A Numerical Study of Crack Arrest in Elastic–Plastic Materials

SÖREN ÖSTLUND

Department of Strength of Materials and Solid Mechanics, Royal Institute of Technology, S-100 44 Stockholm, Sweden

ABSTRACT

Numerical simulations of crack arrest in elastic–plastic materials have been performed by use of the finite element method. A crack propagating dynamically under steady–state conditions is supposed to suddenly arrest and the transient behaviour at the crack tip after arrest is studied. The finite element mesh is subjected to loadings given by the plane strain elasto–dynamic crack tip fields and both rate–dependent and rate–independent constitutive models of elastic–plastic materials have been investigated.

KEYWORDS

Crack arrest; dynamic; elastic–plastic materials; finite element method; moving mesh.

INTRODUCTION

The arrest of rapidly running cracks is an issue of great technical importance. During the last years extensive research has been directed into the phenomenas around crack arrest. However, there still does not seem to exist a generally accepted arrest criterium, especially not if plasticity is included in the analysis. A comprehensive review of the state of the art can be found in the book by Kanninen and Popelar (1985)

Crack arrest in a rate–dependent plastic material has been studied numerically by Aboudi and Achenbach (1981, 1983). They considered a crack initially running in an elastic solid. At a certain time the crack enters a region, where the material properties gradually change from elastic to elastic–viscoplastic, and the crack finally arrests. The idea of a transition zone were also utilized by Xu, Chung and Achenbach (1984) in a numerical study of crack arrest in an elastic perfectly–plastic material.

In the present work no transition region is introduced and it is assumed that the crack arrest is an effect of a sudden change in the toughness of the material. A crack initially propagating dynamically under steady–state conditions is considered. At a time $t = 0$ the crack tip suddenly stops and the transient behaviour after the arrest is studied. Results will be presented for the normal stress and strain in front of the crack tip. These kinds of near tip field values will be of interest in the future research on crack arrest criteria.

NUMERICAL FORMULATION

The simulation procedure includes two different formulations of the governing equations. In the propagation phase it is assumed that a steady–state condition has developed so that the equations in a convective coordinate system no longer explicitly depend on time. For the transient phase after crack arrest, however, the time dependence is included in the usual way. For both formulations it is assumed that the loading condition is plane strain, mode I.

Propagation phase

The governing equations for the growing crack are formulated in a convecting Cartesian coordinate system (x_1, x_2, x_3) fixed to the crack tip. The x_3–axis coincides with the edge of the crack and the crack propagates in the x_1–direction with constant velocity $\dot{a}$. The steady–state assumption permits the material time derivative to be written as $-\dot{a}\frac{\partial}{\partial x_1}(\ )$. The equations of motion then becomes

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \dot{a}^2 \frac{\partial^2 u_i}{\partial {x_1}^2}\ , \qquad (1)$$

where ρ denotes the mass density, σ_{ij} the stress tensor and u_i the displacement vector. It is further assumed that the total strain, ε_{ij}, is the sum of the elastic strains, ε_{ij}^e, and the plastic strains, ε_{ij}^p. This gives a general expression for the constitutive relation

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^p)\ , \qquad (2)$$

where C_{ijkl} is the elasticity tensor.

A virtual work representation of eq.(1) can be obtained by forming the inner product of eq.(1) with a virtual displacement δu_i and integrating over a volume,

$$\int_V \left[\delta\varepsilon_{ij} C_{ijkl}\varepsilon_{kl} - \rho \dot{a}^2 \frac{\partial(\delta u_i)}{\partial x_1}\frac{\partial u_i}{\partial x_1}\right] dV + \int_S \rho \dot{a}^2 \delta u_i \frac{\partial u_i}{\partial x_1} n_1 dS =$$

$$\int_V \delta\varepsilon_{ij} C_{ijkl}\varepsilon_{kl}^p dV + \int_S \delta u_i T_i dS\ , \qquad (3)$$

where T_i denotes the surface tractions and n_i the normal vector of the boundary, S, of the volume. The finite element equations can be derived from eq.(3) by assuming proper interpolation functions for the displacement vector u_i. In the present work linear interpolation functions in a form appropriate for the four–noded isoparametric element were used. The numerical integration of eq.(3) can be performed by using an iterative method which basic ideas stems from the work of Dean and Hutchinson (1980). For steady–state conditions the stress state and the plastic strains can be obtained by integrating the incremental form of the stress–strain relation (2) along lines parallell to the negative x_1–axis. Lam and Freund (1985) and Freund, Hutchinson and Lam (1986) describes the method in a way which is close to the one used in this paper.

