Application of Fracture Mechanics to Strain Localization Mechanism

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ABSTRACT

A microslip model for the strain localization in sand deformation is proposed. The model takes account of the interaction effects between microslips and shows the process of growth of individual microslips. The formulation is based on the method of pseudo-traction, a general method for interaction problems. The strain localization as well as the strain softening phenomenon is predicted by the present model. The effect of local dilatancy, which is an important mechanism in strain localization, is also discussed.

KEYWORDS

Microslip; initial defect; strain localization; strain softening; shear band; local dilatancy; pseudo-traction.

INTRODUCTION

The highly localized deformation of sand under a triaxial loading, in the form of a narrow shear band, is a well observed phenomenon. Because the load carrying capacity is subject to the formation of shear band, the phenomenon, with its appealing academic feature, has been the focus of extensive studies for the last few decades.

Analytical studies of the strain localization problems can be divided into macro approach and micro approach. A firm theoretical basis for the former has been established through extensive studies on the bifurcation theory, which associates strain localization with the loss of ellipticity in the governing incremental equations (Hill, 1962; Rudnicki and Rice, 1975). In this category of study efforts have been made for exploring suitable material models, such as non-normality of the plastic flow rule and a yield surface vertex structure, that activate strain localization or bifurcation into a shear band. As an alternative approach based on the constitutive equations, considerable progress has also been made in the application of the finite element method (De Borst, 1988).

One of the concerns in the strain localization problems is to clarify the microscopic mechanism which causes the changing from uniform deformation to localized deformation. Studies on the bifurcation theory and the numerical approach by FEM are based on constitutive equations which describe the behavior of materials macroscopically and do not consider the microstructure of the material. Therefore, those macroscopic approaches may not provide information on the micromechanism for strain localization. To capture the mechanism, studies from microscopic point of view are inevitable.

Studies on micromechanics of granular materials have attracted attention in recent years. Although the theory is still in its primary stage of development where establishment of the constitutive equation of granular material is the major concern, the effort to study granular behaviors is considered to lead to a better understanding of the micromechanism of shear band formation.

In the present study, a microslip model is proposed. Inelastic deformation of sand is modeled to be caused by slip zones formed at initial defects. The process of growth and interaction of the individual microslips are investigated. The main purpose of this study is to clarify the dominant micromechanism of strain localization in the deformation of sand. The extension of the present work is considered to provide comprehensive answers to the fundamental questions such as, why does shear band occur? what is the effect of the boundaries on strain localization? etc.

In the following, the microslip model is introduced and the formulation for the model is presented. Illustrative examples are given, which show strain localization and strain softening.

THE MICROSLIP MODEL

Sand is one of typical granular materials. Under the action of external loads, contact forces develop between adjacent particles. Once the shear force exceeds the shear resistance at contact points, relative slipping between particles accompanied with their displacements and rotations takes place, causing the rearrangement of the structure of sand particles. This kind of relative movement between granules contributes to the overall deformation of sand. Unlike cohesive soil, the inter-particle shear resistance in sand is frictional in nature. The frictional sliding is understood as the fundamental mechanism of inelastic deformation of sand.

Mechanics of granular material, which studies the behavior of

an assembly of particles, has a long way to correlate the behavior of each particle to the overall deformation and the strain localization. The continuum approach, which is based on constitutive equations, may not clarify the relationship between micromechanism and overall behavior. To study the mechanism of sand deformation, an intermediate approach with a simplified model is necessary.

To study the fundamental problems in strain localizations, we introduce the microslip model. We consider an elastic body containing a certain number of initial defects. Under external loading, an initial defect produces stress concentration which leads to the formation of slip zones; see Fig.1. Here a microslip is defined as an initial defect plus two associated slip zones. Microslips represent inelastic deformations of sand in local regions which are the result of relative motions among the particles inside each region, and the elastic body surrounding microslips takes care of the elastic constraint.

Further, the present model considers local dilatancy of sand as an important factor for strain localization. In metals, shear deformation normally does not cause volume change. In sand, however, shear deformation is commonly accompanied by significant volume change, which results from repacking the grains. Contrast to the overall contraction by shearing in loose sand, local heaving in dense sand contributes to the overall volume expansion. Although the global dilatancy is a well-known phenomenon, the effect of local dilatancy on strain localization has not been studied. When local uplift of sand particles takes place, it causes resistance from the surrounding material. At the same time, it alters the stress field of the surrounding material and thus has strong effect on adjacent particles. This mechanism of local dilatancy is included in the present model to study its effect on the strain localization.

