Changing Deformation Fields Surrounding a Growing Ductile Crack

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ABSTRACT

A SEN specimen made of AL 2024-0 is loaded to failure quasi-statically. A laser-optical method is used to map the deformation field surrounding the crack tip at different stages of loading. At the early stage of loading there exists a HRR field surrounding the crack tip. However, just before the arrest of crack growing the crack tip deformation is not characterizable by any of the known theoretical methods. When crack growth ensures, there seems to be significient experimental evidence to suggest the existence of a logarithmic singularity as proposed by Gao and Hwang(1983).

KEYWORDS

Crack tip deformation; elastic-plastic fracture; HRR field, GHD field.

INTRODUCTION

The deformation field at a crack tip is one of the central concerns in the studies of fracture mechanics. It is well established that in the stage of elastic deformation the crack tip field is characterized by the so-called stress intensity factor K (William, 1957). For a ductile material such as metal the crack tip yields and blunts as the load to the specimen is increased. If the crack remains stationary and is undergoing small scale yielding, the crack tip is characterized by the so-called HRR field (Rice, 1968). As the specimen is further loaded the crack starts to grow. First there is a stage of transient stable crack growth wherein the crack extends disimilarly. Then a stage of steady state growth ensues, wherein the crack grows self similarly. The deformation field of a growing crack has been studied by Gao and Dai and Hwang(1985). The validity of the K field description has been established experimentally by many workers. Recently Chiang et al(1987, 1988a) have also shown experimentally the existence of a HRR field when the crack tip is deformed plastically. An attempt to show the presence of GHD field (after Gao, Hwang and Dai) in a growing crack was also made by Chiang et al(1988b). In this paper we investigate the deformation field of a simple edge notch (SEN) Al 2024-0 specimen as we bring the load gradually up from blunting the crack to the stage of approximately steady state crack propagation. We compare the deformation field with that predicated by HRR and GHD theories. We ideatify the stages at which a particular theoretical description seems to fit the experimental result.

BRIEF DESCRIPTION OF THEORETICAL CRACK TIP FIELDS

The so-called HRR field at the tip of a stationary crack under the assumption of small scale yielding are

$$u_i - \tilde{u} = \alpha\epsilon_o r\left[\frac{J}{\alpha\epsilon_o\sigma_o I_n r}\right]^{\frac{n}{n+1}}\tilde{u}_i(\theta, n) \quad (1)$$

$$\epsilon_{ij} = \alpha\epsilon_o\left[\frac{J}{\alpha\epsilon_o\sigma_o I_n r}\right]^{\frac{n}{n+1}}\tilde{\epsilon}_{ij}(\theta, n) \quad (2)$$

$$\sigma_{ij} = \sigma_0\left[\frac{J}{\alpha\epsilon_o\sigma_o I_n r}\right]^{\frac{1}{n+1}}\tilde{\sigma}_{ij}(\theta, n) \quad (3)$$

where J is Rice's J-integral (Rice, 1968), I_n is a dimensionless integration constant, α and n are the Ramberg-Osgood constants, ϵ_0 and σ_0 yielding strain and stress of the material; $\tilde{u}_i$, $\tilde{\epsilon}_{ij}$ and $\tilde{\sigma}_{ij}$ are the dimensionless functions depending on θ, n and J integral as reported in (Rice, 1968). Equation (1)-(3) can be converted into the following forms

$$\log u_i = C_1 + \frac{1}{n+1}\log r \quad (4)$$

$$\log \epsilon_{ij} = C_2 - \frac{n}{n+1}\log r \quad (5)$$

$$\log \sigma_{ij} = C_3 - \frac{1}{n+1}\log r \quad (6)$$

where

$$C_1 = \log[(\alpha\epsilon_o)^{\frac{1}{n+1}}(\frac{J}{\sigma_o I_n})^{\frac{n}{n+1}}\tilde{u}_i(\theta, n)] \quad (7)$$

$$C_2 = \log[(\alpha\epsilon_o)^{\frac{n}{n+1}}(\frac{J}{\sigma_o I_n})^{\frac{n}{n+1}}\tilde{\epsilon}_{ij}(\theta, n)] \quad (8)$$

$$C_3 = \log[(\alpha\epsilon_o)^{\frac{1}{n+1}}\sigma_0(\frac{J}{\sigma_o I_n})^{\frac{1}{n+1}}\tilde{\sigma}_{ij}(\theta, n)] \quad (9)$$

From Eqs (4), (5) and (6) we can see that if we plot $\log u_i$, $\log \epsilon_{ij}$ or $\log \sigma_{ij}$ against $\log r$, the results are straight lines with 1/(n+1), -n/(n+1) or -1/(n+1) as slopes. Thus we can determine the existence of HRR field by plotting experimentally obtained stress, strain or displacement values along different radial sections from the crack tip in log-log form and observe whether or not there exists a straight line section with the appropriate slope.

