A Probabilistic Approach to Crack Instability

A. CHUDNOVSKY and B. KUNIN
Department of Civil Engineering, Mechanics, and Metallurgy,
University of Illinois at Chicago,
P.O. Box 4348, Chicago, IL 60680, USA

ABSTRACT

A wide scatter of critical crack lengths, critical loads, critical energy release rates, etc., is well known in brittle fracture. It is also recognized that the scatter of such macroparameters is caused by microstructural fluctuations of material morphology. This is typical for critical phenomena, where microscopical fluctuations produce a macroscopical effect. In fracture, microstructural fluctuations are reflected most explicitly in the morphology of fracture surfaces.

Below we consider a probabilistic model of brittle fracture, restricting ourselves to two-dimensional problems.

The essence of the model is illustrated through the following experiment (Mull et al., 1987). Fatigue crack propagation followed by crack instability has been observed in 25 macroscopically identical specimens made of a short fiber reinforced composite. Figure 1 illustrates the ensemble of the

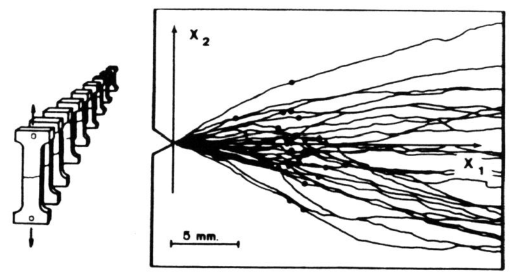

Fig. 1

crack trajectories. Dots on each trajectory indicates the critical crack tip location, i.e., the transition point from stable to unstable crack propagation. Although the specimens were (macroscopically) identical, the crack trajectories were unique for each specimen. More importantly, each of these crack trajectories should be viewed as a priori possible.

For a given path ω, introduce the probability $P(l|\omega)$ of failure along ω provided the crack has already reached the depth l. Also let $P\{\omega\}$ denote the probability that the crack "chooses" ω. Then, for a given specimen, the probability that the critical crack depth will not exceed l can be written as

$$P(l) = \sum_{\Omega} P(l|\omega) P\{\omega\} , \qquad (1)$$

where Ω is the set of all possible crack trajectories. (We assume that only one crack is formed in each specimen.)

In a continuum-based model, the set Ω is innumerable, and (1) is substituted by

$$P(l) = \int_{\Omega} P(l|\omega) \, d\mu(\omega) , \qquad (2)$$

where $d\mu(\omega)$ is an analog of $P\{\omega\}$ in (1) and is constructed on the basis of a statistical analysis of fracture surfaces. Thus the probability measure $d\mu(\omega)$ is a characterization of micromechanisms of fracture reflected in the fracture surface morphology.

The conditional probability $P(l|\omega)$ is defined as the probability that energy release due to an infinitesimal crack extension exceeds the required fracture energy at all points of the path ω beyond the depth l. The probability has been evaluated (Chudnovsky and Kunin, 1987) as

$$P(l|\omega) = \exp\left\{ -\int_{l}^{B} \exp\left[-\left(\frac{J_{\omega}(x)/2 - \gamma_{min}}{\gamma_o} \right)^{\alpha} \right] \frac{dx}{r} \right\} . \qquad (3)$$

Here $J_{\omega}(x)$ is the energy release rate for a crack formed along ω to the depth x; the random γ field of specific fracture energy is assumed statistically homogeneous with pointwise Weibull distribution and a correlation distance r, γ_{min}, γ_o, and α being the parameters of the Weibull distribution; B is the width of the specimen.

After a choice of the measure $d\mu(\omega)$ is made, evaluation of the integral in (2) can be done in different ways: by Monte-Carlo method, stationary phase method, reduction to differential equations, etc. Following (Chudnovsky and Kunin, 1987) we restrict ourselves to a "diffusion" approximation, i.e., we assume that crack trajectories have independent increments. The corresponding probabilistic measure is known as Wiener measure. In this case, evaluation of (2) can be reduced to solving of a diffusion type equation (Chudnovsky and Kunin, 1987; Gelfand, et al., 1956; Beran, 1968; Chudnovsky, et al., 1987). A "crack diffusion coefficient" appears in this equation as an analog of the conventional diffusion coefficient. It reflects the tendency of crack trajectories to deviate from the maximum energy release path.

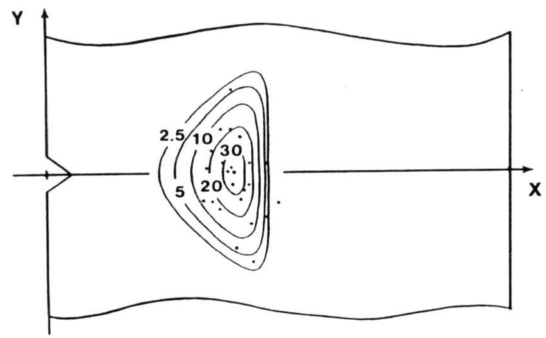

Fig. 2

Figure 2 illustrates predicted contours of equal probability density for the critical crack tip location (the latter are identified as point of transition from stable to unstable crack growth); the dots indicate the critical crack tip locations observed in experiment. A good agreement suggests that the model grasps some important features of the phenomenon of brittle fracture.

The diffusion approximation requires knowledge of the characteristics of the γ-field (Weibull parameters γ_{min}, γ_o, α and the correlation distance r) as well, as the dependency of the diffusion coefficient D on the load-specimen configuration. Then the model provides: (a) predictive formalism for probability distributions of critical crack depths, critical loads and crack arrest depths (the latter in the model's "stable" version (Chudnovsky, et al., 1987), (b) similarity criteria for small scale testing.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the financial support from the Office of Naval Research (Contract No. 3425504) and partial support from the NASA Lewis Research Center (Grant No. NAG3-754).

REFERENCES

Mull, M. A., A. Chudnovsky and A. Moet (1987). A probabilistic approach to the fracture toughness in composites. Phylos. Mag. A., 56 419-493.

Chudnovsky, A. and B. Kunin (1987). A probabilistic model of brittle crack formation. J. Appl. Phys., 62 4124-4129.

Gelfand, I. M. and A. M. Jaglom, (1956). Integration in functional spaces and its applications in quantum physics. Russ. Math. Surv., 11 48-147.

Beran, M. J. (1968). Statistical Continuum Theories. Interscience, NY.

Chudnovsky, A., J. Botsis and B. Kunin (1987). The role of microdefects in fracture. In: Proc. ASME Sump. on Constitutive Modelling for Non-traditional materials, Boston, MA.