and the strain hardening characteristics of a variety of steels. Eftis and Krafft (1965) extended this work in tests on mild steel plates taken from the specimens used in the University of Illinois crack propagation studies. These investigators were able to determine the critical values of the stress intensity factor, K_{IC} , for crack tip loading rates of up to about $\dot{K}_I = 10^3 \text{ MPa}\sqrt{\text{m}}/\text{s}$. Other investigators have used various rapid loading machine techniques to obtain dynamic fracture initiation data on a variety of materials. However, machine testing techniques are limited to maximum loading rates on the order of $\dot{K}_I = 10^3 \text{ MPa}\sqrt{\text{m}}/\text{s}$ which is still quite low when compared with loading rates induced by stress waves.

A somewhat different approach to dynamic testing is the Charpy test which, because of simplicity and low cost, is perhaps the most widely used test for measuring dynamic fracture toughness properties. However, the energy measurements resulting from the standard Charpy test are somewhat difficult to interpret in the framework of elastic or elastic-plastic fracture mechanics and thus it is of greater value in giving relative differences between specimens rather than in providing useful data to apply to current fracture theories. The instrumented Charpy test provides more valuable data, and several empirical correlations have been formulated to interpret Charpy data in terms of elastic-plastic fracture mechanics. Loading times of duration ranging from 10 to 50 μs have been obtained in Charpy tests (e.g., Kobayashi et al., 1986) resulting in loading rates one to two orders of magnitude faster than those achievable in dynamic machine tests. Ruiz and Mines (1985) have modified the instrumented Charpy test so that the load is applied via a Hopkinson pressure bar which then allows a measurement of the load transmitted to the Charpy specimen as a function of time. Nevertheless, plane strain conditions do not prevail along the crack front, and this is a major limitation of the instrumented Charpy test.

Two experimental procedures which are capable of measuring the dynamic fracture toughness of structural metals at stress intensity rates of the order of $10^6 \text{ MPa}\sqrt{\text{m}}/\text{s}$ are reviewed in this paper. These two techniques are those with which the authors have been associated, namely, stress-wave loading of a notched round bar (Costin et al, 1977) and impact loading of a three-point-bend ductile fracture specimen (Nakamura et al., 1986). Comparable rates have been achieved by Corran et al. (1983) and by Klepaczko and Solecki (1984). In their investigations a compact tension specimen is loaded by a compressional split-Hopkinson pressure bar. Both investigations use wedge-loading, but Corran et al. load the specimen through a pair of pins, while Klepaczko and Solecki use the wedge directly to separate the two arms of the specimen. Ravi-Chandar and Knauss (1984) developed a method which applies pressure loading to the opposing crack faces of a machined slit in a large plate. A conducting strip is inserted directly between the parallel machined faces of the crack and then a capacitor-inductor circuit is discharged which applies a sudden direct pressure on the faces of the crack to give \dot{K}_I of about $10^5 \text{ MPa}\sqrt{\text{m}}/\text{s}$. Even higher loading rates have been achieved in the plate impact experiment developed by Ravichandran and Clifton (1988). The specimen is a flat disc with a mid-plane crack grown in halfway across the disc. A sharp crack front is obtained by cyclic loading at low stress levels. A flyer plate strikes the front face of the specimen and the impact causes a sharp compressive pulse that propagates through the thickness of the disc and is reflected at the rear face as a tensile pulse. The tensile pulse, in turn, loads the crack front at a rate \dot{K}_I of about $10^8 \text{ MPa}\sqrt{\text{m}}/\text{s}$. The motion of the specimen's rear face is monitored by laser interferometry. The rear face motion is used to calculate the propagation of the crack.

