A Unified Fracture Theory: the Dual-parameter Fracture-criterion

ZHUNAG TAO* and H. W. LIU**
*The Central Iron and Steel Research Institute, Beijing, PRC
**Department of Mechanical and Aerospace Engineering, Syracuse University, Syracuse, NY 13244, USA

ABSTRACT

Hancock and Cowling (1980) found that the crack-tip opening displacement, δ_f, at fracture initiation varies from 90 to 900 μm. δ_f depends strongly upon specimen geometry. In this study, the crack-tip fields of Hancock and Cowling's specimens are calculated. The measured δ_f decreases with an increase in the calculated crack-tip tensile stress. This is consistent with Bridgman's observation of the decrease in fracture ductility with an increase in hydrostatic compression.

KEYWORDS

Fracture mechanics, non-linear, local criterion, dual-parameter.

INTRODUCTION

Liu (1964, 1983) has shown that the validity of the linear elastic fracture mechanics is based on the capability of the stress intensity factor, K, to characterize the entire crack-tip field, i.e. the crack-tip stress and strain fields. When the condition of *small-scale yielding* prevails, K prescribes all twelve stress and strain components even *within* the *small crack-tip plastic zone*. In other words, at fracture initiation, all twelve crack-tip stress and strain components reach their critical values as prescribed by K_c. Therefore, in reality, K should be considered as a fracture criterion consisting of twelve components. All of these twelve components are characterized by a single parameter K.

Liu (1964, 1983) has given the deductive reasoning for the capability of K to characterize the entire crack-tip field. Liu and Ke (1975), Liu and Hu (1976), and Kuo (1976) have provided the evidence with crack-tip moire strain measurements and finite element calculations. The crack-tip field calculated by Hutchinson (1968) and Rice and Rosengren (1968) provides further support for the validity of the concept that at the same K-value the entire crack-tip field is the same even within the small crack-tip plastic zone.

If fracture initiation is controlled by crack-tip stresses and strains and if K is capable of characterizing the entire crack-tip field, the value of K_c at fracture initiation must be invariant to in-plane geometric variation. On the other hand, according to the theory, even if the values of K or J are the same but the states of the crack-tip stresses differ, the values of K_c or J_c will vary accordingly. There is a substantial amount of experimental evidence showing that fracture toughness, even in the case of small-scale yielding, is dependent on the state of crack-tip stresses. Such evidence includes the thickness effect on fracture toughness, the variation of the fracture toughness under a combined load, and the fact that stable crack growth does take place. As a crack grows in a stable manner, the crack-tip configuration and, thereby, the state of crack-tip stresses change, and the critical value of K for further crack extension changes accordingly.

This paper analyzes the crack-tip fields in general yielding and proposes a dual-parameter fracture-criterion specifically for non-linear fracture mechanics and more generally as an unified fracture theory.

THE ELASTIC-PLASTIC CRACK-TIP FIELDS AND THE DUAL-PARAMETER FRACTURE-CRITERION

Hutchinson (1968) and Rice and Rosengren (1968) analyzed the crack-tip fields in a non-linear elastic solid. The deformation theory is used in their analyses. The deformation theory implies proportional loading, which in turn implies a fixed shear line pattern. However, a crack-tip shear line field may change from the constrained shear line field of small-scale yielding to a non-constrained field of general yielding. The shear line patterns in deep general yielding are characteristics of specimen geometries: constrained flow in bend specimen and double-edge-cracked tensile panel and non-constrained flow in center-cracked and single-edge- cracked tensile panels.

The maximum tensile stress in a constrained flow could be several times the flow stress, while the maximum tensile stress in a non-constrained flow is nearly equal to the flow stress. The higher tensile stress in the constrained flow will give a lower fracture toughness and vice versa.

McMeeking and Parks (1979) used the updated Lagrangian finite-deformation finite-element method based on the incremental plasticity theory, which does not presume proportional loading, to calculate the crack-tip field in center-cracked tensile panels and an edge-cracked bend specimen in general yielding. They started with a semi-circular notch tip having a very small radius. The notch tip was further blunted by the large deformation. A blunted notch tip does not have a stress or strain singularity.