Transient phase

The governing equations for the arrested crack are formulated in a fixed Cartesian coordinate system. The virtual work representation of the equations of motion in this case becomes

$$\int_V \left[\delta\varepsilon_{ij} C_{ijkl}\varepsilon_{kl} + \rho \ddot{u}_i \delta u_i\right] dV = \int_V \delta\varepsilon_{ij} C_{ijkl}\varepsilon_{kl}^p dV + \int_S \delta u_i T_i dS\ , \qquad (4)$$

where $(\dot{\ })$ is the material time derivative. The numerical time integration of the finite element equations derived from eq.(4) were performed with a central difference scheme.

MATERIAL MODELS

Two different kinds of material models have been applied in this study. Both models adopt the von Mises yield criteria, thus the effective stress, $\bar{\sigma}$, can be written as

$$\bar{\sigma} = \left[\frac{3}{2} s_{ij}s_{ij}\right]^{1/2}\ , \qquad (5)$$

where s_{ij} denotes the stress deviator. The first model is an ordinary rate–independent elastic perfectly plastic material. The stress–strain relation for this type of material follows the Prandtl–Reuss equations when yielding has occured. In incremental form they can be written as

$$\dot{s}_{ij} = \frac{E}{1+\nu}(\dot{e}_{ij} - n_{ij}n_{kl}\dot{e}_{kl}) \qquad (6)$$

when $s_{ij}\dot{e}_{ij} \geq 0$ and $\bar{\sigma} = \sigma_y$. In eq.(6) e_{ij} is the strain deviator, σ_y is the tensile yield stress, $n_{ij} = s_{ij}/\sqrt{s_{kl}s_{kl}}$, E is Young's modulus and ν is Poisson's ratio.

The second material model is a non–hardening elastic–viscoplastic model proposed by Perzyna (1963). This model assumes that the plastic strain–rate is of the form

$$\dot{\varepsilon}_{ij}^p = \gamma \left\langle \left[\frac{\bar{\sigma} - \sigma_y}{\sigma_y}\right] \right\rangle^N \frac{3 s_{ij}}{2\bar{\sigma}}\ , \qquad (7)$$

where γ and N are material constants. Throughout this report γ = 4000.0 s^{-1} and N = 2.0 is used. The notation $\langle\phi\rangle$ should be interpreted as ϕ if $\phi > 0$ and 0 if $\phi \leq 0$.

RESULTS

The finite element mesh used in the calculations consisted of 578 rectangular elements with a total number of degrees of freedom of 1260. The elements close to the crack tip were all of equal size. Outside the steady–state plastic zone a coarser mesh were used to save computing time. The mesh is shown in Fig. 1. Although a finer mesh is desirable the limiting factor is the mesh dependent stability limit of the central difference scheme. A finer mesh requires a smaller time step and consequently more computing time.The nodes at the mesh boundaries were subjected to prescribed displacements according to the elasto–dynamic crack tip fields except for a small part of the trailing edge close to the crack surface where instead stress boundary conditions were applied.

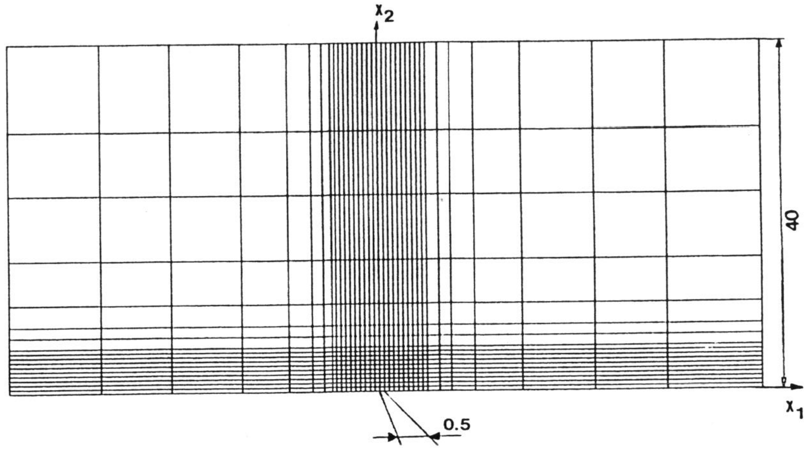

Fig. 1. Finite element mesh.

