# Analysis of Radial Cracks Emanating from an Arbitrarily Loaded Circular Hole

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## ABSTRACT

Stress intensity factors for radial crack(s) emanating from a circular hole in an infinite sheet, with radial and tangential forces acting on the hole perimeter at an arbitrary point, have been obtained by using the weight function method. The results can readily be used in damage tolerance design for structural components with arbitrarily loaded circular holes containing radial crack(s). As an application example, stress intensity factors for fretting condition have been determined, and the effect of the friction between the hole and the pin discussed.

## KEYWORDS

Weight function, strsss intensity factors, cracks at hole, fretting

## INTRODUCTION

The fastener hole is one of the most common sources of cracking in structural components because of its high stress concentration. Stress intensity factors for cracks emanating from a circular hole have therefore been investigated extensively. Various solution techniques have been employed (Bowie, 1956; Newman, 1971; Rooke and Tweed, 1980; Hsu, 1975; Shivakumar and Hsu, 1977). However, there is still a need for new stress intensity factors to cope with new load conditions. One such example is the radial crack(s) at a loaded hole with its edge subjected to both normal and shear forces distributed in an arbitrary manner. This problem is of importance to the damage tolerance analysis of numerous fastener holes and lugs in aircraft structures. Recently Rooke and Hutchins (1984) attacked the single crack problem with localized radial and tangential loads by using the method of integral transform. The method is accurate, but demands substantial computational effort. The aim of the present paper is to provide a more versatile, cost-effective and easy-to-use technique, the weight function method, for solving problems of crack(s) at a hole subjected to arbitrary load distribution. This technique has in fact been used by the first author of the present paper to determine stress intensity factors for crack(s) at holes for various loadings (Wu, 1985). In this paper attention is focused on the load condition of concentrated normal and shear forces acting at the hole perimeter. Both single and double cracks are treated.

## WEIGHT FUNCTION METHOD

It has been proven that the stress intensity factor for a crack subjected to an arbitrary crack line stress $\sigma(X)$ defined as the stress at the prospective crack site in the uncracked body, can be determined by (Bueckner 1970):

$$K=\int_0^A \sigma(X)\cdot m(A,X)\,dX$$
$$m(A,X)=\frac{E'}{K_0}\cdot\frac{\partial U_0(A,X)}{\partial A} \qquad (1)$$

where $E'=E/(1-v^2)$ for plane strain and $E'=E$ for plane stress, $A$ is the crack length , and $m(A,X)$ the weight function, $K_0(A)$ is the stress intensity and $U_0(A,X)$ crack surface displacement for a reference load condition.

For edge cracks, Wu (1984) has derived a closed-form expression for $m(A,X)$:

$$m(a,x)=\frac{1}{\sqrt{2}\pi a f_0(a)}\sum_{i=1}^{3}\beta_i(a)\cdot(a-x)^{i-\frac{3}{2}}\cdot\sqrt{\pi a R}\cdot\frac{1}{W} \qquad (2)$$

where $a=A/W$, $x=X/W$, $W$ being a characteristic length of the cracked body. For radial cracks, we define $a=A/R$, $x=X/R$, (Fig.1). The function $f_0(a)$ is the dimensionless stress intensity factor for the special load case of uniform pressure over the entire crack surfaces, and expressed in a polynomial form:

$$f_0(a)=K_0(A)/(\sigma\sqrt{\pi A})=\sum_0^N \alpha_n a^n \qquad (3a)$$

With $f_0(a)$ available, the functions $\beta_i(a)$ $i=1,2,3$ are completely defined by

$$\beta_1(a)=2f_0(a)\cdot a^{\frac{1}{2}}$$
$$\beta_2(a)=[4f_0'(a)\cdot a+2f_0(a)+\tfrac{3}{2}g(a)]\cdot a^{-\frac{1}{2}}$$
$$\beta_3(a)=[g'(a)\cdot a-\tfrac{1}{2}g(a)]\cdot a^{-\frac{3}{2}}$$
$$g(a)=5\pi\Phi/\sqrt{2}-20f_0(a)/3$$
$$\Phi(a)=\frac{1}{a^2}\int_0^a a\cdot f_0^2(a)\cdot da=\sum_{i,j=0}^{N}\frac{\alpha_i\alpha_j}{i+j+2}\cdot a^{i+j} \qquad (3b)$$

