# Correcting J–R Curve Mismatch Twist with Normalization Analysis

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## ABSTRACT

Automatic methods for measuring crack length in a J-R curve fracture toughness test may have small errors which result in larger "mismatch twisting" of the R curve. A method for determining crack length from normalized load and displacement curves can be used to correct this problem. Examples are presented here showing the "mismatch twist" and the resulting correction. This error in J-R curve evaluation can give rise to several problems including incorrectly inferred size effects and unconservative values of the tearing modulus used for instability prediction. It is also a problem for the case where R curve extrapolation is necessary.

## KEYWORDS

Fracture toughness; J-R curve; crack extension; normalization; mismatch twist.

## INTRODUCTION

Ductile fracture toughness can be measured with the R curve where the crack extension $\Delta a$ is plotted as a function of a mechanical characterizing parameter J or CTOD (1, 2). In order to develop the R curve from a laboratory test, a continuous measure of load, displacement and crack length is needed. Load and displacement can be directly measured on a test specimen but crack length requires an indirect measurement. Several methods have been developed for indirect crack length measurement, most commonly used is the elastic unloading compliance method (3) which has been incorporated into the ASTM standard test method for the J-R curve (E1152).

Elastic compliance measurement of crack length requires some sophisticated gages and recording equipment and can be a difficult task for the novice. Even for the experienced testor, crack length measurement by this method can produce error. The standard test method allows 15 percent error in measurement of crack length change (comparing compliance measured crack length change to a physical measurement on the fracture surface), but this error is often exceeded. For the J-R curve, incorrect crack length measurement can influence the calculated value of J as well as the crack length. The result can be an artificial twisting of the J-R curve, or "mismatch twist".

The R curve is often used for stability analysis where the slope is the important parameter (4). An error in the R curve shape could cause a large error in this slope. Also R curves developed on small laboratory specimens may not result in enough crack extension to analyze larger structures and extrapolation may be desired. A mismatch twist will not give a correct final section of R curve for extrapolation. The R curve can be anchored to a correct final point by physically matching the final crack length. A method of J-R curve analysis using normalized load and displacement (5, 6) can insure a correct final point and eliminate any R curve twist due to mismatch. This paper explains the cause of "mismatch twist", a method for eliminating it and shows some example results of this correction.

## MISMATCH TWIST

The J-R curve is a plot of J versus crack extension and is determined by the standard ASTM method E1152, where J is plotted on the ordinate and crack extension, $\Delta a$, on the abscissa. An error in $\Delta a$ measurement directly changes the value on the abscissa, however, the J calculation contains crack length terms and plastic displacement terms which are also influenced by crack length. Therefore an incorrect $\Delta a$ measurement also changes the value of J. An over estimate of $\Delta a$ reduces J whereas underestimate increases J. The result for a single point is shown in Figure 1a. An accumulation of incorrect $\Delta a$ values causes a twisting of the J-R curve as shown in Figure 1b. Serious mismatch of $\Delta a$ can change the final R curve slope significantly.

## NORMALIZED J-R CURVE EVALUATION

A method for evaluating the J-R curve form normalized load and displacement has been previously presented by the authors (5, 6, 7). This method uses the relationships between load, plastic and elastic displacement and crack length suggested by Ernst (8). These relationships can be given as

$$v = v_{el} + v_{pl} = PC\left(\frac{a}{W}\right) + h\left(P_N\right) \quad (1)$$

where the displacement, v, is separated into elastic and plastic components, $v_{el}$ and $v_{pl}$, which are related to load P, through elastic compliance C(a/W) and a function of normalized load $h(P_N)$. The normalized load $P_N$ is written for deeply cracked bend specimens as

$$P_N = \frac{P_W}{Bb^2 g(b/W)} = H\left(\frac{v_{pl}}{W}\right) \quad (2)$$

Where W is specimen width, b is uncracked ligament (W-a) and B is thickness. The relationship between load, displacement and crack length is given by equation 1. The compliance function C(a/W) is tabulated in Handbooks, therefore if the plasticity function $h(P_N)$ is known, crack length can be continuously determined from values of P and v. The method used by the authors was to assume a functional form with some unknown constants and determine constants at known values of crack length, namely initial and final values. The functional form of $h(P_N)$ used most successfully combined a power law relationship with a straight line equation.