Although the elasto–dynamic fields are functions of the crack tip velocity and a change of velocity takes place at the time of arrest, the calculations presented here were performed with boundary conditions according to the steady–state crack growth. This avoids the introduction of unnecessary numerical disturbances. The remote stress intensity factor, K_I, were choosen so that small–scale yielding could be assumed. This means that the size of the active plastic zone were kept within ten percent of the vertical size of the mesh, see i.e. Freund, Hutchinson and Lam (1986).

When the crack tip velocity suddenly changes the second time derivative of the displacements and the crack tip acceleration will become infinite in a way which can be described by a Dirac function. By introducing this into the complete virtual work representation for a convecting coordinate system, and integrating over an infitesimal time interval $(-\epsilon, \epsilon)$ an additional initial condition on the transient phase computations is obtained. In matrix formulation this condition reads

$$M\Delta\dot{\alpha} = A\Delta\dot{a}\alpha\,, \tag{8}$$

where M denotes the mass matrix, $\Delta\dot{\alpha}$ the correction of nodal velocities, α the nodal displacements, $\Delta\dot{a}$ the jump in crack tip velocity and A is a matrix formed from

$$\int_V \rho\delta u_i \frac{\partial u_i}{\partial x_1}\,\mathrm{d}V\,. \tag{9}$$

This condition was not observed by Östlund and Gudmundson (1987) in their elastic transient crack propagation calculations and this is the main reason why their results, for cracks which suddenly changes velocity, not are very accurate.

The initial velocity used in the crack arrest simulations was $m = 0.4$, where $m = \dot{a}/C_2$ and C_2 is the elastic shear wave velocity. The results presented were computed at the center of the element directly ahead of the crack tip.

In Fig. 2 and 3, ε_{22} and ε_{22}^{p} are plotted as functions of time for the rate–independent and rate–dependent material model respectively. In Fig. 4, σ_{22} are plotted for both material models.

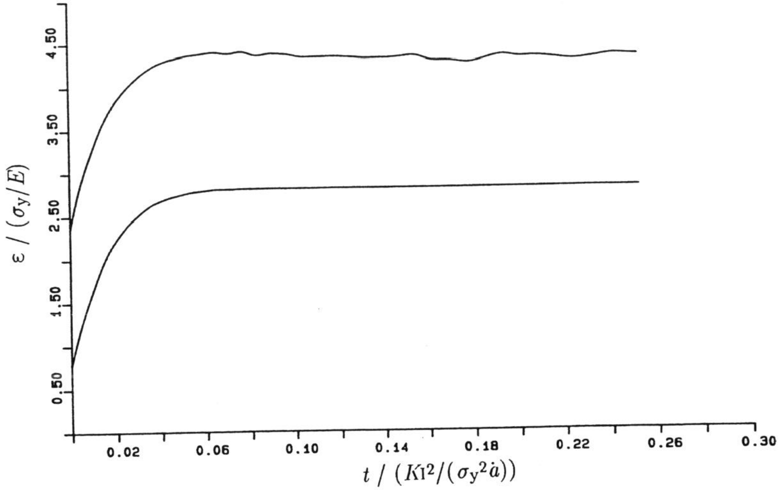

Fig. 2. The strains ε_{22} (above) and ε_{22}^{p} (below) as function of time for the rate–independent material.