For growing cracks the field is characterized by the so-called GHD field as follows:

$$u_i = r(\ln\frac{A}{r})^{\tilde{\alpha}+1}\tilde{u}_i(\theta, E, \nu, n, \alpha, F) \quad (10)$$

$$\epsilon_{ij} = (\ln\frac{A}{r})^{\tilde{\alpha}+1}\tilde{\epsilon}_{ij}(\theta, E, \nu, n, \alpha, F) \quad (11)$$

$$\sigma_{ij} = (\ln\frac{A}{r})^{\tilde{\alpha}}\tilde{\sigma}_{ij}(\theta, E, \nu, n, \alpha, F) \quad (12)$$

where $\tilde{\alpha} = 1/(n-1)$. n and α are constants of the Ramberg- Osgood material; E is the Young's modulus, ν Poisson's ratio and F a constant. A is an integration constnat depending on load; and r,θ are polar coordinates at the growing crack tip; $\bar{u}_i$, $\bar{\epsilon_{ij}}$ and $\bar{\sigma_{ij}}$ are constants having different expressions in various regions. Equation (10)-(12) can be converted into the following forms:

$$(\frac{u_i}{r})^{\frac{1}{\alpha+1}} = (logA - logr)ln0.1[\bar{u_i}(\theta, E, \nu, n, \alpha, F)]^{\frac{1}{\alpha+1}} \quad (13)$$

$$(\epsilon_{ij})^{\frac{1}{\alpha+1}} = (logA - logr)ln0.1[\bar{\epsilon_{ij}}(\theta, E, \nu, n, \alpha, F)]^{\frac{1}{\alpha+1}} \quad (14)$$

$$(\sigma_{ij})^{\frac{1}{\alpha}} = (logA - logr)ln0.1[\bar{\sigma_{ij}}(\theta, E, \nu, n, \alpha, F)]^{\frac{1}{\alpha}} \quad (15)$$

Thus it is seen from Eqs (13)-(15) that if we plot $(\frac{u_i}{r})^{\frac{1}{\alpha+1}}$, $(\epsilon_{ij})^{\frac{1}{\alpha+1}}$, $(\sigma_{ij})^{\frac{1}{\alpha}}$, against $\log r$, these are also straight lines with $(\bar{u_i})^{\frac{1}{\alpha+1}}$, $(\bar{\epsilon_{ij}})^{\frac{1}{\alpha+1}}$, and $(\bar{\sigma_{ij}})^{\frac{1}{\alpha}}$, as slopes, respectively. However, since these quantities are not yet determined, we would not know the exact slope of these straight lines. Nevertheless if experimental data do fall into a straight line, we could argue that there might be such a field as described by Eqs.(10)-(12).

THE OPTICAL METHOD

We recently developed an optical method (Chiang, 1988a), which is capable of obtaining the three dimensional surface deformation field of the area surrounding a crack tip. The optical system is as shown in Fig.1 where a laser beam is first collimated to impinge upon a line grating. A second lens collects the various diffraction beams from the grating and forms the diffraction spectrum in the form of an array of bright spots. All diffraction orders are blocked by a mask except the plus and minus first orders which are collected by a third lens to impinge upon the specimen at an inclined angle β . A standing wave of certain pitch together with a laser speckle pattern is created on the specimen surface which is printed with a moire crossed grating of certain pitch. A camera is situated at some distance away from the specimen and pointing at normal angle towards the specimen. Under load the printed crossed grating, the laser speckle field and the projected line grating are all deformed. The deformed gratings and the speckle field are then recorded on photographic film at different stages of loading. Each developed film is then