McMeeking and Park's calculation for the tensile panel is shown in Fig. (1). Their data as well as those of Shih and German (1981) show that, at the same J-level, the values of stresses and strains in small-scale yielding differ widely from the values in large-scale yielding and general yielding and that the crack field may vary widely from one specimen geometry to another. Although, J does characterize the plane-strain crack-tip field in an elastic-plastic metallic specimen, it has to be subjected to limitations of specimen size and load level as recommended by McMeeking and Parks (1979). The specimen size requirement could be severe for certain geometry, and the recommended specimen size requirements need to be verified empirically.

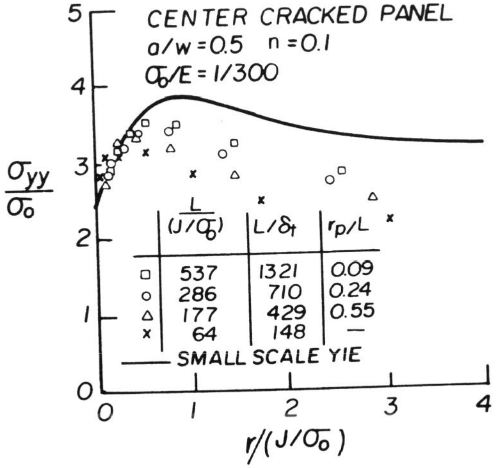

Fig. 1 σ_{yy} distribution ahead of crack tip. Center cracked tensile panel.

Extending the concept of the characteristic crack-tip field of the linear elastic fracture mechanics, J can be used as a single-parameter fracture-criterion only if J is capable of characterizing the entire crack-tip field, i.e. at the same J-value, the entire crack-tip field is the same regardless of the specimen geometry and the extent of the crack-tip plastic deformation. In other words, the applicability of J as a single-parameter fracture-criterion is assured only if the crack-tip field in a small specimen in general yielding corresponds directly to the crack-tip field, at the same J, in a large specimen in small-scale yielding. We call this the principle of direct correspondence for the applicability of J as a single-parameter fracture-criterion. To use J indiscriminately as a single-parameter fracture-criterion may introduce error.

K characterizes all twelve crack-tip stresses and strains; therefore, K can be considered as a fracture-criterion consisting of twelve components. All of these twelve components are characterized by a single parameter K. When a single parameter fails to characterize an entire crack-tip field, in principle, a fracture-criterion in terms of all of the twelve crack tip stress and strain components should be used. However, such a cumbersome fracture-criterion is difficult to apply. Thus, if a single parameter cannot be used, it is desirable to develop the next simplest criterion, i.e., a criterion consisting of two parameters. Our problem is then to select these two parameters.

Bridgman (1952) has shown that the fracture ductility of a smooth tensile bar increases with a superimposed hydrostatic pressure. A "hydrostatic tensile stress" exists at a crack tip and the hydrostatic tensile stress will reduce the local fracture ductility at a crack tip.

The ratio $(\sigma_{yy}/\bar{\sigma})$ reflects the amount of the hydrostatic tensile stress component present in the crack-tip vicinity. The crack-tip opening displacement δ reflects the plastic deformation of the entire crack-tip vicinity. Therefore, we propose to use δ_f at fracture initiation and the local stress (σ_{yy}/σ) in the crack-tip vicinity as a dual-parameter fracture-criterion. δ_f is a measure of fracture ductility, and fracture ductility is a function of the superimposed hydrostatic tensile stress, (σ_{yy}/σ). Since (σ_{yy}/σ) varies from point to point, the location at which to take the reading remains to be determined. We shall study the case of small-scale yielding first then extend our discussion to general yielding.

Prior to the fracture of a ductile and tough material, a crack tip is deformed extensively and a sharp crack tip is blunted to a round "notch". For a blunted crack tip, the maximum tensile stress exists in the interior away from the blunted crack-tip surface.

Rice and Johnson (1970) used the small deformation theory to calculate the stresses ahead of a blunted notch tip in a work-hardening material in the case of small-scale yielding. They calculated the stresses for three values of the ratio σ_o/E (0.0025, 0.0050, 0.0075) and three values of the strain hardening coefficient (n = 0, 0.1, 0.2). Figure (2) shows the plot of $\sigma_{yy}/\sigma_{yy(max)}$ versus $x/(J/\sigma_o)$. At a given value of n, all three sets of data for the three different values of σ_o/E coincide. The quantity (J/σ_o) is related to δ. Therefore, the quantity $x/(J/\sigma_o)$ can be viewed as (x/δ). The value of x/δ at the location of $\sigma_{yy(max)}$ is a function of n only. The values of x/δ in Figure (2) are close to the value of 2 found earlier by Wang (1953). The data in Figure (2) also show that the tensile stress σ_{yy} in the entire blunted crack-tip region can be characterized by the value of $\sigma_{yy(max)}$. Furthermore, even the stress σ_{yy} further away from the crack tip is capable of characterizing the tensile stress field within the entire crack-tip region.