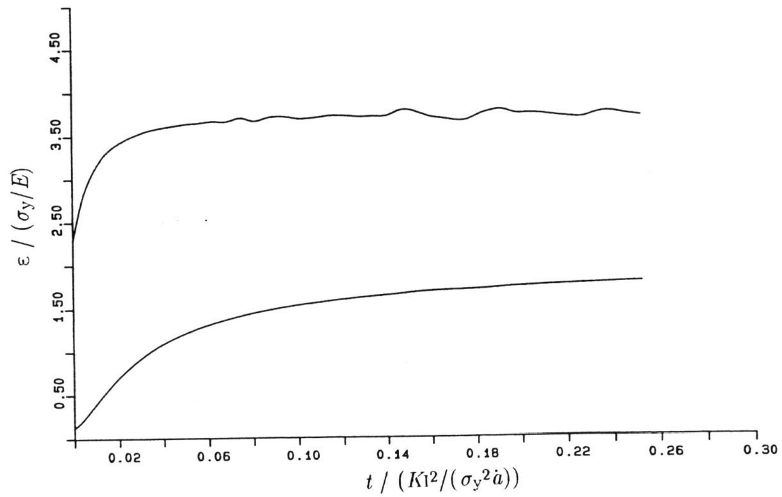

Fig. 3. The strains ε_{22} (above) and ε_{22}^{p} (below) as function of time for the rate–dependent material.

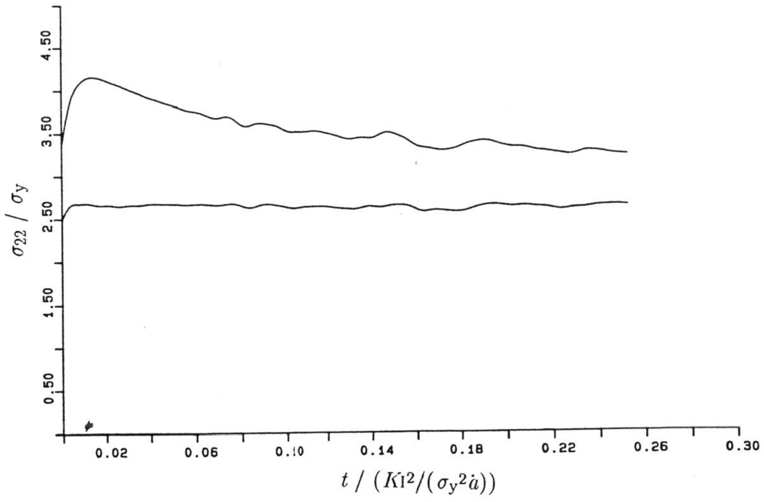

Fig. 4. The stress σ_{22} as function of time for both the rate–dependent material (above) and the rate–independent material (below).

The calculations were all performed for a time which was short enough to avoid any interaction with the mesh boundaries. Except for differences in the numerical values due to the different character of the two material models, there are some basic differences in the response to the sudden arrest of the crack. As can be expected the plastic strains are accumulated during a much shorter time interval for the rate–independent model. After a short time no change is seen for the elastic–perfectly plastic model while the plastic strain calculated with the rate–dependent model is increasing during the whole simulated time. It is also interesting to notice that the increase of plastic strain relative the steady–state value are considerably larger for the rate–dependent model. Also the normal stress, σ_{22}, shows a larger rise, relative the steady–state value for the elastic–viscoplastic model compared to the rate–independent model. However the basic difference between the two models are seen when the stresses have reached their maximum. The stress σ_{22} for the rate–independent model then remains more or less constant while the normal stress for the rate–dependent model slowly relax. The small decrease of the rate–independent stress shown in Fig. 4 are probably caused by numerical effects.

If a crack propagation condition based on a critical value of the strain in front of the crack tip is adopted then it is of interest to study the rise in strain when crack arrest is simulated.

CONCLUSIONS

The numerical procedure described above is a promising tool for simulations of crack arrest in elastic–plastic materials. With some minor extensions it will also be applicable to general non–steady crack growth.

The field variables close to the crack tip are relatively free of numerical noise, which not always is the case for other methods. This can simplify the interpretation of the numerical results.

The moving mesh procedure of course has the drawbacks that it is only applicable to structures with horisontal edges parallell to the crack line and infinite dimensions in the direction of crack growth, i.e. the infinite strip problem. However, it is possible to implement the moving mesh formulation only for the elements just around the crack tip and keep the rest of the mesh stationary.

Despite these drawbacks the procedure is very efficient for basic studies of the fields close to the crack tip. A major advantage is that the crack tip position always is well defined. This is not the case for the nodal relaxation technique commonly used today.

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