2. DYNAMIC LOADING OF A NOTCHED ROUND BAR

The experiment described is, in part, an adaptation of the Kolsky pressure bar to dynamic fracture testing. However, the technique is modified to allow for rapid tensile loading of a fatigued pre-cracked round bar. The specimens are in the form of solid bars one inch in diameter, with a circumferentially fatigued pre-crack (see Fig. 1). A tensile force (caused by an explosive) is applied at one end of the bar, generating a tensile stress pulse which propagates down the bar and loads the precracked section to fracture. The elapsed time between arrival of the pulse at the pre-cracked section and onset of crack growth is approximately 20 to 25 microseconds, corresponding to values of \dot{K} in excess of $10^6 \text{ MPa}\sqrt{\text{m}}/\text{s}$ — nearly three orders of magnitude faster than that achieved in dynamic machine testing. An accurate measure of the average stress on the uncracked ligament as a function of time is provided by the portion of the pulse transmitted through the ligament. Crack mouth opening displacement as a function of time is measured using an optical system. The elimination of time between the load and displacement measurements provides a load-displacement record from the onset of loading to fracture initiation. In summary, each test provides a record of load versus time and of crack opening displacement versus time which can be combined to yield a complete load-displacement record. Fracture toughness, K_{IC} for brittle materials, and the critical value of the J-integral, J_{IC} , for ductile materials, can be determined directly from the experimental data.

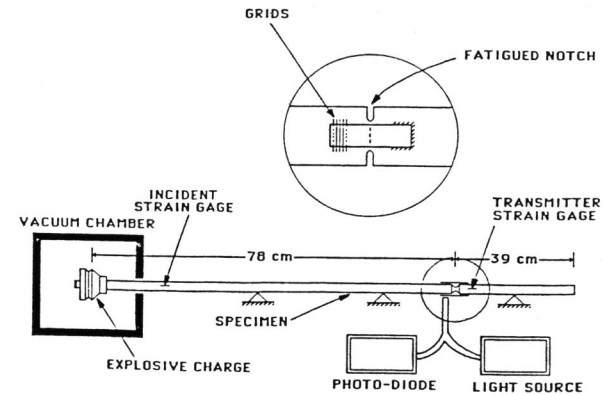


Fig. 1. Schematic diagram of the apparatus employed for the dynamic fracture toughness experiments

This technique has the advantage of achieving extremely high loading rates, while providing a load-displacement record which is directly comparable with the corresponding record obtained in a static test on a similar specimen. The effects of strain rates (at stress wave loading) on fracture toughness are readily investigated by using the method. In each test the data recorded are in the form of oscilloscope traces. A common zero time point is established and an applied load versus crack opening displacement curve is generated. Having determined the load-displacement curve, the fracture initiation parameters can be calculated. For a nominally brittle material the plane strain fracture toughness K_{IC} is determined in the following manner. For the notched round bar, K_I is given by (Tada et al., 1973)

$$K = \frac{P}{\pi R^2} \sqrt{\pi R} F(2R/D) \quad (2.1)$$

where R is the radius of the uncracked ligament, P is the applied load, D is the outer diameter of the bar, and $F(2R/D)$ is a dimensionless size function (F is about 0.48 for the relative crack depth used in the experiments). The load P for use in (2.1) is determined from the load-displacement curve in accordance with ASTM standards by using the 5 percent slope offset procedure. In order to apply linear fracture mechanics concepts, the size of the crack tip plastic zone must be small compared to the nominal dimensions of the specimen. For the geometry under discussion, the size requirement for a valid K_{IC} test is

$$R \geq 2.5(K_{IC}/\sigma_0)^2 \quad (2.2)$$

where σ_0 is the flow stress of the material determined at a strain rate comparable to the strain rate achieved near the crack tip region during the fracture test. The required dynamic flow stresses were determined in shear by means of the Kolsky bar technique, as described by Duffy et al. (1971). The specimens for the shear experiment are obtained from the fracture specimens.