To illustrate the model, first we consider a single microslip inside an infinite plane. The slip zone formed at the initial defect is modeled by a crack-like slit. We assume frictional sliding along the microslip with a simple frictional condition;

$$\tau_s = \sigma_n \cdot \tan\Phi o \qquad \text{on initial defect} \tag{1}$$

$$\tau_s = \sigma_n \cdot \tan\Phi c \qquad \text{along slip zones } (\Phi c < \Phi o) \tag{2}$$

where τ_s is the shear stress and σ_n is the normal stress. On the initial defect, the frictional angle is taken to be Φo. Along the slip zones, the frictional angle is Φc with $\Phi o < \Phi c$. The direction of the microslip is chosen so that the frictional sliding is most likely to occur, with an angle of $45^o + \Phi c/2$ with respect to the horizontal direction.

To introduce the local dilatancy which is the volume expansion due to sliding, we consider a dilatancy stress on the microslips. Instead of prescribing opening displacement associated with frictional sliding, we additionally apply the

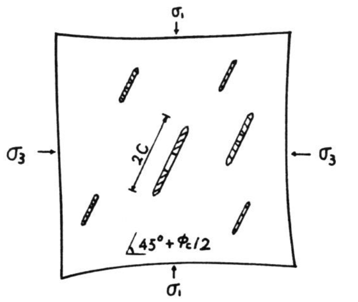

Fig.1 Illustration of micro-slips in an infinite field

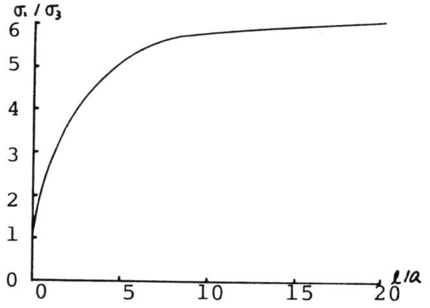

Fig.2 Slip length of a single microslip vs stress ratio

dilatancy stress which acts to open up the faces of the microslips. The amount of the dilatation is a function of the amount of sliding. Here we introduce a simple condition; the dilatancy stress is assumed to be proportional to half length of the microslip, that is,

$$\sigma_d = \alpha_d \cdot C \tag{3}$$

where σ_d is the dilatancy stress, C is the length of the slip zone L plus half length of the initial defect a, and α_d is the dilatancy coefficient.

Since the stress is relaxed by the slip zones, the stress singularity does not exist at the tip of the microslip. This leads to the following condition:

$$K_{II} = 0 \tag{4}$$

where K_{II} is the mode II stress intensity factor at the end of the microslip. For general problems we have a number of microslips along which the conditions (1)-(4) are assumed. For simplicity all microslips are assumed parallel in the critical direction mentioned above. Solving the prescribed problems with conditions (1)-(4), we obtain the length of microslips for given applied load.

For the case of a single microslip, the problem is equivalent to the well-known Dugdale model and the relation between the applied load and the length of the microslip is given by

$$\frac{L}{a} = \sec\left[\frac{\pi\cdot(\tau - \sigma\cdot\tan\Phi_o)}{2\cdot\sigma\cdot(\tan\Phi c - \tan\Phi_o)}\right] - 1 \tag{5}$$

where

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\cdot\sin\Phi c, \qquad \tau = \frac{\sigma_1 - \sigma_3}{2}\cdot\cos\Phi c \tag{6}$$

In deriving the above relation, no dilatancy stress is considered. This relation is shown in Fig.2, where the microslip grows continuously as the load increases.

In the case of multiple microslips, the existence of interaction effects between microslips complicates the problem. The mathematical formulation of the problem is given in the next section.

FORMULATION

To account for the interaction effects among microslips, we apply the method of pseudo-tractions in which unknown tractions, pseudo-tractions, are introduced to the microslips (Horii and Nemat-Nasser, 1985). The pseudo-tractions are determined so that the boundary conditions of the original problem are satisfied. The problem is formulated in the framework of the Muskhelishvili's complex stress potentials Φ and Ψ, and is completely defined by the following equations:

$$\sigma^i_{sr} = \sum_{\substack{j=1\\ j\neq i}}^{N} \Big\{2(\cos4\Phi_{ij} - 2\cos2\Phi_{ij})\cdot\sigma^j_d\cdot\frac{C_j^2}{D_{ij}^2} + \frac{1}{2}(\sin4\Phi_{ij} - \sin2\Phi_{ij})$$
$$\cdot(\tau_h - \sigma_h\tan\Phi c)\,\frac{C_j^2}{D_{ij}^2} + \frac{2}{\pi}(\sin4\Phi_{ij} - \sin2\Phi_{ij})\cdot\sigma_h$$
$$\cdot(\tan\Phi c - \tan\Phi_o)\cdot\frac{a_j\cdot C_j}{D_{ij}}\Big\} \tag{7}$$

