Correlation of Creep-fatigue Crack Growth Rates Using Crack-tip Parameters

BRIAN GIESEKE* and ASHOK SAXENA**
**Graduate Student and **Professor, Mechanical Properties Research Laboratory, School of Materials Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332–0245, USA*

ABSTRACT

Several approaches for correlating creep-fatigue crack growth (CFCG) rates are reviewed which incorporate time-dependent fracture parameters. The parameters C(t), C^*, and C_t have been evaluated for correlating CFCG rates by partitioning the overall growth rates into cycle- and time-dependent contributions. It is shown that the use of C_t is the most appropriate for describing the time-dependent contribution. Further, an additional crack length and geometry dependent factor is introduced by the presence of creep at the crack tip. This factor is not included in correlations with ΔK. Considerable creep-fatigue data from various sources are used to support these conclusions.

KEYWORDS

Creep; Fatigue; Creep-fatigue; Crack Growth; C_t; C^*

INTRODUCTION AND BACKGROUND

Since the early work of James (1972), high temperature fatigue crack growth rates have routinely been correlated using ΔK with full recognition of the fact that creep deformation occurs at the crack tip below certain frequencies. It is now well established that the crack-tip stress fields are affected by creep (Riedel, 1983; Saxena *et al.*,1986), and a fresh look at the validity of using ΔK for characterizing creep-fatigue crack growth (CFCG) rates is needed. In the past few years, the concepts in time-dependent fracture mechanics have evolved yielding several parameters such as C^*, C(t), and C_t which may be used to correlate crack growth under static loading, in other words, creep crack growth (Landes *et al.*, 1976; Riedel *et al.*, 1980; Saxena, 1986). Recently, attempts have also been made to use these parameters for correlating CFCG rates in combination with ΔJ or ΔK (Saxena, 1980; Okazaki *et al.*, 1983; Saxena *et al.*, 1987; Dimopulos *et al.*, 1988; Nikbin *et al.*, 1988; Ohji *et al.*, 1988). Of the various fracture mechanics parameters, C_t appears to show the most promise (Saxena *et al.*, 1987). The various crack-tip parameters which can account for creep deformation are first briefly described with emphasis on their attributes and limitations in regard to correlation of creep-fatigue crack growth behavior. General methodologies for modeling creep-fatigue crack growth behavior are then described. Subsequently, data correlations are presented with the objective of evaluating the various parameters.

Candidate Time-Dependent Fracture Mechanics Parameters

Under static loading, the stress fields near a crack tip in a material undergoing elastic, power-law creep deformation are of the Hutchinson, Rice and Rosengren (HRR) type, the magnitude of which is denoted by C(t) (Riedel *et al.*, 1980; Bassani *et al.*, 1981). C(t) is approximately given by (Ehlers *et al.*, 1981):

$$C(t) = \frac{K^2(1-\nu^2)}{E(n+1)t} + C^* \quad (1)$$

where E is Young's modulus, n is the creep exponent in Norton's creep law, t is time, ν is Poisson's ratio and C^* is a path-independent integral first introduced by Landes *et al.* (1976). For small scale creep conditions (SSC), which are analogous to small scale yielding, the first term in Eq. (1) dominates (Riedel *et al.*, 1980). When extensive creep (SS) conditions exist, $C(t) = C^*$ (Goldman *et al.*, 1975). A useful method for determining when extensive creep conditions exist is to compare the Riedel-Rice transition time, t_1 (the time at which the first term in Eq. (1) equals C^*), with the cycle time, t_c (Riedel, 1983; Saxena, 1988). SSC conditions exist when $t_c \ll t_1$ and extensive creep conditions exist when $t_c \gg t_1$. C^*, in Eq. (1), has also been interpreted as the stress-power dissipation rate in cracked bodies (Landes *et al.*, 1976). This interpretation (or definition) can be used to measure the value of C^* at the loading pins for bodies undergoing dominantly steady-state creep deformation.