When testing more ductile materials, the plastic zone extends over a substantial fraction of the ligament so that (2.2) is not satisfied. In these cases a J-integral approach has been adopted. The relevant size requirement for a valid J_{IC} test is (see review article by Hutchinson, 1983)

$$R \geq 50J_{IC}/\sigma_0 \quad (2.3)$$

The size requirement (2.3) is at least an order of magnitude smaller than that required by (2.2). The specimens employed in the experiments satisfy the size requirement (2.3) for a valid J_{IC} test. For a deeply cracked round bar, the value of the J can be determined from a load-displacement record according to (Rice et al., 1973),

$$J = \frac{1}{2\pi R^2} \left(3 \int_0^{\delta_c} P d\delta_c - P\delta_c \right) \quad (2.4)$$

Here δ_c is the load-point displacement due to the crack and may be approximated by the crack mouth opening displacement. With J_{IC} thus obtained, the equivalent K_{IC} is given by the established relation

$$K_{IC}^2 = \frac{E}{1-\nu^2} J_{IC} \quad (2.5)$$

where E is the elastic modulus and ν is Poisson's ratio. With this value in hand, the loading rate in the dynamic fracture test is

$$\dot{K}_I = K_{IC}/t_C \quad (2.6)$$

where t_C is the time to fracture initiation measured from the instant of the wave's arrival at the crack front.

Clearly the analysis in the above experiment involves a number of assumptions or simplifications which required further study before results could be employed with confidence. A full-field finite element computational simulation of the experiment has been carried out by Nakamura, Shih and Freund (1985). The analysis confirmed that the data are interpretable within the framework of nonlinear fracture mechanics and that measurement method for J_{IC} is accurate. Nakamura et al. noted that the measurement method for J based on (2.4) can underestimate the dynamic fracture toughness by as much as 20 percent. To ensure greater accuracy they recommended that the relative crack depth be greater than about 0.7. The detailed deformation and stress fields obtained in the computational simulation support the assumptions made in the experiments — that the incident (and reflected) pulse and the pulse transmitted through the ligament can be accurately inferred from surface strain measurements taken at a distance of one bar diameter upstream and one bar diameter downstream from the crack plane respectively. In other words, the analysis confirms that the location of

the strain gauges on the test specimens used in the various investigations were appropriate for measuring the transmitted pulse in the notched round bar experiment.

Dynamic fracture initiation by stress wave loading with a circumferentially notched round bar has been employed in a number of experiments to test a variety of materials. The first extensive use was by Costin and Duffy (1979) but since then it has been employed in a detailed study of the effects of microstructure on fracture initiation in a plain carbon steel. The ferrite and prior austenite grain sizes were varied systematically by Couque et al. (1988). Quasi-static and dynamic values of the critical stress intensity factor were determined over the range of temperatures from -150°C to $+150^\circ\text{C}$ thus covering both lower shelf and upper shelf behavior. Other steels have been tested as well, as for instance a low alloy structural steel by Cho et al. (1988) and an AISI 4340 VAR steel by Chi et al. (1988) (see Fig. 2). The technique has also been applied to determine the dynamic fracture behavior of a 2124-T6 aluminum reinforced with 13.2 v/o SiC whiskers by Marchand et al. (1988). Ceramics and ceramic matrix composites have also been tested by making use of a modified version of this technique. Duffy, Suresh, Cho and Bopp (1987) showed that the dynamic fracture toughness is higher than the fracture toughness obtained by quasi-static experiments.