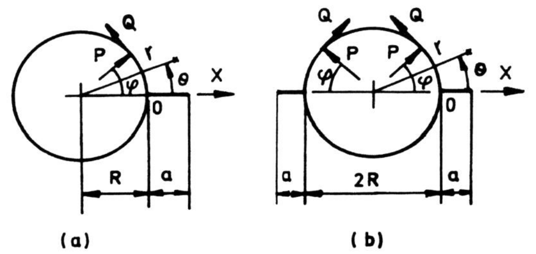


Fig. 1. Radial crack(s) at a circular hole



Knowing the weight function $m(a,x)$ for the cracked geometry, new stress intensity factors can easily be obtained through

$$K=f(a)\cdot\sigma\sqrt{\pi a W}$$
$$f(a)=\frac{1}{\sqrt{2}\pi a f_0(a)}\int_0^a\frac{\sigma(x)}{\sigma}\left(\sum_{i=1}^{3}\beta_i(a)(a-x)^{i-\frac{3}{2}}\right)dx \qquad (4)$$

Since the weight function is determined once and for all, the only work is the analysis of stress $\sigma(x)$ in the crack-free body, and a simple quadrature.

## STRESS INTENSITY FACTORS FOR CONCENTRATED FORCES

The determination of stress intensity factors by using the weight function technique requires the knowledge of stress distribution $\sigma(x)$ along the prospective crack line in the crack-free body, which can be solved in linear elasticity. For the problem considered in the present paper, i.e. a concentrated radial or tangential force acting on the hole perimeter at $\theta=\phi$, the hoop stress at $\theta=0$ is, (Fig 1)

a), Due to radial force P :

$$\sigma_{\theta\theta}^R(r,\phi)=\frac{P}{2R}[g_1^R(r,\phi)+\alpha g_2^R(r,\phi)] \qquad (5a)$$
$$g_1^R(r,\phi)=\frac{1}{\pi}\left[\frac{1}{r^2}-\frac{2r\cos\phi}{S^2}-\frac{(1+4r^2-r^4)\cos\phi}{rS^4}+\frac{4}{S^4}\right] \qquad (5b)$$
$$g_2^R(r,\phi)=\frac{\cos\phi}{\pi}(r^{-1}+r^{-3}) \qquad (5c)$$

b), Due to tangential force Q :

$$\sigma_{\theta\theta}^T(r,\phi)=\frac{Q}{2R}[g_1^T(r,\phi)+\alpha g_2^T(r,\phi)] \qquad (6a)$$
$$g_1^T(r,\phi)=\frac{\sin\phi}{\pi r S^2}\left[2(r^2+1)-\frac{(r^2-1)^2}{S^2}\right] \qquad (6b)$$
$$g_2^T(r,\phi)=-\frac{\sin\phi}{\pi}(r^{-1}+r^{-3}) \qquad (6c)$$

where

$$r=\rho/R=1+x\,,\qquad S^2=r^2-2r\cos\phi+1$$

$$\alpha=\begin{cases}(3-4\nu)/[2(1-\nu)] & \text{plane strain}\\ 3-4\nu & \text{plane stress}\end{cases}$$

Substituting eq. (5) and (6) into eq.(4), and integrating, one obtains the non-dimensional stress intensity factor f(a). In doing so, care should be taken with the singular term $1/\sqrt{(a-x)}$. An alternative to overcome this is to make "piece-linearization" of $\sigma(x)$. For a linear stress segment acting at an arbitrary part on the crack surface

$$\sigma(x)=\left[\sigma_j-\frac{x_j(\sigma_{j+1}-\sigma_j)}{x_{j+1}-x_j}\right]+\frac{x(\sigma_{j+1}-\sigma_j)}{x_{j+1}-x_j}\qquad x_j\le x\le x_{j+1} \qquad (7a)$$

the stress intensity factor is

$$f_j=\left[\sigma_j-\frac{x_j(\sigma_{j+1}-\sigma_j)}{x_{j+1}-x_j}\right]f_{jc}+\frac{\sigma_{j+1}-\sigma_j}{x_{j+1}-x_j}\cdot f_{jL}$$
$$f_{jc}=\frac{\sqrt{2}}{\pi a f_0(a)}\left[\sum_{i=1}^{3}\frac{1}{2i-1}\beta_i(a)(a-x)^{i-\frac{1}{2}}\right]_{x_{j+1}}^{x_j} \qquad (7b)$$
$$f_{jL}=\frac{\sqrt{2}}{\pi a f_0(a)}\left[\sum_{i=1}^{3}\frac{\beta_i(a)[2a+(2i-1)x]}{(2i-1)(2i+1)}(a-x)^{i-\frac{1}{2}}\right]_{x_{j+1}}^{x_j}$$