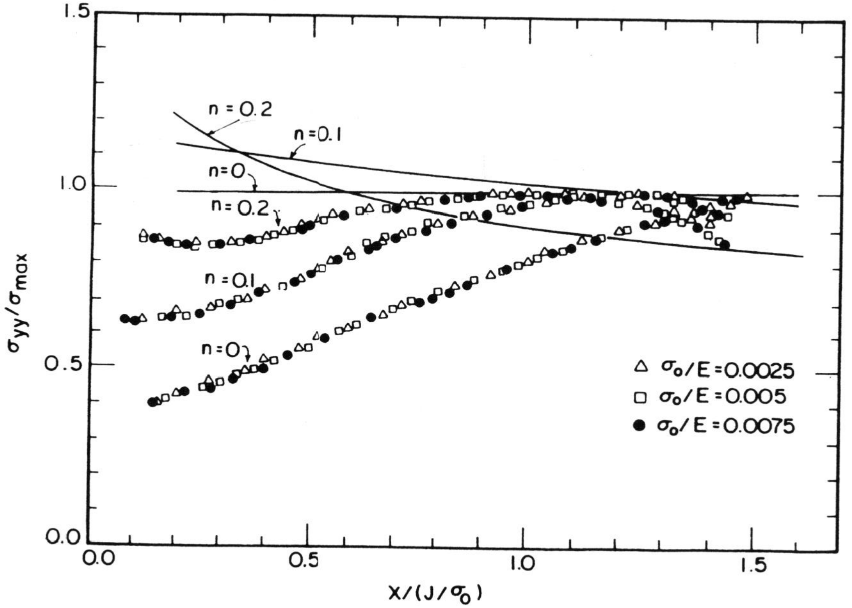

Fig. 2 σ_{yy} distribution ahead of crack tip. Solid curves for sharp crack tip. Data points for blunted crack tip.

Hutchinson (1968) and Rice and Rosengren (1968) made their crack-tip analyses using the small deformation theory based on the initial geometry of a sharp crack tip. They have not taken the effects of large deformation and crack-tip blunting into account. The solid curves in Figure (2) are the HRR field for a sharp crack tip. The stress readings are normalized by the σ_{max} for $\sigma_o/E = 0.005$. It is interesting to note that, for n equal to 0 and 0.1, the calculated σ_{yy} values at $r = 2\delta$ for both sharp and blunted crack tips are very close to each other. At n = 0.2, the worst deviation is only slightly more than 10%.

There is a maxima for each of the σ_{yy}-curves at a blunted crack tip as calculated by McMeeking and Parks, Figure 1. For the case of small-scale yielding, the maxima occur at $r/(J/\sigma_o) = 1.6$ and 1.0 for n = 0 and 0.1 respectively. With the relation between crack-tip opening displacement, δ, and (J/σ_o), as given by Shih and German (1981), $\sigma_{yy(max)}$ occurs at $r/\delta = 2.0$ and 2.5 for n = 0 and 0.1, respectively. These values are close to those found by Wang (1953) and Rice and Johnson (1970).

The data in Figure (2) as well as those calculated by McMeeking and Parks (1979) indicate that, for small-scale yielding, $\sigma_{yy(max)}$ takes place at r close to 2δ and a single parameter σ_{yy} at $r = 2\delta$ is capable of characterizing the tensile stress in the entire blunted crack-tip region. Therefore, we suggest using σ_{yy} at $r = 2\delta$ as one of the two parameters of the proposed dual-parameter fracture-criterion.