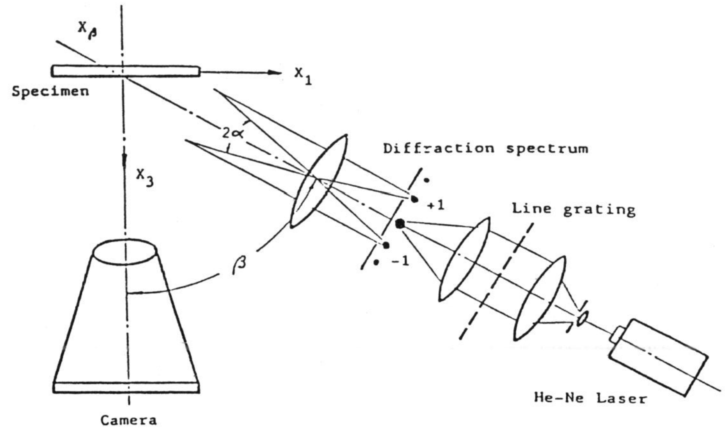


Fig.1 Arrangement of the optical method

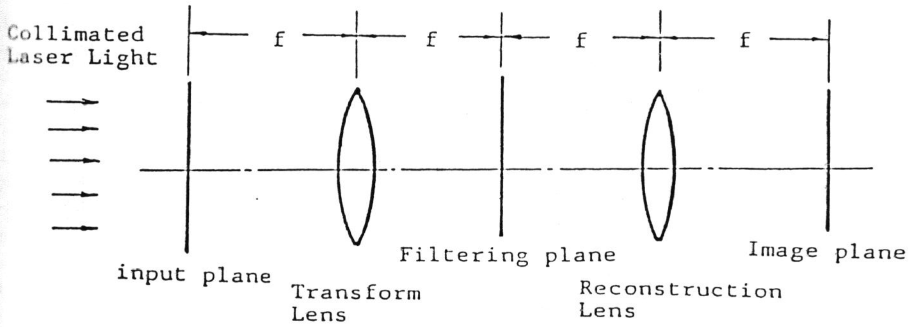


Fig.2 Optical-spatial filtering arrangement for separating u_1, u_2, & u_3 displacement fields

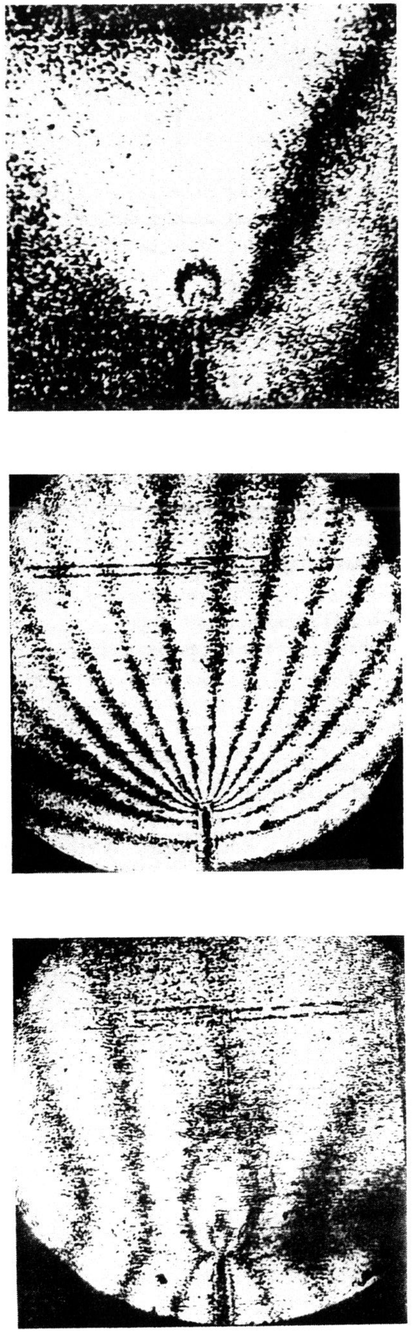
inserted into an optical Fourier filtering system as shown in Fig. 2. By properly select a filter to let pass a certain spatial frequency to be collected by the reconstrution lens a fringe pattern represneting contours of displacement component along x, y, or z direction can be obtained. A typical set of three fringe patterns thus obtained are shown in Fig. 3.

