Determination of Stress Intensity Factor in a Crack-stiffened Panel Under Equibiaxial Loading

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ABSTRACT

A method for the determination of stress intensity factor(SIF) in a cracked plate stiffened by symmetrically bonded strip-patches (or stiffeners) under biaxial loading is presented. The patches are modeled by considering their longitudinal and transverse stiffnesses. Sample problems are sloved and the results are compared with those for the case of uniaxial loading. The shear stresses developed in the adhesive, and their dependence on patch location, patch length, shear modulus of adhesive and thickness of adhesive, are shown. It is observed that the reduction in SIF due to the stiffener is less under biaxial loading than under uniaxial loading.

KEY WORDS

Crack-stiffened panel, biaxial load, and stress intensity factor.

NOMENCLATURE

A_i, B_i	Arbitrary constants.
a	Half crack length.
B	Stiffener width.
E	Young's modulus.
F_1, F_2	Integrals defined in the text.
G	Shear modulus.
K_I	Mode I stress intensity factor.
l	Half stiffener length.
N	Number of collocation points.
R_1, R_2	Functions defined in the text.
S_1, S_2, S_3, S_4	Functions defined in the text.
t	Thickness.
u	Displacement in x direction.
v	Displacement in y direction.
τ_{zx}	Transverse shear stress.
τ_{zy}	Longitudinal shear stress.
σ_o	Boundary load.
μ	Poisson's ratio.

Subscripts

a Adhesive.
p Stiffener or patch.
s Sheet.

INTRODUCTION

Stiffeners are widely used in structures. Fracture study of stiffened structures has assumed a great importance in view of the application of damage tolerance concepts in design.

Arin(1974) analysed the effect of a partially debonded infinite stiffener on the SIF. He obtained the integral equation from the compatibility of displacements between the sheet and the stiffener and this was finally solved numerically. Swift(1978) analysed the similar problem by taking the stiffener as an assemblage of number of discrete noninteracting patches. Chandra et al.(1985) studied the case of an infinite plate stiffened by symmetrically bonded finite strips. They analysed by assuming the stiffener to have only the longitudinal stiffness. All these approaches are useful only for a mode I situation. Recently Sethuraman and Maiti(1988) have proposed a method of analysis, which can be applied to solve problem involving bonded finite strips and mixed mode loading. This has been possible by considering the stiffener to have both longitudinal and transverse stiffnesses. A Mode I and a Mode II problems have been examined earlier by Sethuraman and Maiti(1988). In the present paper the method is applied to examine cases under biaxial loading.

METHOD OF ANALYSIS

A typical problem with biaxial loading is shown in Fig.1. Under the action of the external loading, the sheet is subjected to longitudinal(τ_{zy}) and transverse(τ_{zx}) shear stresses at the location of the stiffener(Fig.2). These stresses can be assumed constant along the width of the stiffener. The basic unknowns here are these shear stresses. These shear stresses can be initially assumed in the form of polynomials involving a finite number of arbitrary constants. The deformations of the patch and the sheet can then be expressed in terms of these unknown shear stresses. These deformations are interrelated because they represent the deformation of the adhesive . A compatibility condition can be written based on this fact. This finally leads to an integral equation. Using a point collocation scheme, the integral equation can be reduced to a set of simultaneous equations , solving which the arbitrary constants are determined. This method is elaborated in what follows.

Determination of τ_{zx} and τ_{zy}

The adhesive is usually assumed to undergo a pure shear deformation(Arin, 1974; Swift, 1978; Jones, 1979; Chandra et al., 1985). The shear stresses τ_{zx} and τ_{zy} in the sheet and the patch vary in their thickness direction. Assuming this variation to be linear, Jones and Callinan(1979) related the adhesive shear stresses to sheet and stiffener centre displacements by

$$\tau_{zx} = \frac{(u_p - u_s)}{\left(\frac{t_a}{G_a} + \frac{t_s}{4G_s} + \frac{3t_p}{8G_p}\right)} \quad (1)$$

$$\tau_{zy} = \frac{(v_p - v_s)}{\left(\frac{t_a}{G_a} + \frac{t_s}{4G_s} + \frac{3t_p}{8G_p}\right)} \quad (2)$$

These relations are the compatibility conditions.