For distributed pressure over the crack surfaces,

$$f = \Sigma f_i \tag{7c}$$

## Single crack

We consider a single crack emanating from a circular hole, which is subjected to a radial force P, or a tangential force Q on the hole perimeter at $\theta=\phi$, Fig.1a. The stress intensity factor of a single crack due to the radial force P can be expressed as

$$K^R = f(g^R)\cdot\frac{P}{2R}\sqrt{\pi a R} \tag{8}$$

where $f(g^R) = f(g_1^R) + \alpha f(g_2^R)$ , $f(g_2^R) = \frac{\cos\phi}{\pi}[f(r^{-1}) + f(r^{-3})]$

$f(g_1^R)$ and $f(g_2^R)$ denote the non-dimensional stress intensity contributed by $g_1^R$ and $g_2^R$ in eq.(5b) and (5c) respectively.

Correspondingly, for tangential force Q, we have

$$K^T = f(g^T)\cdot\frac{Q}{2R}\sqrt{\pi a R} \tag{9}$$

where $f(g^T) = f(g_1^T) + \alpha f(g_2^T)$ , $f(g_2^T) = -\frac{\sin\phi}{\pi}[f(r^{-1}) + f(r^{-3})]$

$f(g_1^T)$ and $f(g_2^T)$ denote the non-dimensional stress intensity contributed by $g_1^T$ and $g_2^T$ in eq.(6b) and (6c) respectively. The terms $f(g_1^R)$ and $f(g_1^T)$ can be determined by either numerical integration or the "piece-linearization" technique, while the terms $f(g_2^R)$ and $f(g_2^T)$ can be obtained by integrating analytically (Wu, 1985). The results so obtained are in very good agreement (less than 2%) with those of Rooke and Hutchins (1984) obtained by using integral transform. Numerical values are not repeated here for brevity.

## Double cracks

We consider the problem of double cracks, i.e. two diametrically opposed cracks with equal length emanating from a circular hole, which is subjected to two concentrated radial forces P and tangential forces Q acting symmetrically on the hole perimeter, Fig.1b. The crack line stress $\sigma(x)$ in this case becomes the sum of the two $\sigma(x)$-components due to Ps or Qs. The results are, (the terms involving $\alpha$ vanish because of symmetry)

$$\sigma_{\theta\theta}^R = \frac{P}{2R}h^R(r,\phi)$$

$$\sigma_{\theta\theta}^T = \frac{Q}{2R}h^T(r,\phi) \tag{10}$$

where

$$h^R(r,\phi) = g_1^R(r,\phi) + g_1^R(r,\pi-\phi) \tag{11}$$

$$h^T(r,\phi) = g_1^T(r,\phi) + g_1^T(r,-(\pi-\phi)) \tag{12}$$

Functions $g_1^R$ and $g_1^T$ are given in eq.(5) and (6) respectively. The stress intensity factor for double cracks then becomes

$$K^R = f(h^R)\cdot\frac{P}{2R}\sqrt{\pi a R}$$

$$K^T = f(h^T)\frac{Q}{2R}\sqrt{\pi a R} \tag{13}$$

where $f(h^R)$ and $f(h^T)$ can be determined by substituting $h^R$ and $h^T$ of eq. (11) and (12) into eq.(4). Numerical values are presented in Table 1. The accuracy is estimated to be better than 2% .