EXPERIMENTAL RESULTS AND COMPARISON

We selected for this study a SEN specimen made of Al 2024-0. The geometry and material property of the specimen are shown in Fig. 4. We assume that the material behaves according to Ramberg-Osgood law

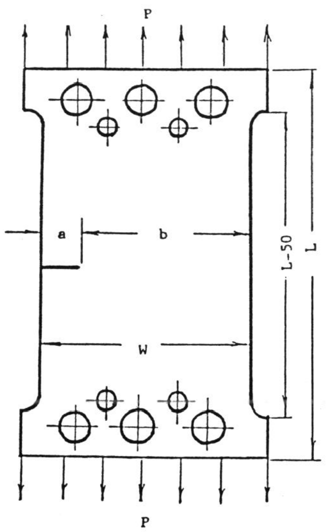
$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha(\frac{\sigma}{\sigma_0})^n, \qquad (16)$$

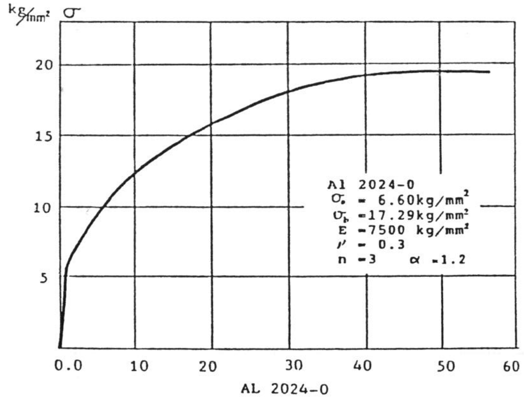
where ϵ_0 and σ_0 are the yield strain and stress, respectively. α is a constant and n is the hardening exponent. The values of these constants together with Young's modulus E and Poisson's ratio ν are listed in Fig. 4. The specimen surface was printed with a 20 l/mm crossed grating and illuminated with a laser system as shown in Fig. 1. It was loaded gradually in an Instron machine and the load-displacement curve is shown in Fig. 5. At each loading point marked on the curve the machine was stopped and the deformed grating photographed. The developed grating specklegram was then optical Fourier filtered to yield fringe patterns representing displacement contours. For this particular study we only retained the u_2-field (opening displacement) fringe patterns for easier comparison. A series of four fringe patterns of u_2-field under different loading are shown in Fig. 6. During the experiment we watched the onset of crack growth with a 10x magnifier from the back surface of the specimen. Point C on the load-displacement curve was the load at which we first noticed the initiation of a new crack emanating from the blunt initial edge notch. Fig.7 depicts the $\log u_2 - \log r$ plot along different radial directions for all six loading points (A,B,C,D,E,and F) on the load-displacement curve. The slopes of these curves start to rise as the crack is about to grow. After the crack is extended the slops start to



(c) (b) (a)

Fig.3 Typical fringe pattern representing contours of displacemence component of (a)u_1 (b)u_2 and (c)u_3