However, our primary concern is the fracture-criterion for non-linear fracture mechanics. We need to examine the crack-tip field in general yielding. The data in Figure (1) are in general yielding and they show that, as the load is increased, the crack tip loses its constraint, σ_{yy}

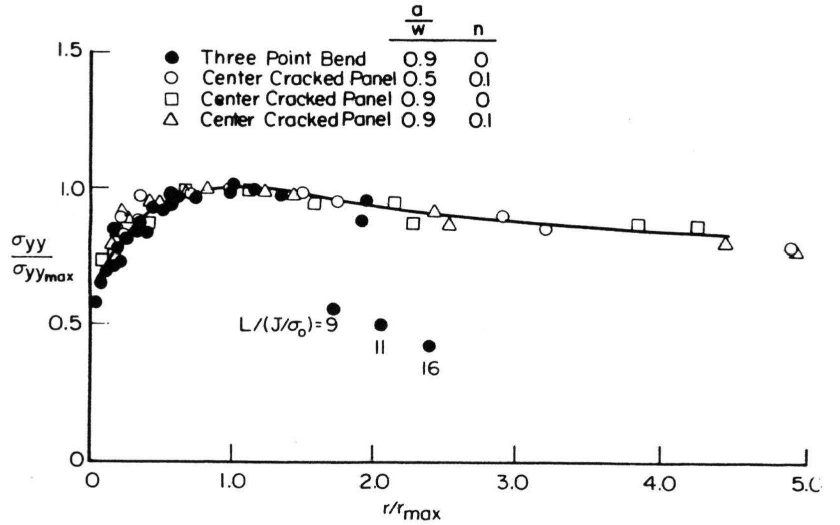

Fig. 3 σ_{yy} is characterized by $\sigma_{yy(max)}$ and r_{max} at a blunted crack tip.

decreases, and the location of $\sigma_{yy(max)}$ shifts toward the crack tip. The data in Figure (1) are replotted in Figure (3) as $(\sigma_{yy}/\sigma_{yy(max)})$ vs. (r/r_{max}). r_{max} is the location of $\sigma_{yy(max)}$. All the data fall within a narrow band. The quantity $(\sigma_{yy}/\sigma_{yy(max)})$ correlates well with the ratio (r/r_{max}). The data indicate that, in a blunted crack-tip region, $(\sigma_{yy}/\sigma_{yy(max)})$ is a function of (r/r_{max}) regardless of the degree of constraint and the amount of stress triaxiality.

$$(\sigma_{yy}/\sigma_{yy(max)}) = f(r/r_{max})$$

The tensile stress, σ_{yy}, in a blunted crack-tip region are characterized by two parameters: $\sigma_{yy(max)}$ and r_{max}. In other words, for the same set of values of $\sigma_{yy(max)}$ and r_{max}, the values of σ_{yy} in a blunted crack-tip region are the same.

Figure (4) shows the plot of (σ_{yy}/σ_o) vs. $[r/(J/\sigma_o)]$ calculated by McMeeking and Parks (1980) for two center-cracked panels in general yielding. The solid symbols are for the crack length a/W = 0.9 and the open symbols are for a/W = 0.5. The solid and open symbols with the same $\sigma_{yy(max)}$ values fit together very well. As the applied load increases, the quantity $(\sigma_{yy(max)}/\sigma_o)$ shifts downward, and the ratio (r/r_{max}) shifts to the left. The data indicate that the shifts in $\sigma_{yy(max)}$ and r_{max} are coupled together: i.e., for the same amount of downward shift of $\sigma_{yy(max)}$ in two different specimens, the amounts of shifts of r_{max} in these two specimens are the same.

At a higher value of J, r_{max} shifts closer to the crack tip, and the $\sigma_{yy(max)}$ takes place at $r < 2\delta$. However, the data in Figures (2) and (3) indicate that even the σ_{yy} at r equal to 3 to 4 times r_{max} is capable of characterizing the tensile stress in a blunted crack-tip region with the exception of the three-point bend specimen. The ligament of the bend specimen is too small, and the steep drop of σ_{yy} is caused by the bending.