$$\tau^i_p = \sum_{\substack{j=1\\ j\neq i}}^{N} \Big\{-\frac{1}{2}\cdot(\sin4\Phi_{ij} - \sin2\Phi_{ij})\cdot\sigma^j_d\cdot\frac{C_j^2}{D_{ij}^2} + \frac{1}{2}\cdot\cos4\Phi_{ij}$$
$$\cdot(\tau_h - \sigma_h\tan\Phi c)\cdot\frac{C_j^2}{D_{ij}^2} + \frac{2}{\pi}\cdot\cos4\Phi_{ij}\cdot\sigma_h\cdot(\tan\Phi c - \tan\Phi_o)\cdot\frac{a_j\cdot C_j}{D_{ij}}\Big\} \tag{8}$$

$$C_i\cdot[\tau_h + \tau^i_p - (\sigma_h + \sigma^i_{sr} + \sigma^i_d)\tan\Phi c] + \frac{2}{\pi}\cdot a_i\cdot(\sigma_h + \sigma^i_{sr} + \sigma^i_d)$$
$$\cdot(\tan\Phi c - \tan\Phi_o) = 0 \tag{9}$$

where $i = 1,2,\cdots,N$, Φ_{ij} and D_{ij} are the relative angle and distance between the ith and jth defect respectively. In the above equations, τ^i_p is the pseudo-traction and σ^i_{sr} is the normal stress on the ith microslip; τ_h and σ_h are the homogeneous stresses defined by (6) and σ_d is given by (3). The above equations are derived under the conditions $|C_j/D_{ij}|<<1$ and $|a_j/D_{ij}|<<1$. The system of non-linear equations are solved by Newton-Raphson method. Eigenvalue analysis is carried out to find possible bifurcation points so as to obtain the bifurcated solutions. When the growth of microslips is not highly localized, the so-called arc length control technique is employed.

RESULTS AND DISCUSSION

The system of equations derived in the previous section is

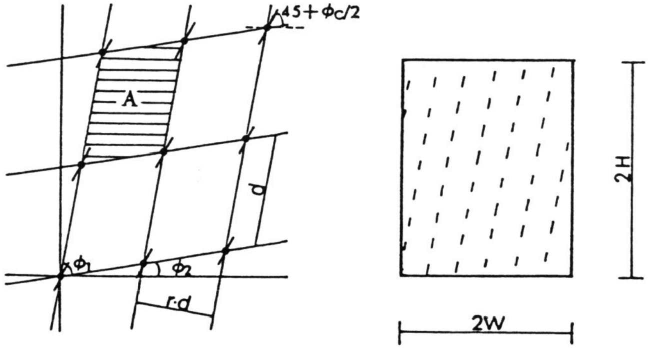

Fig.3 Illustration of arrangement of initial defects

solved for given distribution of initial defects (D_{ij} and Φ^{ij}), with the given values of the frictional angle Φ_o and Φ_c, the dilatancy coefficient α_d, the size of initial defects a_i and the confining pressure σ_3. Note that quantities with the dimension of length and stress are nondimensionalized with the initial defect size a_0 and the stress σ_3 respectively. Random distribution of the initial defects may be the straightforward assumption. However this may create the problem of having too closely positioned defects, which results in strange behavior dominated by these microslips. In the present study, we arrange the initial defects in the way specified in Fig.3, where the initial defects are distributed inside the given rectangular region. By the illustrated way different arrangements of defects are obtained by changing Φ_1 and Φ_2 with the density of defects unchanged (d and r are kept constant).

Since little information on local dilatancy is available from experimental observations, the value of the parameter α_d is assigned somewhat arbitrarily. The range of variation suggested by the present calculations, however, is 0.05 N/mm³ < α_d < 0.10 N/mm³.