Typical σ – ε Curve

geometry

material	a mm	b mm	W mm	L mm	t mm
2024-0	21	55.2	76.2	254	3.18

Material properties of Al 2024-0

ϵ_0	σ_0	E	ν	α	n
0.8×10^{-3}	$6.6 kg/mm^2$	$7500 kg/mm^2$	0.3	1.2	3

Fig.4 Specimen geometry & material properties

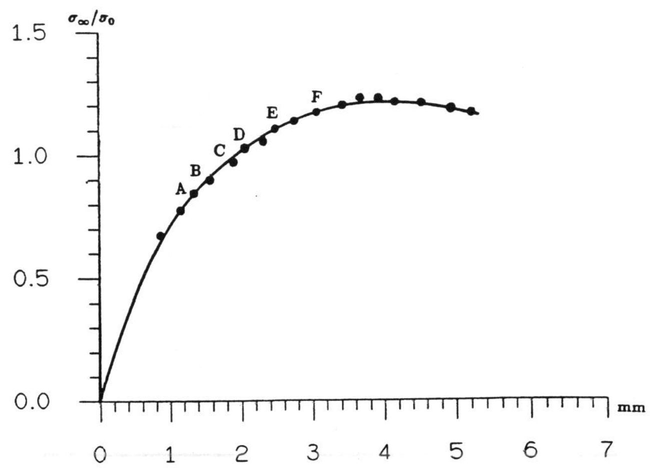
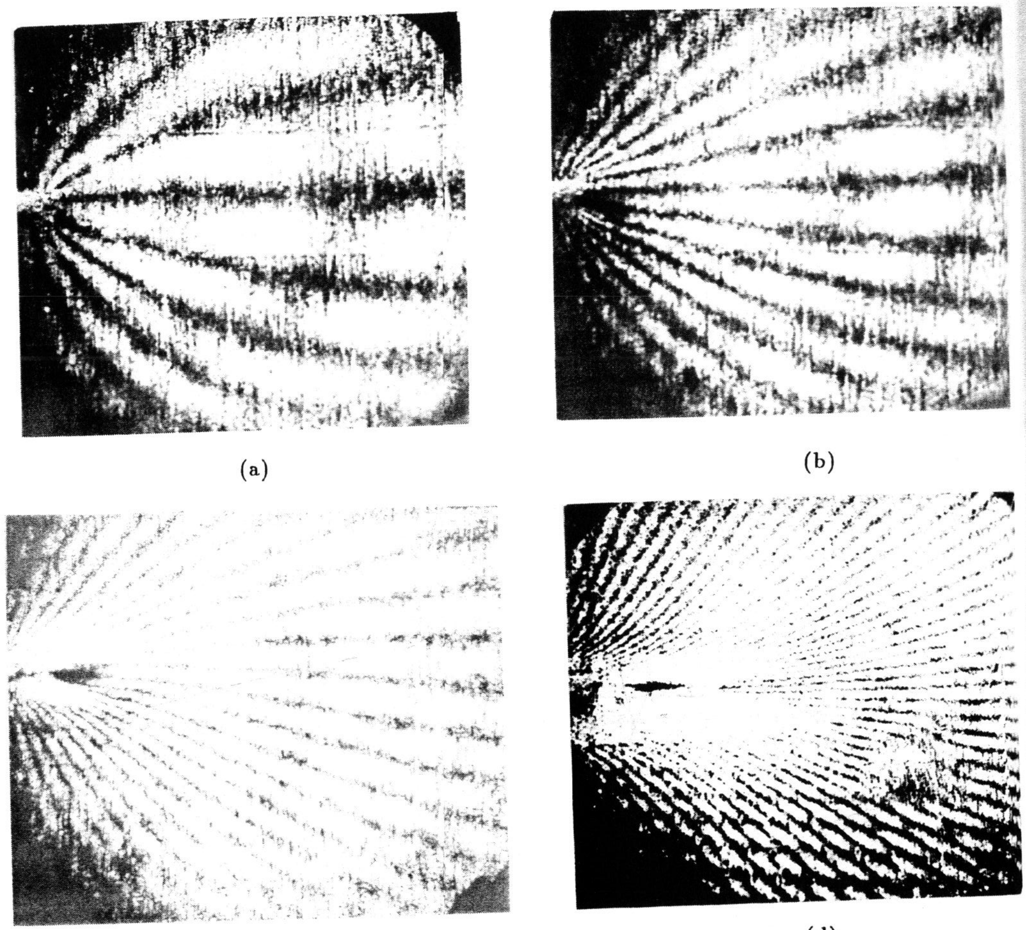


Fig.5 Load-displacement curve

drop and they become quite flat when the crack growth approaches steady state. Theoretical slopes of K-field (1/2) and HRR field (1/4) are also plotted on these drawings. It is interesting to note that at loading point A the displacement curves show a region whose slope is very close to the HRR field slope. To the right of this region there seems to exist a section whose slope can be approximated by 1/2, i.e. the K-field slope. At loading point B, however, the slopes of these curves are raised that one can resonably state that there exist no HRR field. Curiously though a certain region of these curves still posseses a 1/2 slope, but one cannot term them K-field because substantial yielding has occurred at the crack tip at this stage. When crack growth initiates the slopes of these curves start to drop down, passes the slope of HRR field and become very flat as the crack propogation approches steady state. HRR field does not exist in these situations because unloading has occurred which violates the basic assumption of the HRR field theory. Fig. 8 shows the corresponding plots of $(\frac{u_2}{r})^{2/3}$ vs $\log r$ for the six loading points.