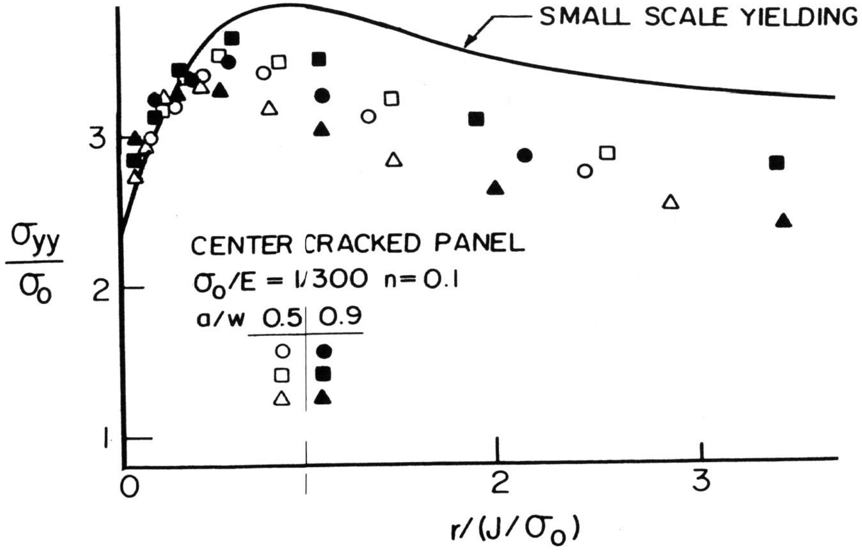

Fig. 4 The coupled shifts of $\sigma_{yy(max)}$ and r_{max} at a blunted crack tip.

There is an added advantage for using the value of σ_{yy} at $x = 2\delta$ as one of the parameters for the fracture criterion. In the region closer to the crack tip, i.e. to the left of the maximum, the exact values of the stresses and strains depend strongly on the formulation of the finite element program. The stresses and strains calculated with one finite element program may differ considerably from those calculated using a different program. However, at a distance $r = 2\delta$, the stresses calculated with different finite element programs should be close to each other as shown in Figure (2). Therefore, the values calculated at various laboratories can be used with a reasonable reproducibility.

Hancock and Cowling (1980) measured the critical values of the crack tip opening displacement, δ_f, at the fracture initiation in HY80 steel specimens of six different geometric configurations. The ligament sizes, L, of their specimens are rather small (5 to 12.5 mm) and HY80 steel is very ductile and tough. At fracture initiation, deep general yielding took place. The critical values of δ_f at fracture initiation vary by a factor of 10 from 90 to 900 µm. δ_f is defined as the crack-tip opening displacement, when a crack advances by 0.05mm (0.002 inches).

Liu and Zhuang (1987) have calculated the plane-strain crack-tip fields of the specimens used by Hancock and Cowling. Figure (5) shows δ_f as a function of the calculated value of $(\sigma_{yy}/\bar{\sigma})$ at $r = 2\delta$ for the specimens tested by Hancock and Cowling. The measured "fracture ductility", δ_f, decreases as the calculated tensile stress triaxiality (reflected by the ratio $\sigma_{yy}/\bar{\sigma}$) increases. This consistent qualitative trend is the extension of Bridgman's observation of the increase in fracture ductility with an increase in hydrostatic compression.

Fracture toughness is a measure of the tensile fracture ductility of a material under a superimposed hydrostatic tension. The magnitude of the crack-tip hydrostatic tension is affected by the in-plane geometric

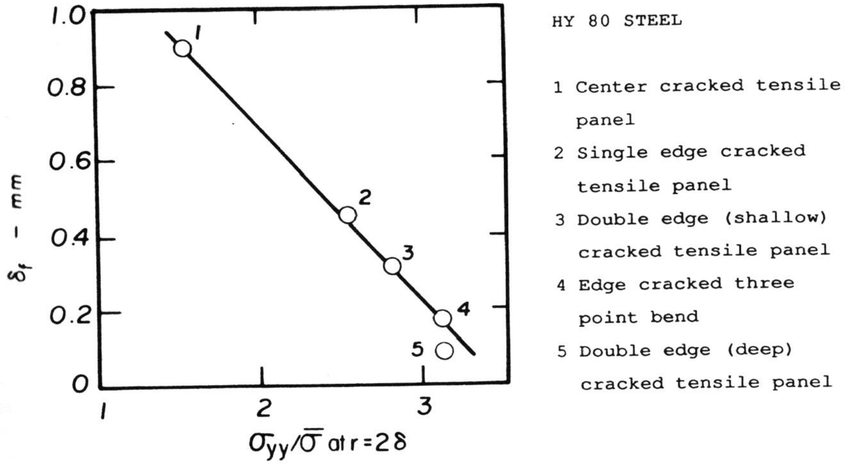

Fig. 5 Fracture ductility diagram for HY 80 steel

constraint and the thickness constraint to crack-tip plastic deformation; it is also affected by the mixed-mode loading. The fracture ductility diagram in Figure (5) offers an unifying concept to cover all three of these effects.

In summary, a dual-parameter fracture-criterion for non-linear fracture mechanics is proposed. The diagram of fracture ductility versus crack-tip tensile stress correlates the fracture data of HY-80 steel consistently. A higher hydrostatic tensile stress gives a lower fracture ductility.

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