Table 1. Dimensionless stress intensity factors for double cracks

a). $f(h^R)$ for normal forces b). $f(h^T)$ for shear forces

| | $\phi°$ \ $\alpha$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.07 | 0.08 | 0.09 | 0.1 | 0.2 | 0.3 | 0.5 | 0.7 | 0.9 | 1.0 | 1.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(h^R)$ | 0.5 | -38.376 | -23.681 | -15.633 | -11.515 | -8.967 | -6.013 | -5.189 | -4.481 | -3.986 | -1.617 | -.813 | -.342 | -.214 | -.095 | -.097 | -.050 |
| | 1.5 | -9.153 | -12.593 | -11.815 | -10.050 | -8.396 | -5.996 | -5.164 | -4.498 | -3.959 | -1.561 | -.834 | -.344 | -.187 | -.120 | -.102 | -.048 |
| | 2.5 | -2.833 | -5.648 | -6.838 | -6.944 | -6.527 | -5.305 | -4.730 | -4.220 | -3.777 | -1.551 | -.828 | -.337 | -.179 | -.113 | -.094 | -.042 |
| | 3.5 | -.829 | -2.629 | -3.805 | -4.406 | -4.586 | -4.280 | -3.993 | -3.688 | -3.389 | -1.518 | -.817 | -.329 | -.171 | -.105 | -.086 | -.036 |
| | 5.0 | 0.289 | -.714 | -1.521 | -2.110 | -2.495 | -2.796 | -2.790 | -2.723 | -2.618 | -1.412 | -.786 | -.315 | -.158 | -.093 | -.074 | -.026 |
| | 7.0 | 0.828 | 0.278 | -.209 | -.622 | -.955 | -1.396 | -1.522 | -1.597 | -1.632 | -1.184 | -.713 | -.291 | -.140 | -.076 | -.058 | -.013 |
| | 11.0 | 1.167 | 0.926 | 0.698 | 0.488 | 0.297 | -.023 | -.153 | -.263 | -.355 | -.625 | -.480 | -.218 | -.096 | -.041 | -.024 | 0.012 |
| | 15.0 | 1.274 | 1.135 | 1.001 | 0.873 | 0.753 | 0.537 | 0.442 | 0.354 | 0.275 | -.150 | -.209 | -.117 | -.040 | 0.0 | 0.012 | 0.038 |
| | 19.0 | 1.321 | 1.227 | 1.136 | 1.048 | 0.964 | 0.809 | 0.737 | 0.670 | 0.607 | 0.189 | 0.035 | -.002 | 0.025 | 0.046 | 0.052 | 0.063 |
| | 22.5 | 1.344 | 1.271 | 1.201 | 1.132 | 1.067 | 0.943 | 0.885 | 0.830 | 0.778 | 0.397 | 0.210 | 0.097 | 0.085 | 0.088 | 0.090 | 0.086 |
| | 32.5 | 1.373 | 1.328 | 1.285 | 1.242 | 1.201 | 1.123 | 1.086 | 1.050 | 1.015 | 0.728 | 0.538 | 0.337 | 0.251 | 0.210 | 0.197 | 0.152 |
| | 42.5 | 1.384 | 1.350 | 1.317 | 1.284 | 1.253 | 1.193 | 1.164 | 1.136 | 1.109 | 0.877 | 0.708 | 0.496 | 0.382 | 0.316 | 0.292 | 0.213 |
| | 50.0 | 1.388 | 1.358 | 1.329 | 1.301 | 1.273 | 1.220 | 1.195 | 1.170 | 1.146 | 0.938 | 0.782 | 0.575 | 0.454 | 0.379 | 0.350 | 0.253 |
| | 70.0 | 1.393 | 1.368 | 1.343 | 1.319 | 1.296 | 1.252 | 1.231 | 1.210 | 1.189 | 1.013 | 0.877 | 0.686 | 0.564 | 0.481 | 0.448 | 0.327 |
| | 90.0 | 1.394 | 1.370 | 1.347 | 1.324 | 1.302 | 1.259 | 1.239 | 1.219 | 1.200 | 1.031 | 0.900 | 0.715 | 0.595 | 0.511 | 0.477 | 0.351 |
| $f(h^T)$ | 0.5 | 79.186 | 40.038 | 26.088 | 19.225 | 15.162 | 10.573 | 9.150 | 8.049 | 7.171 | 3.289 | 2.055 | 1.146 | 0.809 | 0.645 | 0.590 | 0.406 |
| | 1.5 | 47.233 | 35.331 | 25.952 | 19.757 | 15.683 | 10.892 | 9.398 | 8.244 | 7.328 | 3.327 | 2.073 | 1.153 | 0.813 | 0.646 | 0.591 | 0.406 |
| | 2.5 | 30.623 | 26.702 | 22.314 | 18.429 | 15.313 | 11.031 | 9.575 | 8.421 | 7.491 | 3.377 | 2.095 | 1.161 | 0.816 | 0.647 | 0.592 | 0.406 |
| | 3.5 | 22.368 | 20.646 | 18.455 | 16.194 | 14.106 | 10.760 | 9.486 | 8.429 | 7.548 | 3.430 | 2.120 | 1.170 | 0.820 | 0.649 | 0.592 | 0.405 |
| | 5.0 | 15.834 | 15.117 | 14.157 | 13.065 | 11.937 | 9.825 | 8.903 | 8.081 | 7.355 | 3.497 | 2.159 | 1.184 | 0.825 | 0.651 | 0.594 | 0.405 |
| | 7.0 | 11.350 | 11.020 | 10.585 | 10.074 | 9.515 | 8.353 | 7.789 | 7.251 | 6.747 | 3.526 | 2.203 | 1.203 | 0.834 | 0.654 | 0.595 | 0.404 |
| | 11.0 | 7.196 | 7.063 | 6.900 | 6.713 | 6.506 | 6.052 | 5.813 | 5.573 | 5.333 | 3.348 | 2.219 | 1.233 | 0.848 | 0.660 | 0.598 | 0.401 |
| | 15.0 | 5.227 | 5.147 | 5.057 | 4.957 | 4.848 | 4.611 | 4.485 | 4.356 | 4.226 | 2.999 | 2.132 | 1.239 | 0.856 | 0.662 | 0.598 | 0.398 |
| | 19.0 | 4.069 | 4.014 | 3.953 | 3.888 | 3.819 | 3.670 | 3.592 | 3.512 | 3.430 | 2.625 | 1.980 | 1.216 | 0.852 | 0.659 | 0.594 | 0.392 |
| | 22.5 | 3.384 | 3.340 | 3.293 | 3.244 | 3.193 | 3.084 | 3.028 | 2.970 | 2.912 | 2.325 | 1.824 | 1.175 | 0.837 | 0.650 | 0.586 | 0.386 |
| | 32.5 | 2.201 | 2.174 | 2.147 | 2.120 | 2.092 | 2.034 | 2.005 | 1.976 | 1.947 | 1.655 | 1.391 | 0.996 | 0.751 | 0.597 | 0.541 | 0.358 |
| | 42.5 | 1.530 | 1.512 | 1.494 | 1.476 | 1.458 | 1.422 | 1.404 | 1.386 | 1.368 | 1.195 | 1.038 | 0.792 | 0.625 | 0.510 | 0.466 | 0.315 |
| | 50.0 | 1.177 | 1.163 | 1.149 | 1.136 | 1.123 | 1.096 | 1.083 | 1.069 | 1.056 | 0.932 | 0.822 | 0.646 | 0.522 | 0.434 | 0.399 | 0.274 |
| | 70.0 | 0.510 | 0.505 | 0.499 | 0.493 | 0.488 | 0.477 | 0.471 | 0.466 | 0.461 | 0.412 | 0.369 | 0.302 | 0.253 | 0.217 | 0.202 | 0.144 |
| | 90.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