It is interesting to note that at points A and B no straight line portion exists in any of the curves. At point C where we observed the initiation of crack growth, the plots start to show straight line and portions this trend continues throughout the rest of the loading points (D,E, and F). It should be noted



(a) (b) (c) (d)

Fig.6 u_2-field fringe of the SEN specimen under different load:
(a) $\sigma_\infty/\sigma_0 = 0.882$ (b) $\sigma_\infty/\sigma_0 = 1.057$ (c) $\sigma_\infty/\sigma_0 = 1.163$ (d) $\sigma_\infty/\sigma_0 = 1.226$

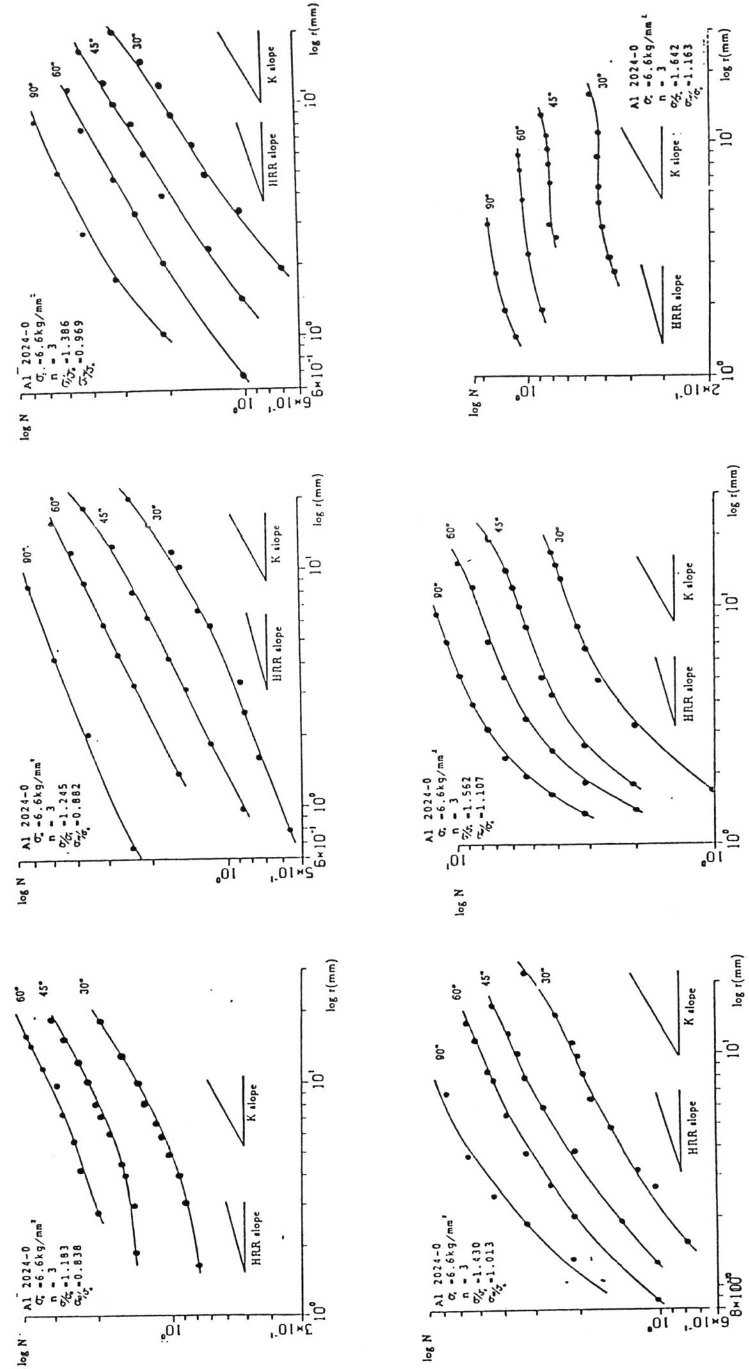


Fig.7 Displacement plots for the determination of HRR Field

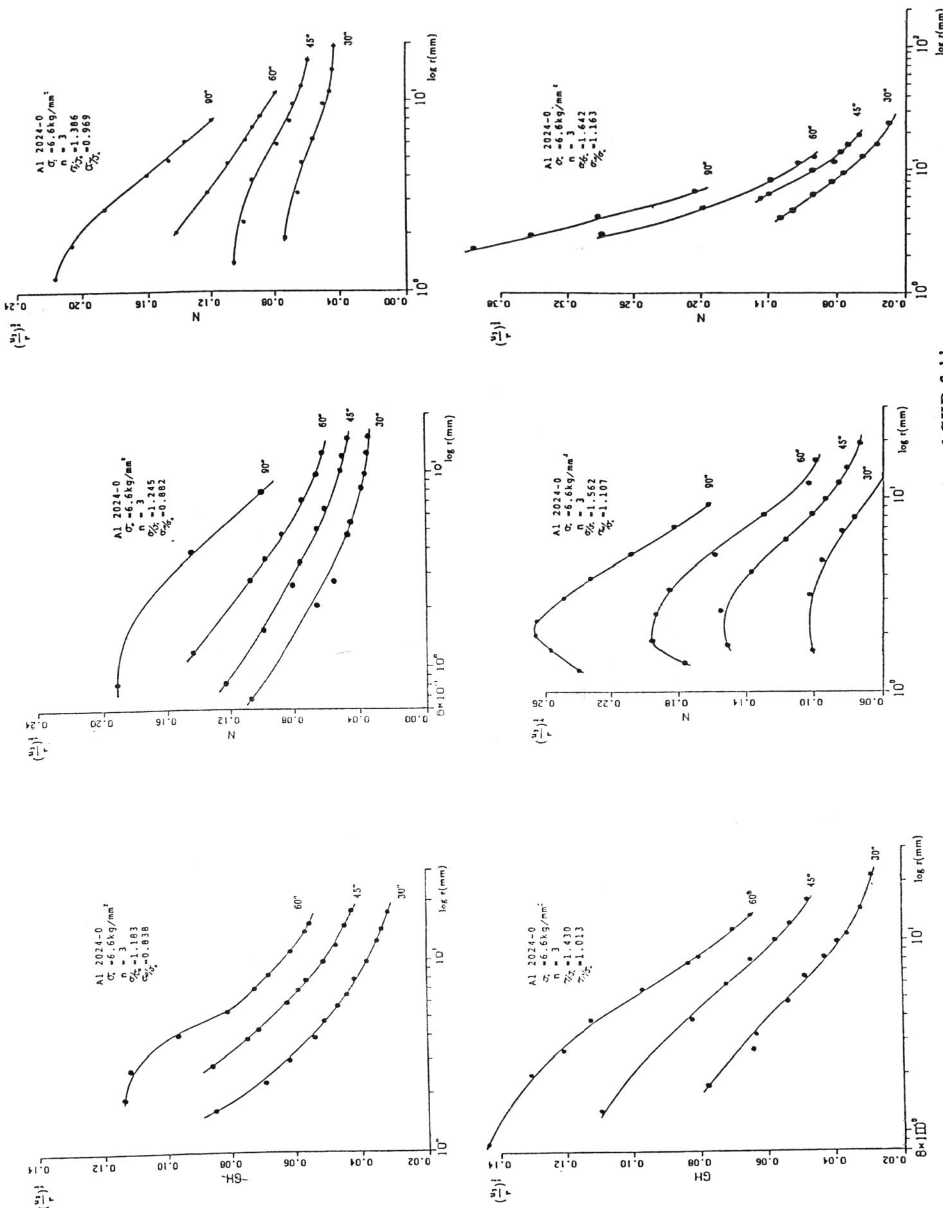


Fig.8 Displacement plots for the determination of GHD field

that in contrast to the HRR field the straight line portions of the plot do not posses the same slopes. And since the theoretical slopes are not avaiable we cannot make direct comparisons. Thus, the experimental evidence only qualifatively indicates the validity of the GHD field as described by Eqs (10)-(12).

CONCLUSION AND DISCUSSION

By using an optical technique we have been able to follow the deformation field of a ductile crack from its stationary stage to steady state propagation stage. At the begining there exists a HRR field at the crack tip surrounded by a K-field. As the crack tip blunts significantly but before the onset of crack growth the singularity changes drastically and it can no longer be described as a HRR field. As the crack starts to grow the singularity changes to that of logrithmic one as described by the GHD field. Since we do not have the explicit forms of the GHD field the comparison between theory and experiment is only qualitative; although experimental evidence strongly suggests the existence of such a field. More experiments are needed to buttress this asertion.

ACKNOWLEGEMENT

The authors would like to gratefully thank the financial support for this work provided by the Office of Naval Research, Mechanics Division, throught Grant No. N0001482K0566 and the encouragement of Dr.Y. Rajapakse, the Scientic Officer. The completion of this manuscript has been help by X.M. Hu and Y.Y. Wang.

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