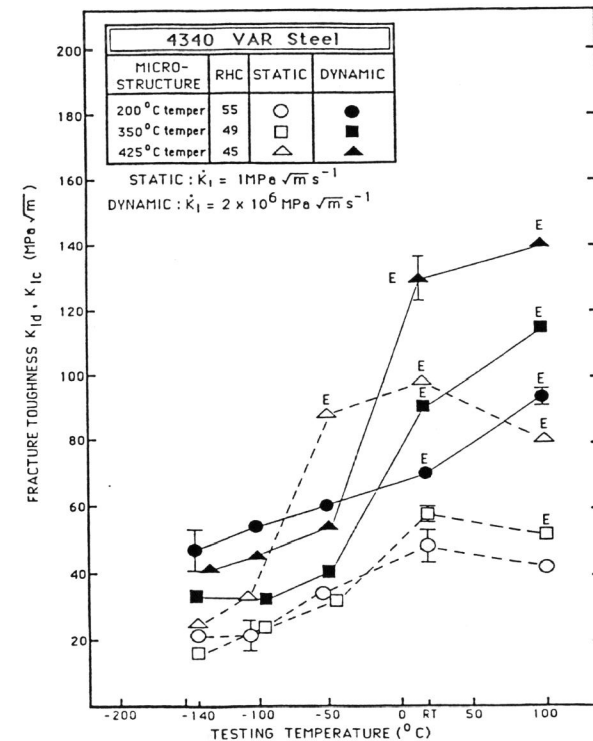


Fig. 2. Fracture toughness as a function of testing temperature for three different temperers of 4340 VAR steel. The letter E identifies those tests in which the ASTM criterion for a valid K_{IC} is not met and an equivalent K_{IC} value is calculated from J_{IC} .

3. DYNAMICALLY LOADED THREE-POINT-BEND BAR

The dynamically loaded three-point-bend bar has been proposed by Joyce and Hackett (1984,1989) for the measurement of the dynamic fracture toughness J_{IC} of ductile materials. The specimen is rapidly loaded by means of a concentrated transverse force applied at mid-span on the uncracked surface of the specimen. Fracture initiation occurs after substantial plastic deformation has developed in the uncracked ligament. They proposed that the value of J at the onset of fracture can be estimated from a deep-crack formula based on measured load-deflection record or from an appropriate quasi-static calibration of the specimen. Nakamura, Shih and Freund (1986,1989) have analyzed the complete transient response of the specimen for a range of loading rates using two-dimensional and three-dimensional full field finite element calculations. They introduced a transition time t_T to provide an estimate of the time beyond application of the loading at which a J -dominated field is established in the crack tip region and a deep-crack J -formula is applicable. They also noted that the proposed measurement techniques for J_{IC} to be very accurate when the time to fracture is larger than $2t_T$.

A schematic of the specimen is shown in Fig. 3. The specimen has a through-thickness planar crack of length a (the crack front being parallel to the x_3 -axis) and is supported by rollers separated by a distance of $2l$. A load P is applied on the surface $x_1 = 0$ at the center span. If the loading is applied very rapidly, then some time must elapse before the resulting sharp fronted waves are dispersed due to geometrical dispersion and material dissipation. It is clear that during the early stages of a dynamic loading process, the near tip region is not dominated by a J -controlled nonlinear field and therefore any estimate of J based on remote loads and/or displacements has no relevance to the near tip fields.

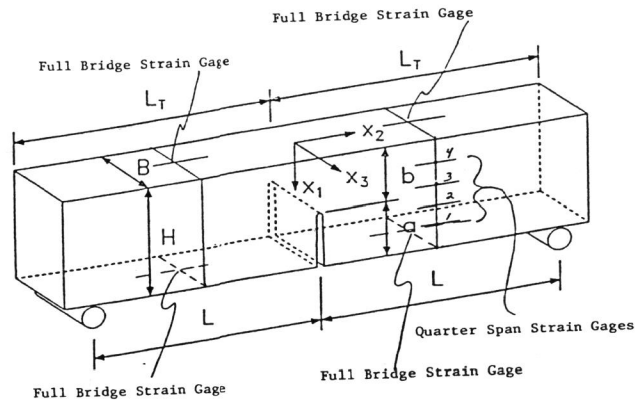


Fig. 3. Schematic of the dynamically loaded three-point-bend specimen showing placement of the strain gauges.

3.1 TRANSITION TIME

The J estimation procedure employed by Joyce and Hackett (1984,1989) cannot take into account discrete waves as they travel across the section of the specimen, but it can be modified to account for inertial effects at the structural level. Nakamura et al. (1986) observed that the response of the specimen can be conveniently characterized by a short-time response

dominated by discrete waves and a long-time response dominated by deformation energy (or more appropriately, the stress work in the case of an elastic-plastic material). At the transition between short-time and long-time response, structural inertial effects are important. At longer times inertial effects diminish relative to the overall energy absorbed by the body. To distinguish the short-time response from the long-time response, the transition time concept was introduced. The transition time provides a practical bound on the time range over which a J -dominated near tip field is established and an appropriately modified deep-crack formula (which has its origin in quasi-static loading) is applicable under transient loadings. Nakamura et al. (1986) gave an estimate of the transition time using the time history of the relative magnitudes of the total kinetic energy and the deformation energy (or stress work) of the specimen. A summary of the key steps in Nakamura, Shih and Freund's derivation of the transition time is given below.