## STRESS INTENSITY FACTORS FOR DISTRIBUTED LOADS

When using the weight function method to tackle crack problems involving distributed loads at the hole perimeter, either of the following two procedures can be used.

a), Determine the crack line stress $\sigma(x)$ first and then use eq.(4) to calculate f(a). This procedure is very efficient when $\sigma(x)$ is known. Table 2 shows results obtained in this way for a normal load distribution $p=p_o \sin\theta$. Also listed are results from the literature. The agreement is seen to be excellent.

Table 2. Comparison of stress intensity factors for normal loads $p=p_o \sin\theta$

| a | present | Rooke & Hutchins (1984) | Rooke & Tweed (1980) | Shivakumar & Hsu (1977) |
|---|---|---|---|---|
| 0.1 | 0.524 | 0.522 | 0.521 | 0.531 |
| 0.2 | 0.420 | 0.417 | 0.416 | 0.422 |
| 0.5 | 0.255 | 0.256 | 0.255 | 0.259 |
| 1.0 | 0.148 | 0.150 | 0.149 | 0.149 |

b), Use the non-dimensional stress intensity factors $f(g^R)$, $f(g^T)$, $f(h^R)$ and $f(h^T)$ as Green's functions, denoted by $f(\phi)$, and make summation over the hole perimeter, as illustrated in the following. For a point force at $\theta=\phi$, we have

$$K(\phi)=f(\phi)\cdot\frac{F}{2R}\sqrt{\pi a R} \tag{14}$$

For distributed loads, since $\Delta F = p_o \cdot p(\phi)\cdot\Delta\phi$

$$K=\int_0^{2\pi} dK(\phi) = f_{p_o}\cdot p_o\cdot\sqrt{\pi a R}$$

$$f_{p_o}=\frac{K}{K_{p_o}} = \frac{1}{2}\int_0^{2\pi} p(\phi)\cdot f(\phi)\cdot d\phi \;, \qquad K_{p_o}=p_o\sqrt{\pi a R} \tag{15}$$