Sheet displacements u_s and v_s

Using the Westergaard stress function(Westergaard, 1939) the displacement in the sheet due to the external biaxial loading can be determined. The displacement due to the shear stresses τ_{xz} and τ_{yz} are obtained by using the solution for a point load.

$$u_s = 2B \int_o^1 (S_1\tau_{zx} + S_2\tau_{zy})dy_o + R_1 , \quad (3)$$

$$v_s = 2B \int_o^1 (S_3\tau_{zx} + S_4\tau_{zy})dy_o + R_2 , \quad (4)$$

where

S_1, S_2 give the displacement in x direction due to a set of four unit point loads P_x, P_y (Fig.3),
S_3, S_4 give the displacement in y direction due to a set of four unit point loads P_x, P_y (Fig.3),
and R_1,R_2 give displacement of the sheet in x and y directions respectively due to the external biaxial load.

These functions S_1,S_2,S_3,S_4,R_1 and R_2 are given by Sethuraman and Maiti(1988).

Stiffener deformation

The longitudinal displacement v_p due to the longitudinal shear stress τ_{zy} is obtained using the condition of equilibrium and stress-displacement relationship. That is,

$$v_p = \frac{1}{E_p t_p} \int_o^y \int_y^1 \tau_{zy}(dy)^2 \quad (5)$$

The transverse displacement u_p due to the shear stress τ_{zx} is obtained by treating the stiffener as a beam. From the elementary beam theory

$$\frac{d^4u_p}{dy^4} = - \frac{B}{E_pI}\tau_{zx} \quad (6)$$

The final expression for u_p is given by Sethuraman and Maiti(1988).

Integral Equation

The compatibility conditions(eqns.1 and 2) finally give

$$C\tau_{zx} = u_p - 2B \int_o^1 (S_1\tau_{zx} + S_2\tau_{zy})dy_o - R_1 \quad (7)$$

$$C\tau_{zy} = \frac{1}{E_p t_p} \int_o^y \int_y^1 \tau_{zy}(dy)^2 - 2B \int_o^1 (S_3\tau_{zx} + S_4\tau_{zy})dy_o - R_2 \quad (8)$$

where

$$C = \frac{t_a}{G_a} + \frac{t_s}{4G_s} + \frac{3t_p}{8G_p}$$

Numerical Solution of Integral Equations

It is convenient to assume τ_{zx} and τ_{zy} in in the form of polynomials:

$$\tau_{zx} = \sum_{i=1}^{N} A_i y^{i-1} \quad (9)$$

$$\tau_{zy} = \sum_{i=1}^{N} B_i y^{i-1} \quad (10)$$

where A_i's and B_i's are arbitrary constants. The problem then reduces to the determination of these constants. After substituting Eqns.9 and 10 in Eqns. 7 and 8, the resulting integral equations can be collocated at N number of points. This gives 2N simultaneous equations solving which the unknowns A_i's and B_i's are obtained.

Stress intensity factor

The stress intensity factor is calculated using the following relationship

$$K_I = \sigma_o\sqrt{\pi a} + 2B \int_o^1 (F_1\tau_{zx} + F_2\tau_{zy})dy_o \quad (11)$$

where

$$F_1 = \frac{1}{\sqrt{\pi a}} \int_{-a}^{+a} \sigma_{y1}(x,o)\left(\frac{a+x}{a-x}\right)^{1/2}dx$$

$$F_2 = \frac{1}{\sqrt{\pi a}} \int_{-a}^{+a} \sigma_{y2}(x,o)\left(\frac{a+x}{a-x}\right)^{1/2}dx$$

σ_{y1} and σ_{y2} are normal stresses along the crack line in an uncracked infinte plate due to a set of four unit point loads acting at $(\pm x_o, \pm y_o)$ in x and y directions respectively. The functions σ_{y1} and σ_{y2} are given in Sethuraman and Maiti(1988).