The C_t parameter is an extension of the stress-power dissipation rate definition of C^* into the SSC regime (Saxena, 1986). It has been proposed for correlating creep crack growth for a wide range of conditions ranging from small-scale to extensive creep. C_t is also uniquely related to the rate of expansion of the crack-tip creep zone under SSC conditions (Bassani *et al.*, 1986; Leung *et al.*, 1988), and $C_t = C^*$ when extensive creep conditions exist. C_t can be measured at the loading pins for all deformation conditions including those involving primary creep (Leung *et al.*, 1988). There is also considerable evidence that C_t can correlate creep crack growth data for conditions ranging from small-scale to extensive creep (Saxena, 1986).

The crack-tip stress fields in creep-fatigue are more complicated than in creep. However, Riedel (1983) has numerically investigated the crack-tip stress fields for a few simple loading waveforms and his results are of interest here. He has shown that if rapid load variations occur within otherwise slow cycles, crack-tip stress peaks follow the peaks of the load variations. On the other hand, for waveforms with hold times and a rise time of approximately one half t_1, he has shown that the stress peak can be as much as 75% higher than the steady-state value. For waveforms with hold times, the stresses decay with time according to Eq. (1) and C^* does not characterize the stress fields except for very long hold times ($t_h \gg t_1$). Hence, C^* is not a valid parameter for characterizing the crack growth rates for short hold times. On the other hand, both C(t) and C_t are valid in the transient regime and can be considered as candidate parameters.

Methodologies for Modeling Creep-Fatigue Crack Growth

Creep-fatigue crack growth rates as a function of frequency, for a fixed ΔK range and waveform, show three regimes of behavior. At high frequencies, a cycle-dependent region exists where crack growth is controlled by fatigue processes. At very low frequencies, crack growth is entirely controlled by time-dependent processes. The crack growth rates in each regime are characterized by a different crack driving force. In the intermediate frequency regime, where the creep-fatigue interactions occur, two approaches have emerged to model crack growth rates.

Partitioning of Crack Growth Rates. Time-dependent fracture mechanics parameters are typically incorporated into creep-fatigue analysis by decomposing the CFCG rates into cycle-dependent and time-dependent contributions and using superposition to determine the overall rate, ie. (Saxena, 1980; Okazaki *et al.*, 1983; Saxena *et al.*, 1984; Saxena *et al.*, 1987; Dimopulos *et al.*, 1988; Nikbin *et al.*, 1988; Ohji *et al.*, 1988):

$$da/dN = (da/dN)_f + (da/dN)_t \quad (2)$$

where $(da/dN)_f$ and $(da/dN)_t$ are the fatigue and time-dependent components, respectively. $(da/dN)_f$ is uniquely characterized by either ΔK_{eff} (Dimopulos *et al.*, 1988) or ΔJ (Ohji *et al.*, 1988) depending upon the amount of instantaneous plasticity. Here ΔK_{eff} is the effective stress intensity factor range and ΔJ is the cyclic J-integral of Dowling *et al.* (1976). Thus $(da/dN)_f$ is given by:

$$(da/dN)_f = C(\Delta K_{eff})^m \quad (3a)$$

or by:

$$(da/dN)_f = C_0(\Delta J)^{m_o} \quad (3b)$$

where C, C_0, m and m_o are material constants of which only two are independent. For example, $m = 2m_o$. For estimating $(da/dN)_t$, the time rate of crack growth is first estimated using:

$$da/dt = D(A)^{\phi} \quad (4)$$

where A is a time-dependent fracture parameter. This has been chosen as C(t) by some researchers (Saxena, 1980; Saxena *et al.*, 1981; Saxena *et al.*, 1984; Saxena, 1988), as C^* by others (Dimopulos *et al.*, 1988; Nikbin *et al.*, 1988; Ohji *et al.*, 1988), and finally as C_t by Saxena *et al.* (1987). The time rate of crack growth can be converted to a growth rate per cycle by a simple integration over the cycle time.

