# SOME THEORETICAL AND EXPERIMENTAL ASPECTS OF MIXED MODE FRACTURES

## H. A. Richard

University of Kaiserslautern, Institute of Technical Mechanics, 6750 Kaiserslautern, Federal Republic of Germany

#### ABSTRACT

A compact tension shear specimen (CTS specimen) and a new loading device will be presented which have been developed for rewieving the brittle fracture criteria for superimposed Mode I and Mode II stresses applied to cracks. Experiments will be described for determining the fracture toughness values  $K_{\overline{L}}$  (for Mode I),  $K_{\overline{L}\overline{L}}$  (for Mode II) as well as  $K_{\overline{L}}^{C}$  and  $K_{\overline{L}\overline{L}}^{C}$  (for Mixed Mode). The experimental results will be compared with the predictions of the fracture criteria.

## KEYWORDS

Mixed Mode loading; fracture criteria; new loading device; CTS specimen; experimental results; fracture toughness.

## INTRODUCTION

In various fields of engineering a large number of crack problems occur which are caused by mixed-mode stresses at the crack tip. Such stresses may occur as a result of superimposed stresses on construction components, oblique or curved cracks, sharply bent or ramified cracks, multiple cracks, cracks occurring in the vicinity of notches, at welded and bonded joints, on composite materials, as a result of dynamic or thermal stresses or of superimposed load, thermal and internal stresses.

The Mixed Mode stresses occurring at the crack tip are characterized by stress intensity factors  $K_{\overline{1}}$  and  $K_{\overline{1}\overline{1}}$ . With the aid of these factors, the following fracture criteria were determined on the basis of, for example, stresses and energies:

- Criterion of maximum tangential stress (Erdogan and Sih, 1963);
- various criteria of the energy release rate: resultant energy release rate (Broek, 1974), criteria according to Hussain, Pu and Underwood (1974), Nuismer (1975), Amestoy, Bui and Dang Van (1980);

- various criteria of the energy density: strain energy density (Sih, 1974), volume-changing energy density (Radaj and Heib, 1978), Shape-chanching energy density (Jayatalaka, Jenkins and Prasad, 1977):

- criterion according to Di Leonardo (1979);

- J-integral criterion (Yu, 1982);

- principal strain criterion (Fischer and Göldner, 1981)

With all these criteria, a comparative stress intensity factor K is equated with the fracture toughness  $K_{{
m Ic}}$ :

$$K_{\mathbf{v}}(K_{\mathbf{I}}, K_{\mathbf{II}}) = K_{\mathbf{IC}} \tag{1}.$$

These criteria make it possible to give a statement on the beginning and the angle of the fracture. Some of the predictions are very contradictory.

Experimental investigations of the Mixed Mode problems were mostly made with the following specimen configurations:

- tensile stress bar with oblique inside crack,

- pressed disc with oblique inside crack,

- twisted pipe with oblique inside crack.

The tensile stress bar with oblique inside crack was used most frequently as a Mixed Mode specimen, although it does not permit a transition from Mode I to Mode II. It is very problematic with all these specimen to initiate a fatigue crack.

In this paper a compact tension shear specimen (CTS specimen) and a new loading device will be presented for a simple determination of the fracture toughness values  $K_{Ic}$  (Mode I),  $K_{IIc}$  (Mode II) as well as  $K_{I}^{C}$  and  $K_{II}^{C}$  (Mixed Mode).

# CTS SPECIMEN AND LOADING DEVICE

The CTS specimen (see Fig. 1) has an edge crack extending at right angles to the edge in the mean cross section of the specimen. The optimal dimensions of the specimen were determined in extensive stress-analytical investigations carried out with the photo-elastic method (see Fig. 2) and the finite-element method (see Fig. 3).

The load is applied to the specimen by a loading device (see Fig. 4) which makes it possible to generate various stresses by applying a tensile force to the device. The line of application of force F extends at an angle  $\alpha$  to the longitudinal axis of the specimen. Depending on the size of angle  $\alpha$ , pure tensile stresses, pure shearing stresses or superimposed tensile and shearing stresses can be applied to cross section AB of the specimen. On specimens having a crack in cross section AB a Mode I loading occurs (at  $\alpha$  =  $0^{\circ}$ ), a Mode II loading (at  $\alpha$  =  $90^{\circ}$ ) and Mixed Mode loading (at  $0 < \alpha < 90^{\circ}$ ).

The loading device and the specimen are connected by six studs, for which purpose the specimen has been provided with round holes and the loading device with elongated holes so that a statically applied load transfer is ensured.

For a CTS specimen having these dimensions stress intensity factors  $K_{\overline{1}}$  and  $K_{\overline{1}\overline{1}}$  were then computed by the finite-element method as a function of  $\alpha$  and  $\alpha/w$ . In the range from 0.5  $\leq \alpha/w \leq$  0.7, the stress intensity factors are determined by the following equations:

$$\frac{K_{\rm I}}{\frac{F}{wt}\sqrt{\pi a}} = \frac{\cos\alpha}{1 - \frac{a}{w}} \sqrt{\frac{0,26 + 2,65 \frac{a}{w-a}}{1 + 0,55 \frac{a}{w-a} - 0,08 \left(\frac{a}{w-a}\right)^2}}$$
(2)

$$\frac{K_{II}}{\frac{F}{wt}\sqrt{\pi a}} = \frac{\sin\alpha}{1 - \frac{a}{w}} \sqrt{\frac{-0.23 + 1.40 \frac{a}{w-a}}{1 - 0.67 \frac{a}{w-a} + 2.08 \left(\frac{a}{w-a}\right)^2}}$$
(3)

mit a/(w-a) = (a/w) / (1-a/w).