CASE STUDIES

A problem with the following dimensions and material properties has been studied.

2a = 25 mm	t_s = 2 mm	G_s = 27100 Nmm^{-2}	E_p = 72100 Nmm^{-2}
2l = 20 mm	t_a = 0.1 mm	G_a = 1155 Nmm^{-2}	E_s = 72100 Nmm^{-2}
B = 2 mm	t_p = 1 mm	G_p = 27100 Nmm^{-2}	σ_o = 2.5 Nmm^{-2}
μ = 0.33			

The creck-stiffened plate subjected to equibiaxial load as shown in Fig.1 is considered for the study. For different patch locations,along the length of the stiffener the variation of longitudinal(τ_{zy}) and the transverse(τ_{zx}) shear stresses are shown in Figs.4 and 5. The SIFs of the stiffened panel subjected to uniaxial and biaxial loading is plotted against the stiffener location in Fig.6. The results concerning the uniaxial load are taken from the reference of Sethuraman and Maiti(1988). It is seen that when stiffener is located close to the crack tip, and on the side opposite the ligament, the stiffener is the most effective. The influence of stiffener length on SIF is shown in Table.1. Increasing the length is advantageous upto a certain limit, but it does not give rise to much further benefit when l/a is more than 1.20. The effects of the adhesive shear modulus and the its thickness are shown in Tables.2 and 3. These are quite sensitive parameters; the reduction in thickness and increase in shear modulus causes more reduction in the SIF.From the results we observe that the stiffener is less effective under the biaxial loading than the uniaxial loading.

CONCLUDING REMARKS

1. An equbiaxial loading leads to a slight reduction in the effect of the stiffener compared with an uniaxial load.
2. Stiffener is the most effective when it is located close to the crack tip and on the side opposite the ligaments.
3. Beyond a certain limit, increasing the patch length does not have any significant beneficial effect on the SIF.
4. The SIF is substantially influenced by the shear modulus of adhesive and its thickness.

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REFERENCES

Arin, k. (1974). A plate with a crack stiffened by a partially debonding stringer. Engg. Fract. Mech. 6, 133-140.
Chandra, R., Murthy,M.V.V., Ramamurthy, T.S. and Rao, A.K. (1985). Analytical estimation of stress intensity factors in patched cracked plates. Engg. Fract. Mech. 21, 479-484.
Jones, R. and Callinan, R.J. (1979). Finite element analysis of patched cracks. J. Struct. Mech. 7, 107-130.
Sethuraman, R. and Maiti, S.K. (1988). Determination of mixed mode stress intensity factors for a crack-stiffened panel. Due to appear in Engg. Fract. Mech.
Swift, T. (1978). Fracture analysis of adhesively bonded cracked panels. Trans. ASME, J. Engg. Mats Tech. 100, 10-15.
Westergaard, H.M. (1939). Bearing pressures and cracks. Trans. ASME, J. Appl. Mech. 16, A49-A53.

Table 1. EFFECT OF PATCH LENGTH ON SIF K_I

X_o = 8.00mm t_a = 0.1mm G_a = 1155Nmm^{-2} a = 12.5mm

PATCH	UNIAXIAL LOADING(Sethuraman et al., 1988)			BIAXIAL LOADING		
LENGTH (2l) mm	SIF ONLY DUE TO EXTERNAL LOAD	SIF ONLY DUE TO STIFFENER REACTION	TOTAL SIF $Nmm^{-1.5}$	SIF ONLY DUE TO EXTERNAL LOAD	SIF ONLY DUE TO STIFFENER REACTION	TOTAL SIF $Nmm^{-1.5}$
8	15.666	-5.196	10.470	15.666	-5.021	10.645
12	15.666	-5.779	9.887	15.666	-5.532	10.134
16	15.666	-5.999	9.667	15.666	-5.717	9.949
20	15.666	-6.078	9.588	15.666	-5.786	9.880
30	15.666	-6.087	9.579	15.666	-5.814	9.852