Partitioning of Crack Driving Forces. An alternate approach to that of partitioning the crack growth rates has been to partition the crack driving force into cycle-dependent and time-dependent contributions (Taira *et al.*, 1979; Ohji *et al.*, 1988). For elastic-plastic-creep conditions, this has resulted in the so-called "total J-integral", ΔJ_T, which is defined as (Taira *et al.*, 1979):

$$\Delta J_T = \Delta J_f + \Delta J_c \quad (5)$$

ΔJ_f applies to fatigue only and ΔJ_c is the time-integral of C^* over one stress cycle. Crack growth rates are then correlated using Eq. (3b) with the substitution of ΔJ_T for ΔJ. The combined ΔJ_T parameter does not currently have a mechanics interpretation because it is not related to crack-tip stress, strain, or any other related crack-tip quantity.

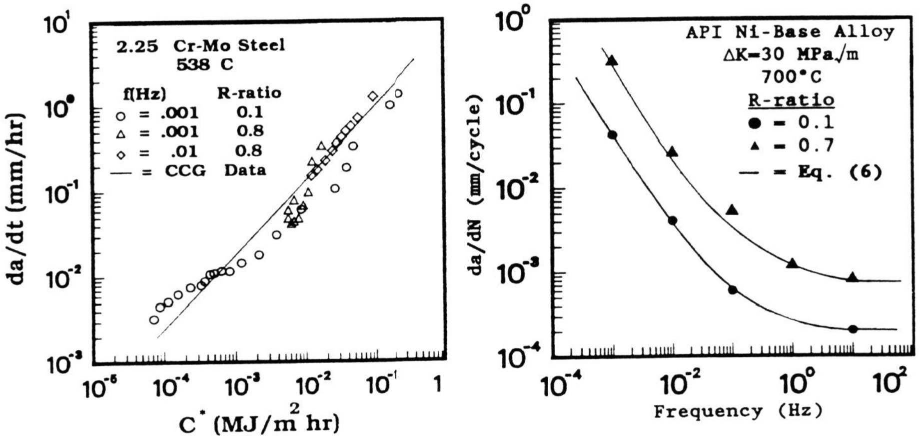
DATA CORRELATIONS

Correlations Using C^* and ΔJ_T

In the time-dependent regime, under cyclic loading, Dimopulos *et al.* (1988) and Nikbin *et al.* (1988) have shown that CFCG rates fall on the same trend as creep crack growth data when converted to time rates of crack growth, da/dt, and plotted versus an experimentally measured value of C^*. They have observed this behavior in a nickel-base superalloy, a creep-brittle steel, and creep-ductile steel using a square loading waveform. An example of the correlation for a 2.25 Cr-Mo steel is shown in Fig. 1a. Their version of Eq. (2) is as follows (Dimopulos *et al.*, 1988):

$$da/dN = C\Delta K^m_{eff} + D(C^*)^\phi/(3600\nu) \qquad (6)$$

where C^* is experimentally determined at the maximum load in the trapezoidal cycle. Figure 1b shows an example of the experimental and predicted CFCG rates determined via Eq. (6) for one material (Dimopulos *et al.*,1988).



(a.) (b.)

Fig. 1. (a) Dependence of crack growth/cycle on frequency. (b) Low frequency crack growth rates as a function of C^*. (Dimopulos *et al.*, 1988)

There is in fact a good correlation which has led Dimopulos *et al.* (1988) to conclude that the crack growth rates for creep-fatigue loading can be determined from tests carried out under static loading and high frequency fatigue conditions to determine the time- and cycle-dependent components of crack growth. It should be noted that the Riedel-Rice transition times in many of these tests were on the order of one hour (Nikbin *et al.*, 1988). Since the longest reported cycle times were only 1000 seconds, SS conditions could not have developed during each cycle. The use of C^* can then be questioned. However, experimentally measured values of C^* for CT specimens are nearly equal to the values of C_t for a wide range of crack sizes (Saxena,

1986). Therefore, these data support the use of C_t as the time-dependent crack-tip parameter instead of C^*. The data correlations with C_t will be discussed later.