The functional form of h is shown graphically in Figure 2. Where normalized load $P_N$ is plotted versus normalized plastic displacement $v_{pl}/W$. This plot can be used to directly calculate J (6, 9) from



$$J = J_{el} + J_{pl} = \frac{K^2}{E} + b\eta g A_N \quad (3)$$

where E is effective elastic modulus $\eta$ is a function of a/W used in J calculation and $A_N$ is area under the normalized plot of Figure 2. This calculation of J can be made directly from known values of load, displacement and crack length and differs from the traditional J calculation which is made incrementally.

The direct determination of J and the use of the final crack length in determining $h(P_N)$ causes the crack length to go through a final point which agrees with the physically measured crack length. Therefore, the initial and final parts of the R curve are correct when analyzed by the normalization method. The only questionable part lies in the center section where all crack length values are inferred by the relationship in equations 1 and 2. Numerous examples were checked where the R curve determined from the normalization method was compared with the one determined from the compliance method (6). Examples where the compliance method predicted well the final measured crack length were used for comparison. A typical example illustrating the good agreement is shown in Figure 3. This shows a crack length difference of about 0.015 inch (0.4 mm) which occurs in the middle of the R curve. Average differences were usually in this order and affect only the crack length prediction not the J calculation.

## MISMATCH CORRECTION

Since the normalization procedure gives the most accurate determination of the J-R curve at the large $\Delta a$ values (where compliance seems to be most inaccurate), this method can be used to correct some of the compliance determined R curves. The mismatch occurs both in crack length and J values. Equation 3 shows the parameters used in the determination of the plastic component of J. From these both b and $A_N$ contribute most to error in J since both are strong functions of crack length. The other parameters g and $\eta$ vary weakly with crack length and do not contribute significantly. Figures 4 and 5 show some examples of J-R curves where the final crack length was incorrectly predicted by compliance. In Figure 4, a compact specimen of width 1 inch (25 mm) had an error in final crack length of 22% with compliance overestimating the physical crack length. The top part of the figure shows the result for the normalized load and plastic displacement plot comparing compliance prediction with the normalized prediction. The bottom shows the resulting J-R curves. For this case, the over prediction of crack length resulted in an underprediction of ligament b and an underprediction of normalized area $A_N$. The latter is largely due to the error in prediction of plastic displacement that results from incorrect crack length. The combined result of the two is a large underprediction of the final J and a "mismatch twist" of the final part of the R curve that is great enough to cause a negative slope. The normalized method corrects this part of the R curve and shows the correct continually increasing slope.

Figure 5 shows another example of a compact specimen with W = 4 inches (102 mm). In this case, the final crack length was underpredicted by compliance as compared with the final measured value by 38%. The resulting J-R curve Figure 5 shows the opposite result from Figure 4b in that the compliance predicted R curve has an upward twist because the final J is overestimated. This causes J-R curves to take a final slope which is too large.

## MISMATCH CONSEQUENCE

The examples shown in Figures 4 and 5 were both part of individual studies to examine effects of size on J-R curve (10). The error in the final crack length prediction led to some incorrect conclusions. The R curve example of Figure 4 is

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plotted with a specimen of the same material but 20 times larger in Figure 6. The conclusion here from the compliance evaluation of the R curve was that the smaller specimen had an apparent size effect at the larger values of crack extension. This was then corrected by introducing a new correlating parameter, the modified J, which eliminated the apparent size effect (11, 12). However, all that was really needed to correct this was an analysis to correct the final crack length. This is provided by the normalization method. As shown in Figure 6 the apparent size effect is eliminated when correct final values of crack extension are used in the J-R curve evaluation.

The example in Figure 5 shows the effect of mismatch on the final R curve slope. This slope is used in a tearing modulus, T, instability analysis (4). The prediction of tearing modulus from compliance R curve is about 12. The normalization procedure was used to correctly predict the tearing modulus of about 5. this shows an overestimate of tearing modulus of a factor of 2.4. Since the usual mismatch in final crack length is to underpredict with the compliance estimate, as in this example, R curve data measured by compliance would typically overestimate the materials resistance to tearing as estimated by T. This is an unconservative error and may account in part for unconservative applications of R curves to structural analysis.