Table 2. EFFECT OF ADHESIVE SHEAR MODULUS ON SIF K_I

X_o = 8.00mm t_a = 0.1mm 2l = 20.00mm a = 12.5mm

SHEAR MODULUS OF ADHESIVE (G_a) Nmm^{-2}	UNIAXIAL LOADING (Sethuraman et al., 1988)			BIAXIAL LOADING		
	SIF ONLY DUE TO EXTERNAL LOAD	SIF ONLY DUE TO STIFFENER REACTION	TOTAL SIF $Nmm^{-1.5}$	SIF ONLY DUE TO EXTERNAL LOAD	SIF ONLY DUE TO STIFFENER REACTION	TOTAL SIF $Nmm^{-1.5}$
770	15.666	-5.727	9.939	15.666	-5.434	10.232
1155	15.666	-6.078	9.588	15.666	-5.786	9.880
1540	15.666	-6.278	9.388	15.666	-5.989	9.677

Table 3. EFFECT OF ADHESIVE THICKNESS ON SIF K_I

X_o = 8.00mm 2l=20.00mm G_a = 1155Nmm^{-2} a = 12.5mm

ADHESIVE THICKNESS (t_a) mm	UNIAXIAL LOADING (Sethuraman et al., 1988)			BIAXIAL LOADING		
	SIF ONLY DUE TO EXTERNAL LOAD	SIF ONLY DUE TO STIFFENER REACTION	TOTAL SIF $Nmm^{-1.5}$	SIF ONLY DUE TO EXTERNAL LOAD	SIF ONLY DUE TO STIFFENER REACTION	TOTAL SIF $Nmm^{-1.5}$
0.03	15.666	-6.695	8.971	15.666	-6.415	9.251
0.06	15.666	-6.408	9.258	15.666	-6.121	9.545
0.10	15.666	-6.078	9.588	15.666	-5.786	9.880

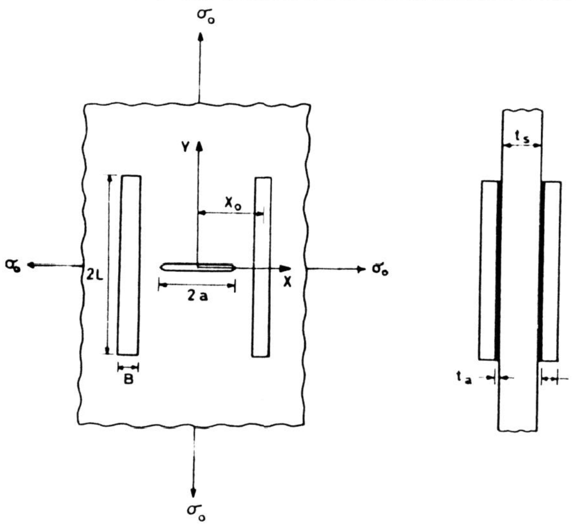

FIGURE 1. SYMMETRICAL CRACK-STIFFENED RECTANGULAR PANEL UNDER BIAXIAL LOADING.

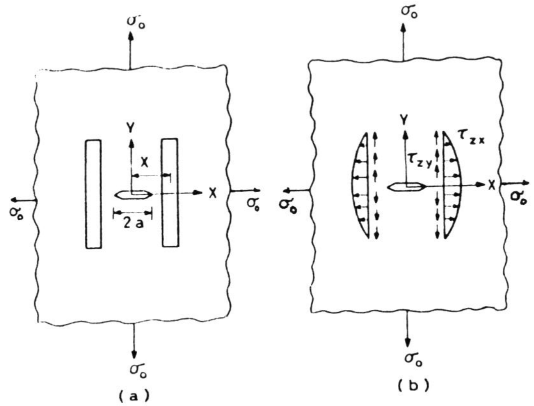

FIGURE 2, THE LOADING AND ITS BREAK-UP.