Creep-fatigue crack growth in 304 stainless was approached differently by Ohji *et al.* (1988). In their method, the term $(da/dN)_t$ in Eq. (2) is given by:

$$(da/dN)_t = AC^*/\nu = A\Delta J_c \qquad (7)$$

where A is a material constant. In these studies also, C^* also was experimentally measured. The overall fatigue crack growth rate was given by:

$$da/dN = A(\Delta J_c + B\Delta J_f) \qquad (8)$$

where A and B are experimentally determined constants. Using this relationship, they have obtained good correlation of fatigue data on 304 stainless tested using sinusoidal waveforms. Equation (8) is in fact a special case of Eq. (2) with constants m_o and ϕ in Eqs. (3b) and (4) being one.

Okazaki *et al.* (1983) have used the total J-integral approach in correlating data generated using balanced and unbalanced triangular, strain-controlled waveforms on 304 stainless at 600 and 700°C. Fatigue data plotted versus ΔJ_T collapsed into a band that lies between cyclic- and time-dependent data. However, a unique relationship between da/dN and ΔJ_T was not observed for all waveforms. Because of this non-unique relationship between da/dN and ΔJ_T, these researchers have suggested that ΔJ_f and ΔJ_c cannot be added linearly. As a result they have proposed a somewhat modified form of the ΔJ_T approach which does differentiate somewhat between waveforms. However, discussion of this technique is beyond the scope of this short paper.

Correlations Using C_t and $C(t)$

Saxena and co-authors (Saxena, 1980; Saxena *et al.*, 1981; Saxena *et al.*, 1984; Saxena, 1988) have used C(t) to correlate the time-dependent contribution to CFCG rates. Initially Saxena (1980) noted that the stress and strain fields at the crack tip, and therefore the crack growth rates, were dependent upon K^2/t, which is proportional to C(t), for SSC. Thus in Eq. (2), the time-dependent crack growth could be described by:

$$da/dt = b(K^2/t)^m = b_1[C(t)]^m \qquad (9)$$

Equation (9) can be integrated over the fatigue cycle and the result can be expressed in the form which relates $(da/dN)_t$ to ΔK by a power-law relationship including terms which relate to cyclic frequency. Reasonable correlations were obtained by this approach.

Recently Saxena *et al.* (1987) evaluated whether time-dependent contributions to CFCG rates can be better correlated with C_t instead of C(t) in Eq. (9). Creep-fatigue data from hold time tests on a 1.0Cr-1.0Mo-.25V rotor steel were re-evaluated using the C_t and C(t) parameters. In these tests, the hold times were much shorter than the smallest calculated transition time, t_1, which was 585 hours. Thus these tests were conducted in a dominantly small scale creep regime.

The term $(da/dN)_t$, from Eq. (2), was taken to be the additional crack growth per cycle due to the hold time. This was converted to an average crack growth rate during the hold time, $(da/dt)_{avg}$, as follows:

$$(da/dt)_{avg} = [da/dN - (da/dN)_f]/t_h \qquad (10)$$

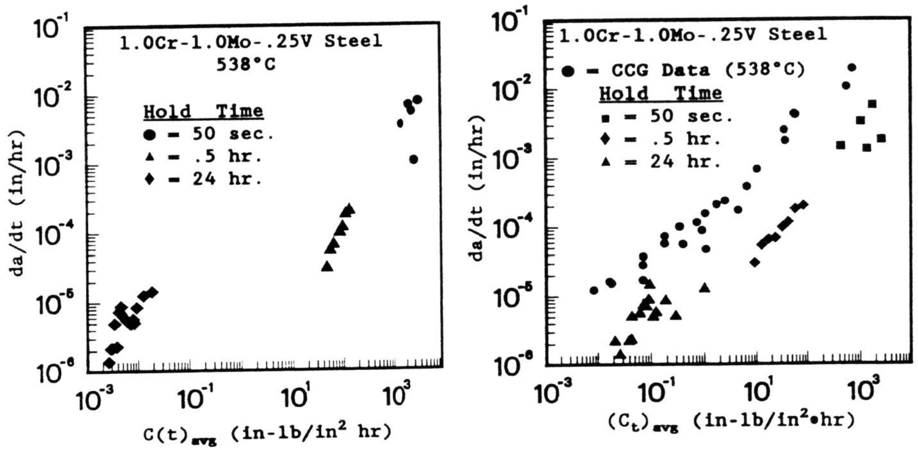
where da/dN is the overall crack growth rate, t_h is the hold time, and $(da/dN)_f$ is the crack growth rate per cycle associated with tests having zero hold time.