Direct measurements of the total kinetic and deformation energies in a laboratory specimen is not possible. An estimate of the kinetic energy was obtained by means of a model based on elastic Bernoulli-Euler beam theory with an assumption that the kinetic energy at the early stage is dominated by elastic structural response. To approximate the deformation energy, a quasi-static elastic 2-D three-point-bend model was considered. Using these models, the ratio of kinetic energy to deformation energy at time t is given by

$$\left(\frac{K}{W}\right)_{\text{model}} = \left(S \frac{H}{c_o} \frac{\dot{\Delta}(t)}{\Delta(t)}\right)^2 \quad (3.1)$$

Here H is the width of the specimen, c_o is the sound speed in the specimen (i.e., longitudinal bar wave speed) and the (time-independent) dimensionless shape factor S depends on the dominant mode shape and elastic compliance C_S of the specimen. In the above model analysis the effect of discrete stress waves was ignored. Obviously, the values of estimated energies are not valid until after elastic waves make several passes within the specimen. The limitation is of no consequence since such wave effects are unimportant at the transition time. The model analysis also assumed that the effects of plastic deformation is not significant prior to the transition time. They also introduced a displacement coefficient D defined by

$$D = \frac{t \dot{\Delta}(t)}{\Delta(t)} \Big|_{t_T} \quad (3.2)$$

For example, if the time variation of the displacement can be represented by $\Delta = \beta t^\gamma$ then $D = \gamma$. With t_T defined as the time when the ratio K/W is equal to unity, (3.1) and (3.2) are combined to give

$$t_T = D S \frac{H}{c_o} \quad (3.3)$$

Nakamura et al. (1989) evaluated the accuracy of the estimate of the transition time based on (3.1) and (3.3) by appealing to the results of full-field 3-D transient finite element simulation of the dynamic fracture experiment. With $\Delta(t)$ and $\dot{\Delta}(t)$ obtained from the results of the 3-D finite element calculations, and values for S , H and c_o appropriate to the specimen, they calculated the time variation of the ratio K/W according to (3.1). The ratio of the actual total kinetic and deformation energies from the finite element model (obtained by summing the respective energies over all the elements) is also calculated and were found to be in good agreement. The close agreement attests to the accuracy of (3.1) and hence the estimate of the transition time using (3.3).

3.2 THE DEEP-CRACK ESTIMATION PROCEDURE FOR J

On the basis of a transient 2-D and 3-D finite element analysis a formula for the computation of dynamic J from measurable quantities has been proposed by Nakamura et al. (1986,1989). The formula is a modified version of a deep-crack formula for calculating the value of J under essentially equilibrium conditions (Rice et al., 1973). Under high-rate of loading, the inertial resistance of the specimen screens the crack tip region from the applied loads. To minimize this effect, the variables in the quasi-static formula are replaced by equivalent variables which characterize the near crack region of the body. Thus the moment is taken to be the net moment M_L carried by the ligament, and the corresponding rotation is replaced by the crack mouth opening displacement divided by the distance between the crack mouth and the hinge axis on the ligament and is denoted by θ_L . With these changes, the formula

$$J(t) = \frac{2}{bB} \int_0^{\theta_L(t)} M_L(t') d\theta_L(t') \quad (3.4)$$

is proposed for estimating the value of J in a dynamically loaded three-point-bend ductile fracture specimen. In the above expression, b is the ligament length and B is the specimen thickness. At the level of beam approximation, the integral of the moment and angle in (3.4) represents the work done on the ligament.