As an illustrative example, we consider a case with 15 initial defects, shown in Fig.4. At the initial stage, the microslips extend almost uniformly as the load increases. As the load exceeds a certain level, the central microslip becomes more active than the rest, taking a leading role in the deformation process. Before reaching the maximum load, except for the microslips in the diagonal region, the microslips stop growing. This indicates the process of strain localization, or the formation of shear band. At and after the peak load these microslips start to shrink while those active microslips in the diagonal region accelerate their elongation. The shear band patterns shown in Fig.4 are in qualitative agreement with the experimental observations, with the solid lines for the

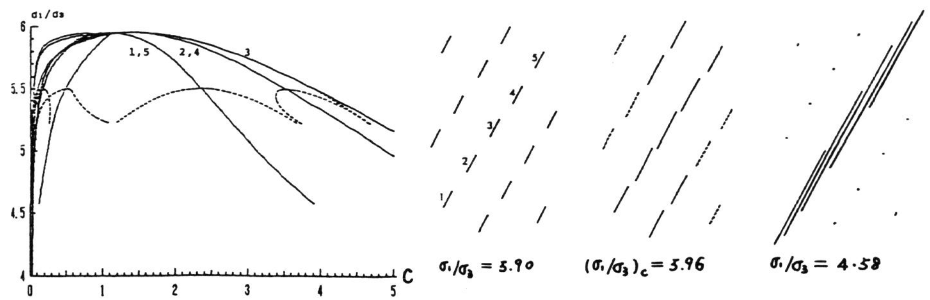

Fig.4 Microslip growth pattern and microslip length vs stress ratio in case of 15 microslips (dotted line is the bifurcation path)

microslips in extension and the dotted lines for those in shrinkage. It is worth of mentioning that upon reaching the maximum stress ratio, unloading begins while the shear band continues to develop. The present model predicts strain softening behavior with extending microslips along which no softening conditions are assumed.

Eigenvalue analysis was carried out in the case study of the microslip model to find bifurcation points. After the bifurcation point has been fixed, the solution can be continued on the bifurcation path. In Fig.4 the bifurcation path is illustrated by the dotted lines. However, the bifurcated solution is out of our present interest because at the point of bifurcation, shear band has already formed by highly elongated microslips and the condition $|C_i/z_i| \ll 1$ for the microslip model is no longer observed at this stage.

In the example shown above, we did not mention the depedance of microslip growth on the arrangement of initial defects. As explained before, we consider the arrangement as shown in Fig.3. For a given density of initial defects we change the angles Φ_1 and Φ_2 to find the critical arrangement with the minimum critical load. Investigation shows that strain localization occurs only with certain arrangement of initial defects. We plot the critical loads for various combinations of Φ_1 and Φ_2 in Fig.5 with the number of initial defects fixed. It is found that shear bands form at about 40° < Φ_1 < 65° and 0° < Φ_1 < 10°. Otherwise active microslips occur in scattered regions and do not form a shear band.

As discussed earlier, local dilatancy is an important micromechanism in sand deformation. To illustrate its effect on strain localization we consider one more case. Keeping all the initial conditions in the above example except that α_d = 0, we reevaluate the relation between the microslip length and the load. The result is shown in Fig.7. As the load increases,

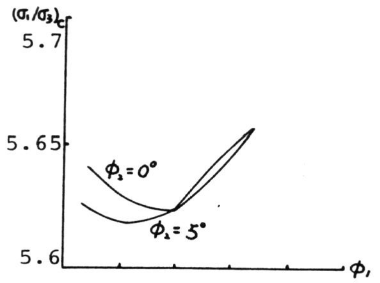

Fig.5 Critical load by different combination of ϕ_1 and ϕ_2

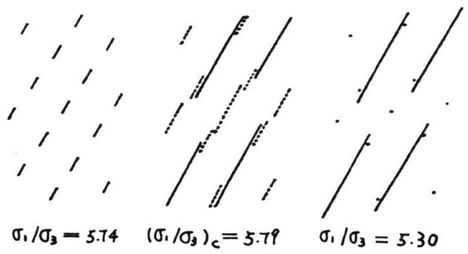

Fig.6 Pattern of microslip growth without dilatancy stress

the active microslips occur in scattered regions and no shear band is formed.

CONCLUDING REMARKS

To explain the phenomena of strain localization in sand deformation from microscopic point of view, a microslip model is proposed. The deformation of sand is modeled by an elastic body containing a certain number of microslips. Slip zones develop from initial defects under the action of applied loading. The interactions between these microslips with the dominant effect of local dilatancy lead to accelerated extension of the microslips in an inclined band and shrinkage of the other microslips, i.e., the formation of shear band.

The strain localization in the form of a narrow shear band as well as the strain softening feature have been reproduced by the present model. The effect of local dilatancy on strain localization is discussed. Studies of some related problems such as the effect of confining pressure on strain localization by the microslip model are possible. An extension of the present work including the external boundaries of a finite specimen is considered important for discussion of the boundary effects on strain localization.

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