The integration can be replaced by summation if necessary:

$$f_{p_o}=\frac{1}{2}\sum_{i=1}^{N} p(\phi_i)\cdot f(\phi_i)\cdot\Delta\phi_i$$

## APPLICATION TO FRETTING

For fastener holes with load transfer it is known that fretting forces between the hole edge and the pin can accelerate crack initiation and growth, particularly when cracks are short. The problem of fretting fatigue is of importance in damage tolerance design of fastener holes and lugs. To quantify the effect of fretting on fatigue growth life, accurate stress intensity factors induced by the corresponding normal and shear load distributions between the hole and the pin must be known.

For fretting, the load distribution is generally assumed as,

$$\sigma^R = p_o \sin\theta$$

$$\sigma^T = \mu p_o \cos^3\theta \cdot \sin\theta \tag{16}$$

where $\mu$ is the coefficient of friction, $0\le\mu\le1.0$.



Stress intensity factors due to the load of eq.(16) can be obtained by summation according to eq.(15). To show the effect of shear forces, we introduce a non-dimensional stress intensity factor $f_P$ :

$$f_P=\frac{K}{K_P} \tag{17}$$

$$K_P=\frac{P}{2R}\sqrt{\pi a R}$$

where P is the resultant force in the direction perpendicular to the crack line. For the load of eq.(16), integration leads to

$$P = 2\cdot\left(\frac{\pi}{4}+\frac{\mu}{5}\right)p_o R \tag{18}$$

and therefore

$$K_P = \left(\frac{\pi}{4}+\frac{\mu}{5}\right)\cdot K_{p_o} \tag{19}$$

The stress intensity due to the combination of $\sigma^R$ and $\sigma^T$ becomes

$$f_P = \frac{K^R+K^T}{K_P} = \frac{f^R_{p_o}+f^T_{p_o}}{\frac{\pi}{4}+\frac{\mu}{5}} \tag{20}$$

where $f^R_{p_o}$ and $f^T_{p_o}$ can be obtained by substituting $p(\theta)=\sin\theta$, and $p(\theta)=\cos^3\theta\sin\theta$ together with the relevant values of $f(\phi_i)$ from eq.(8), (9) and (13) into eq.(15). Figure 2 shows the results of $f_P$. It is noted that for a given load P, although due to the presence of shear, the normal force between the pin and the hole is reduced, and so is the corresponding stress intensity $K^R$, the increase of $K^T$ due to the shear force drives the resultant stress intensity, $K^R+K^T$ up appreciably. The increase is particularly pronounced for small normalized crack lengths. Since the crack growth life is mostly spent while cracks are short, the effect of shear is therefore very important in making realistic life predictions. For larger cracks, the stress intensity is insensitive to the detailed load distribution on the hole perimeter as expected, and hence, friction can be ignored.

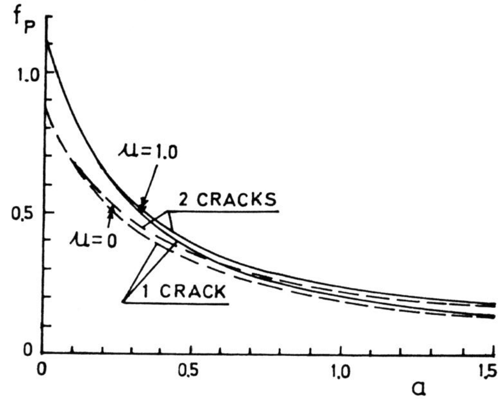


Fig. 2. Stress intensity factors for a loaded hole, effect of shear loads between hole and pin

## CONCLUSIONS

A weight function technique has been used to determine stress intensity factors for crack(s) at a circular hole, which is subjected to concentrated radial and/or tangential forces on its edge. Both single crack and diametrically opposed two equal-length cracks have been considered. The results can be used , by summation, for the calculation of stress intensity factor(s) at an arbitrarily loaded hole. As an application example, one combination of radial and shear forces, which simulates the fretting condition between the pin and the hole, has been evaluated, and the effect of friction on K quantified. The versatility, simplicity, accuracy and cost-effectiveness of the weight function method has been demonstrated. The technique offers a powerful tool for solving crack problems involving complex load situations with minimal effort.

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