To calculate $J(t)$ according to (3.4), the values of $\theta_L(t)$ and $M_L(t)$ must be known. In an experiment, the angle θ_L can be determined from the measurement of the crack mouth opening displacement δ and an estimate of the hinge axis location necessary to convert this displacement to a rotation. The opening displacement at the crack mouth is essentially constant through the thickness. Thus δ is a well-defined measurable quantity. The hinge axis is an effective line on the ligament plane (and parallel to the x_3 -axis) where the axial strain (and stress) vanishes. Under fully yielded conditions, the hinge axis can be rather accurately estimated using the Green and Hundy slip-line solution (1956) for a plane strain rigid-perfectly plastic material. The rotation about the ligament based on the latter hinge axis is denoted by θ_L^* .

An accurate measurement of the moment carried by the uncracked ligament is essential to the success of the proposed procedure. The direct measurement of the net moment M_L carried by the ligament is extremely difficult, due in part to severe plastic deformation between the load-point and the crack tip. Furthermore the three-dimensional nature of the deformation fields near the crack tip and in the vicinity of the ligament preclude the use of surface measurements in these regions for the determination of the net moment carried by the uncracked ligament. Joyce and Hackett (1989) proposed that the instantaneous values of M_L be inferred from measurements at some distance away from the cracked section and from the roller support. At a location remote from the cracked section and the roller support, (e.g. the section mid-way between the crack plane and the roller support), the section should remain elastic and the fields should not vary through the thickness. Nakumara et al. examined the through thickness variation of the stresses at several transverse sections along the beam axis and found negligible through-thickness variation of the axial and shear stresses at the quarter span (mid-way between the crack plane and the roller supports) for times larger than the transition time. In other words, the fields in the vicinity of the quarter span are essentially 2-dimensional in nature and plane stress. The variation of the axial stress with distance from the neutral axis is very nearly in agreement with the distribution according to the Euler-Bernoulli beam theory. They conclude that the load carried by the ligament is most accurately estimated from axial strain measurements made at the quarter span. Further details on experimental test procedure and the calibration of the specimen under quasi-static loadings so that the moment carried by the ligament during the dynamic fracture experiment can be extracted from readings of appropriately placed strain gauges are discussed by Joyce and Hackett (1989).

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REFERENCES

- Chi, Y.C., S.H. Lee, K. Cho, and J. Duffy (1988). The Effects of Tempering and Test Temperature on the Dynamic Fracture Initiation Behavior of an AISI 4340 VAR Steel. Brown University Report, August.
- Cho, K., J.P. Sklenak and J. Duffy (1988). The Dynamic Fracture Behavior of a Structural Steel. presented at 21 National Fracture Mechanics Conference, Annapolis, Md., June, to be published in ASTM-STP.
- Corran, R.S.J., F. Garcia-Benitez, J. Harding, T. Nojima, and C. Ruiz, (1983). Towards the Development of a Dynamic Fracture Initiation Test. Technical Report, University of Oxford.
- Costin, L.S., J. Duffy and L.B. Freund (1977). Fracture initiation in metals under stress wave loading conditions. *Fast Fracture and Crack Arrest*, ASTM STP 627, 301-318.
- Costin, L.S. and J. Duffy (1979). The Effect of Loading Rate and Temperature on the Initiation of Fracture in a Mild, Rate-Sensitive Steel. *Journal of Engineering Materials and Technology*, 101 258-264.
- Couque, H., R.J. Asaro, J. Duffy and S. Lee (1988). Correlations of Microstructure with Dynamic and Quasi-Static Fracture in Plain Carbon Steel. *Metallurgical Transactions, A* 19A, 2179-2206.
- Duffy, J., Campbell, J.D. and Hawley, R.H. (1971). On the Use of a Torsional Split Hopkinson Bar to Study Rate Effects in 1100-0 Aluminum. *Journal of Applied Mechanics* 38, 83-91.
- Duffy, J., S. Suresh, K. Cho and E.R. Bopp (1987). A Method for Dynamic Fracture Initiation Testing of Ceramics. Brown University Report, ONR Contract N00014-85-K-0687, May, 1987. To be published in *Journal of Engineering Materials and Technology*.
- Eftis, J and J.M. Krafft (1965). A Comparison of the Initiation with the Rapid Propagation of a Crack in a Mild Steel Plate. *Journal of Basic Engineering, Trans., ASME*, 257-263.
- Green, A.P. and B.B. Hundy (1956). Initial plastic yielding in notch bend tests. *Journal of the Mechanics and Physics of Solids*, 4, 128-144.
- Hutchinson, J.W. (1983). Fundamentals of the phenomenological theory of nonlinear fracture mechanics. *Journal of Applied Mechanics*, 50, 1042-1051.
- Joyce, J.A. and E.M. Hackett (1989). An advanced procedure for J-R curve testing using a drop tower. in press, ASTM STP 995, 1989.
- Joyce, J.A. and E.M. Hackett (1984). Dynamic J-R Curve Testing of a High Strength Steel using the Multispecimen and Key Curve Techniques. ASTM STP 905, 741-774.
- Kalthoff, J.F. (1985). On the measurement of dynamic fracture toughness - a review of recent work. *International Journal of Fracture*, 27, 227-298.