The average C_t parameter was calculated by integrating the analytical expression for estimating C_t and dividing it by the hold time. The expression for C_t in the small-scale-creep regime assumes elastic, power-law creep deformation and SSC conditions and is given by (Bassani *et al.*, 1988):

$$(C_t)_{ssc} = \frac{4\alpha\beta K^4}{E(n-1)W} (1-\nu^2) \frac{F'}{F} (EA)^{2/(n-1)} t^{-(n-3/n-1)} \tilde{r}_c(\theta) \qquad (11)$$

In Eq. (11), E is Young's modulus, F is the K-calibration function for the cracked body of interest, F'=dF/d(a/W) where W is the body width, a is the crack length, and the other symbols are described elsewhere (Saxena, 1986).

$(da/dt)_{avg}$ was plotted versus $(C(t))_{avg}$ in Fig. 2a and versus $(C_t)_{avg}$ in Fig. 2b. The correlation with $(C_t)_{avg}$ is considerably better than the correlation with $(C(t))_{avg}$. In Fig. 2b, creep crack growth data are also plotted for



(a.) (b.)

Fig. 2. Average crack growth rate versus a.) $C(t)_{avg}$. and b.) $(C_t)_{avg}$. (Saxena *et al.*, 1987)

comparison. It is noted that the periodic loading/unloading events of fatigue have reduced the crack growth rates during the hold time. Excellent correlation between $(da/dt)_{avg}$ and $(C_t)_{avg}$ has also been observed in a 1.25Cr-0.5Mo steel waveforms with hold times of 10, 98, and 600 seconds preceded by an initial 100 percent overload (Yoon *et al.*, 1988). In this case, however, $(da/dt)_{avg}$ vs. $(C_t)_{avg}$ followed a trend similar to the creep crack growth data.

The correlation between $(da/dt)_t$ and $(C_t)_{avg}$ during the fatigue hold time in the approach described above has several interesting implications. First, it is evident from Eq. (11) that for a given waveform and cycle time (constant t_h),

the average C_t is not uniquely determined by K because of the additional crack size and geometry dependent term F'/F. However, in the past, fatigue crack growth data have been routinely correlated with ΔK for constant loading waveforms and cycle times (James, 1972; Saxena, 1980; Saxena *et al.*, 1981; Saxena *et al.*, 1984). This represents an apparent contradiction which is resolved as follows. It is noted that these correlations were obtained usually with a single specimen geometry, mostly CT specimens, for which F'/F does not vary significantly over a wide range of crack sizes (Saxena, 1986). In this case, $(C_t)_{avg}$ is approximately related to K for a constant waveshape and frequency. Also for short cycle times, the contribution of the time-dependent crack growth is small and the overall crack growth per cycle (da/dN) can be dominated by the cycle-dependent portion which correlates uniquely with ΔK. Hence the observed correlations with ΔK are not surprising, but at the same time should be viewed with caution especially for geometries other than the compact type.

Correlations of crack growth rates for waveforms other than trapezoidal have not yet been satisfactorily addressed. This is an area of future study.

SUMMARY AND CONCLUSIONS

Several approaches for correlating creep-fatigue crack growth rates have been reviewed in this paper. Time-dependent fracture parameters have been incorporated into the correlation of CFCG rates under elastic-creep and elastic-plastic-creep conditions with reasonable success. The presence of time-dependent creep deformation at the crack tip during cyclic loading at elevated temperatures introduces an additional crack length and geometry dependent factor which is not included in crack growth rate correlations with ΔK even for constant loading frequency and waveform. The implication is that a non-unique relationship is expected between da/dN and ΔK at elevated temperatures at loading frequencies where creep occurs. It is shown that the use of C_t is most appropriate parameter for describing the time-dependent contribution to the overall crack growth rates. Considerable creep-fatigue crack growth data from several sources are used to support these arguments.

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