An additional case where a correct final section of the J-R curve is essential is in the extrapolation of this curve. Very often the length of the R curve generated from the test specimen does not have sufficient crack extension for the intended application. Use of larger specimens is often not an alternative as in the case of nuclear surveillance specimens where the specimen size is dictated by the size of the surveillance capsule, and an extrapolation of the R curve is necessary. For an accurate extrapolation the final section of the R curve must be correct. The method of normalization gives a correct final crack length which insures that the R curve slope should be nearly correct. This then provides a good basis for R curve extrapolation.

## SUMMARY

The "mismatch twist" caused by incorrectly predicting final crack length on a J-R curve can cause several problems including unconservative estimates of tearing modulus and apparent size effects which are not correct. The normalized method for evaluating the J-R curve eliminates the error in final crack length and eliminates this "mismatch twist". A correct final region of J-R curves can be especially important where an extrapolation of the R curve is necessary.

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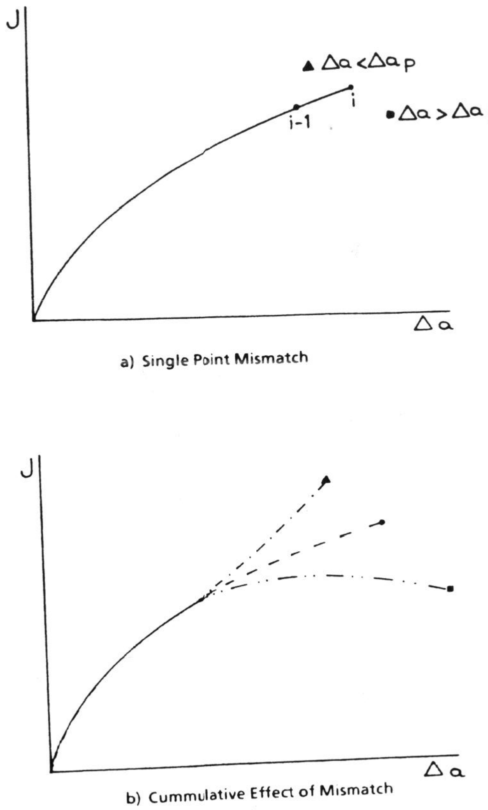