- Klepaczko, J.P. and A. Sroka (1984). Effect of Tempering on Quasi-Static and Impact Fracture Toughness and Mechanical Properties for 5140H Steel. *Met. Trans. A*, 15A, 901-911.
- Kobayashi, T., I. Yamamoto and M. Ninomi (1986). Evaluation of Dynamic Fracture Toughness Parameters by Instrumented Charpy Impact Test. *Engineering Fracture Mechanics*, 24, 773-782.
- J.M. Kraft (1965). Correlation of Plane Strain Crack Toughness with Strain Hardening Characteristics of a Low, a Medium, and a High Strength Steel, *Applied Materials Research* 88-101.
- Marchand, A., J. Duffy, T.A. Christman and S. Suresh (1988). An Experimental Study of the Dynamic Mechanical Properties of an AlSiC Composite. *Engineering Fracture Mechanics*, 30, 295-315.
- Nakamura, T., C.F. Shih and L.B. Freund (1985). Elastic-Plastic Analysis of Dynamically loaded Circumferentially Notched Round Bar. *Engineering Fracture Mechanics* 22, 437-452.
- Nakamura, T., C.F. Shih, and L.B. Freund, (1986). Analysis of a Dynamically Loaded Three-Point-Bend Ductile Fracture Specimen," *Engineering Fracture Mechanics*, 25, 323-339.
- Nakamura, T., C.F. Shih, and L.B. Freund (1989). Three-Dimensional Transient Analysis of a Dynamically Loaded Three-Point-Bend Ductile Fracture Specimen," in press, ASTM STP 995, 1989.
- Ravi-Chandar, K. and W.G. Knauss (1984). An Experimental Investigation into Dynamic Fracture: I Crack Initiation and Arrest. *International Journal of Fracture*, 25, 247-262.
- Ravichandran G. and R.J. Clifton (1988). Dynamic Fracture under Plane Wave Loading. in press *International Journal of Fracture*
- Rice, J.R. (1968). A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks. *Journal of Applied Mechanics*, 35, 379-386.
- Rice, J.R., P.C. Paris and J.C. Merkle (1973). Some Further Results of J-Integral Analysis and Estimates. *Progress in Flaw Growth and Fracture Toughness Testing* American Society for Testing and Materials, STP 536, 231-245.
- Ruiz, C. and R.A.W. Mines (1985) The Hopkinson Pressure Bar: an Alternative to the Instrumented Pendulum for Charpy Tests. *International Journal of Fracture*, 29, 101-109.
- Shockey, D.A. (1985). Dynamic Fracture Testing. in *Metals Handbook, Ninth Edition*, 8, 259-298.
- Tada, H., P.C. Paris and G.R. Irwin (1973). The Stress Analysis of Cracks Handbook, Del Research Corporation, Hellertown, Pennsylvania.