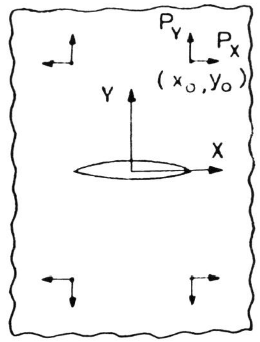

FIGURE 3. FOUR POINT LOADED CRACKED SHEET.

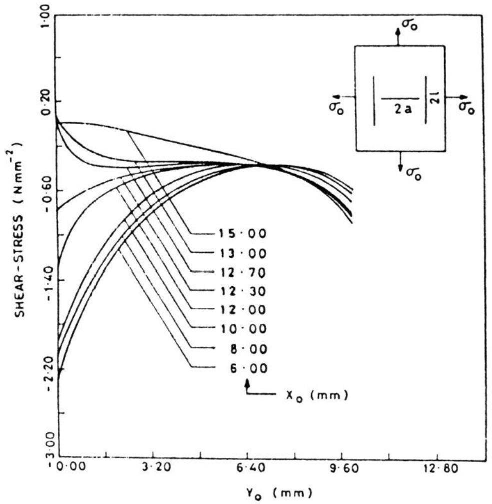

FIGURE 4 VARIATION OF LONGITUDINAL SHEAR STRESS ALONG STIFFENER.

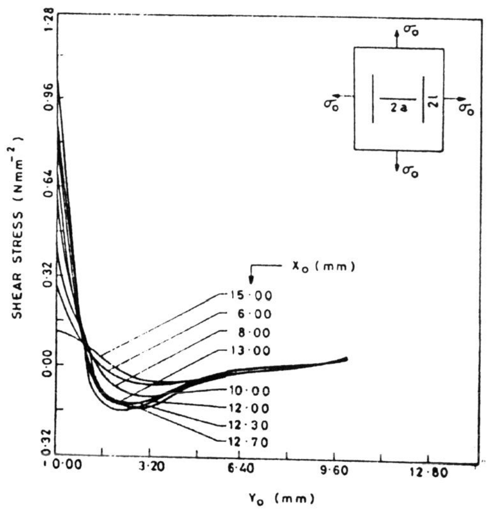

FIGURE 5 VARIATION OF TRANSVERSE SHEAR STRESS ALONG STIFFENER.

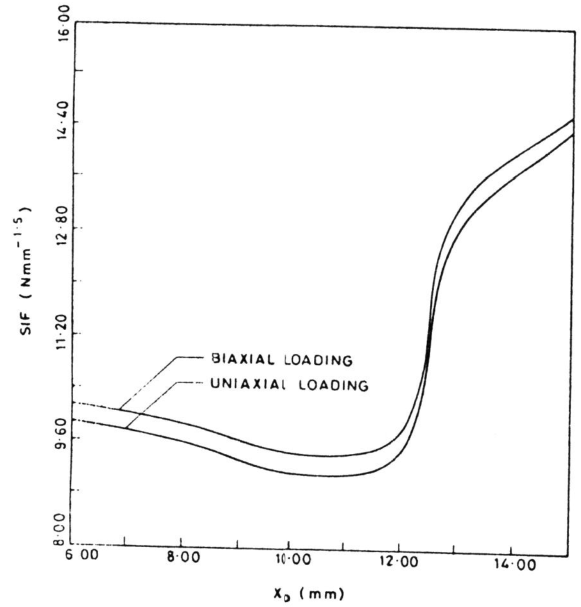

FIGURE 6 VARIATION OF SIF WITH STIFFENER LOCATION FOR UNIAXIAL AND BIAXIAL LOADINGS