Figure 1 Schematic Effect of Mismatching Final Δa on the J R Curve



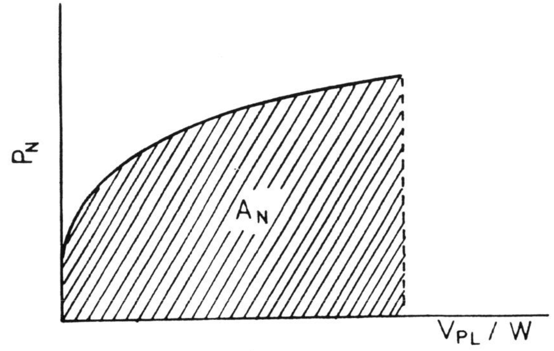



Figure 2 Normalized Load versus Plastic Displacement

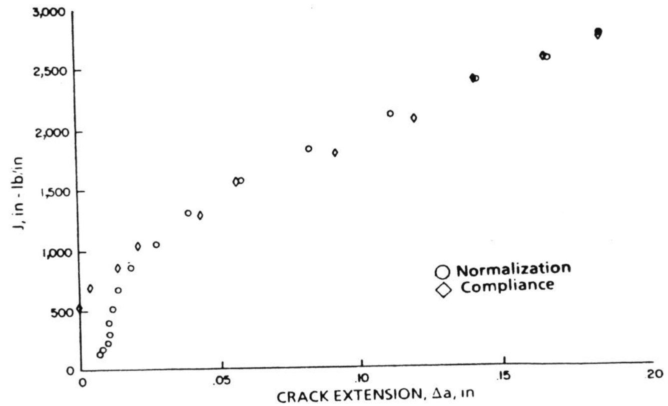
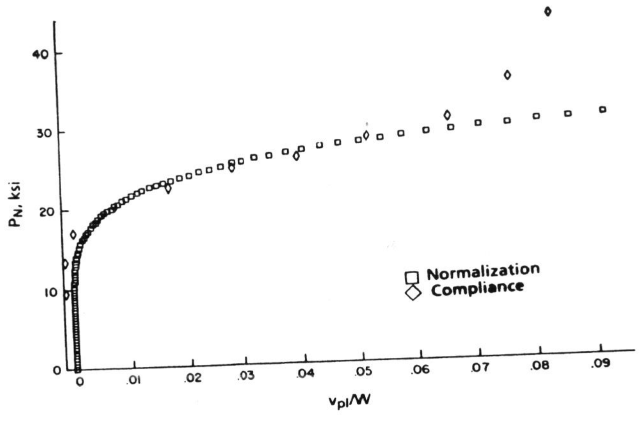



Figure 3 J-R Curve Normalization versus Compliance for A508 Steel



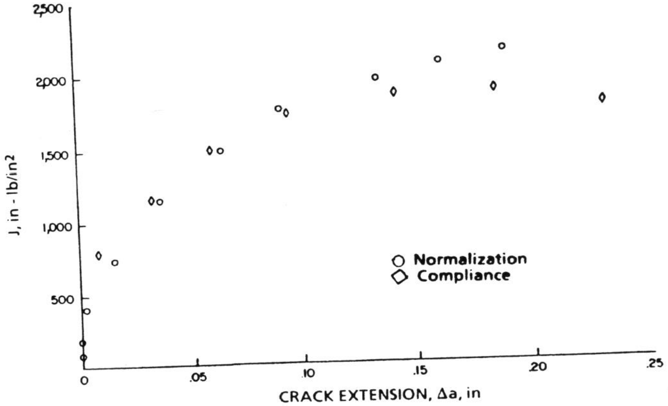




Figure 4 Normalized Load Versus Displacement (Top) and J-R Curve Normalization and Compliance (Bottom) for an A508 Steel.



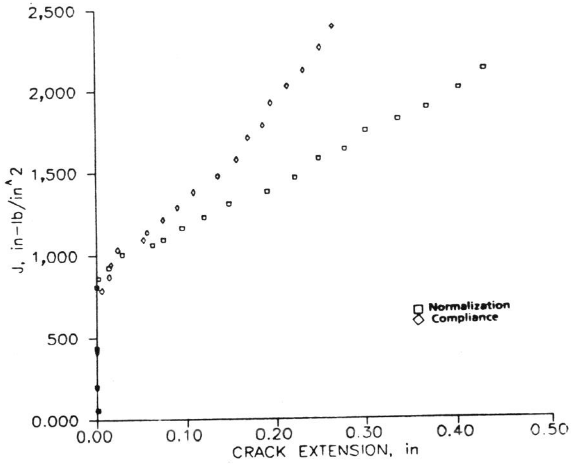



Figure 5 J-R Curve Normalization and Compliance Methods for a Ni Steel.

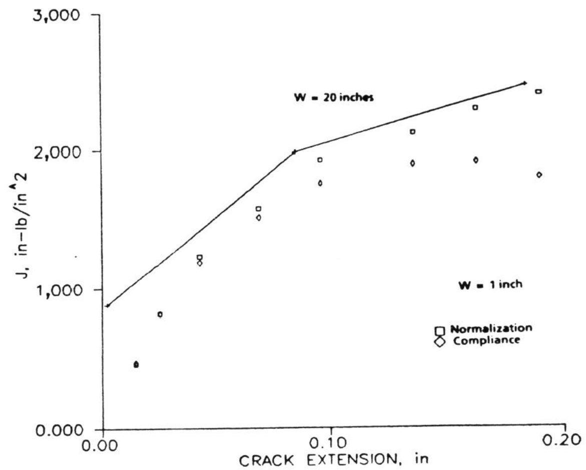



Figure 6 J-R Curve Size Comparison for a Large (20 inches) and Small (1 inch) Specimen of A508 Steel.