### EXPERIMENTS AND RESULTS

The following procedure was used for determining the fracture toughness values and crack deflection angles on specimens exposed to mixed-mode stresses:

a) Making a specimen including a starter notch.

b) Making a fatigue crack by applying combined tensile and vibrating stresses ( $\alpha = 0^{\circ}$ ).

c) Applying the knife edges for measuring the crack edge displacement (see Fig. 5).

d) Tensile test: The specimen is turn at a constant load application velocity; during the tensile test the force displacement curve is plotted and the fracture load F<sub>C</sub> determined.

e) Putting fracture load  $F_C$ , the crack length  $a_C$  measured after fracture, load application angle  $\alpha$ , and specimen width w in equations (2) and (3), fracture toughness values  $K_L^C$  and  $K_{LL}^C$  for mixed mode,  $K_L^C$  for mode I as well as  $K_L^C$  for mode II can be determined.

f) Crack deflection angle  $\varphi$  can be measured on the broken specimen directly (see Fig. 6).

For the experimental investigation of fracturing processes CTS specimens made of PMMA (Plexiglas) were used. These specimen were w = 80 mm wide and t  $_1$  = 20 mm and t  $_2$  = 30 mm thick. Starting with a starter notch (chevron notch) the fatigue cracks were initiated by combined tensile and vibrating stresses (frequency 10 Hz). The total crack length (starter notch and fatigue crack) on the specimen was between 51 and 55 mm. The tensile tests were made on a testing machine (type PSA of Schenck AG, Darmstadt, West Germany) at a constant load application velocity F = 4kN/s. At this velocity no precritical crack growth was observed. The fracture toughness values  $K_{\rm IC}$  and  $K_{\rm II}^{\rm C}$  as well as  $K_{\rm II}^{\rm C}$  were computed by means of equations (2) and (3), using fracture load  $F_{\rm IC}^{\rm C}$  for F and the crack length  $a_{\rm C}$  at the beginning of the fracture for a.

The experimental results are shown in Fig. 7 and Fig. 8. The dispersion of the values is comparatively small both at the fracture deflection angle and at the beginning of the fracture. As shown in Fig. 7, the values measured for the crack deflection angle confirm especially the criteria according to Erdogan and Sih (1963), Nuismer (1975) and Hussain, Pu and Underwood (1974). The experimental results for the beginning of the fracture agree with the criteria according to Erdogan and Sih (1963), Nuismer (1975), Sih (1974) and Di Leonardo (1979).

## CONCLUDING REMARKS

The experimental results show that the CTS specimen used in conjunction with the loading device described above is highly suitable for investigating crack problems occurring under mixed-mode stresses. In order to obtain further findings regarding the correctness of the brittle fracture hypotheses, additional experiments will have to be made using metal specimens too.

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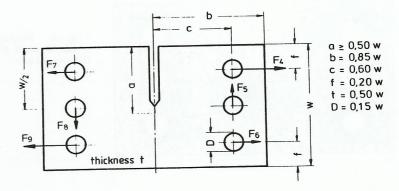


Fig. 1. Compact tension shear specimen (CTS specimen)

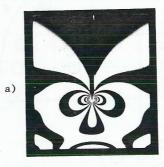






Fig. 2. Isochromatic fringe pattern for the CTS specimen

- a) Mode I
- b) Mixed Mode
- c) Mode II

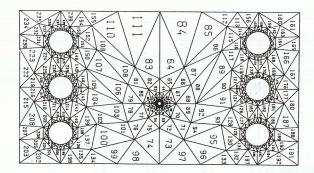
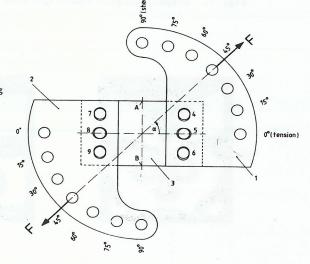
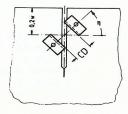


Fig. 3. Finite element mesh for the CTS specimen

Fig. 4. Device for generation superimposed normal and shear loading in a specimen (3)





α	Л
00	00
15°	30
30°	6,5
45°	110
60°	190
70°	38°
900	900

Fig. 5. Arrangement of the knife edges for location of the displacement gauges in relation to load application angle  $\alpha$ 

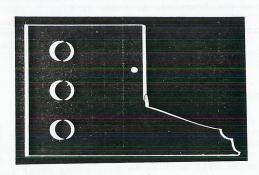


Fig. 6. Fracture of CTS specimen under Mode II loading (Material: PMMA)

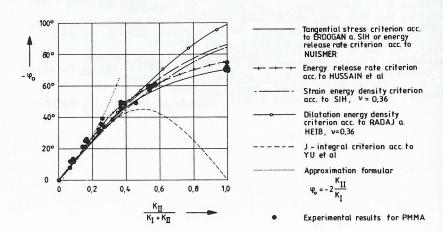


Fig. 7. Fracture angel for Mixed Mode loading - Comparison of the experimental results with some fracture criteria

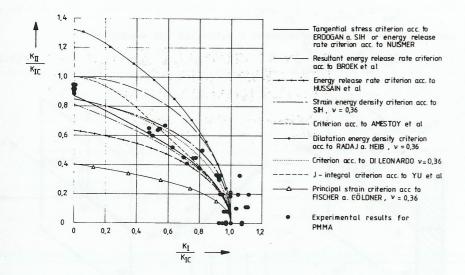


Fig. 8. Fracture boundary for Mixed Mode loading - Comparison of the experimental results with some fracture criteria