PREDICTION OF CRITICAL SIZE OF DEFECTS IN TURBINE SHAFTS WITH OR WITHOUT A CENTRAL BORE

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ABSTRACT

Propagation of semi-elliptical defects in a rotor shaft of 600 MW at 3000 R.P.M. is studied in this paper. The shaft may be with or without bore and carries blades of 1m. on a diameter of 1.65m. The half length of the cracks ranges from 20 to 350 mm.

The stress-intensity factors are determined either by 3D analyses based on the Boundary Element Method or by using 2D semi-analytical formulas. For bored shaft, 3D analyses appear to be absolutely necessary. The evolution of shape and size of the defects is obtained by applying PARIS's propagation law at the two apex of the ellipse, and by taking into account the instantaneous ratio of crack depth over crack length.

As far as fatigue behaviour is concerned, the technological conclusion of this study is that shafts without bore are more reliable than the bored ones.

KEYWORDS

3D fracture mechanics analysis ; industrial analysis ; fatigue crack propagation ; prediction of critical size of defects in shafts.

INTRODUCTION

Among the components of a steam turbine, the rotors are the most important. At present, large size rotors can be obtained as an integral forging. This has been made possible by the progress achieved in forging technique, which involves chemical analysis of steel in which impurities have been nearly totally eliminated, production means such as quenching methods and also inspection means (ultrasonic testing). Along this line, two questions arise for the turbine manufacturer :

- Is it necessary to bore the center of the rotors to remove the largest inclusions ?
- Wich is the largest acceptable inclusion which does not involve a risk for the integrity of the rotor along its service life in case of rotor with bore or without bore ?

In an attempt to answer these questions, AA and CETIM have employed the fracture mechanics theories which make use of the boundary integral equations method. The originality of this study is to take into account the fact that the stress field around a defect is a three-dimension one and that the defect shape evolves as it propagates.

FRAMEWORK OF THE STUDY

The rotor that was selected for this study is a low pressure (LP) rotor of a turbine of 6000 MW at 3000 R.P.M. This choice is due to the large dimensions of this forged mechanical component and to the high level of stress which lies inside it. Two cases were studied : shaft with bore and shaft without bore.

The 3D analyses were performed with the use of CA.ST.OR. This computer code is developped by Cetim, and the option E3D which performs 3D elastic analyses is based on the Boundary Element Method.

The stress intensity factors are determinated by using the displacement field around the crack tips and under a plain strain state assumption.

By using a propagation law, it is then possible to determine the evolution of the size of defects and to calculate the number of cycles leading to the ruin of the component.

COMPUTATION OF K_I INSIDE THE SHAFT

The analyses of the behaviour of the cracked shafts were restricted to the zone around the defects. The dimensions of the modelled structure were chosen in such a way that the boundary conditions were not perturbated by the presence of defects (fig.1). The shape of the defects was semi-elliptic (bored shafts) or elliptic (plain ones), they lay in a plane going through the center line. The defects are characterized by their ratio b/a (fig.5) and one of their half-axes (b or a).

Stress field inside the bored and plain shafts

Stress fields inside shafts without defects were obtained by performing 2D axisymetrical analysis based on the Finite Element Method. The boundaries of the idealised structures are shown on figure 1. Since the boundary conditions don't represent the exact influence of the other parts of the structures, the stress fields obtained by these numerical analyses are slightly different compared to the real ones. Anyhow the error remains small enough to be able to use these results in the following analyses. Figure 2 shows the circumferential stress distribution in the plane of symetry of the last disk and emphasizes the influence of the bore.

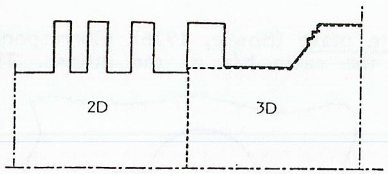


Fig.1 : Modelised structures

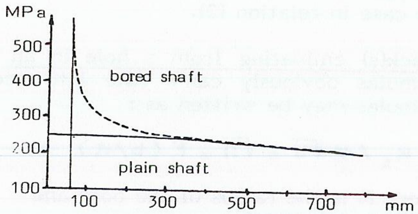


Fig.2 : circumferencial stress

Three dimensional analysis of the shafts

The idealised structure is restricted to the zone where defects would be localised (fig.1 and 3). A special care was paid to the boundary conditions in order to obtain the same stress field by 2D and 3D analyses on the radial surface limiting the structure on the opposite side of the defects.

Shafts without defects. Symetries are taken into account by imposing a normal displacement equal to zero on each surface of symetry. The load case was composed of the rotation of the shaft itself and by external forces due to the rotation of the blades and due to the rotation of the part of the shaft that was not modelised. The difference between the stress field obtained by 2D and 3D analyses is never over 5 % in the center part of the shaft, thus validating the 3D model.

Shafts with defects. The dimensions of the elliptical or semi-elliptical defects were chosen around a critical size which was analytically determined and which corresponds to K_I ranging for 50 to 160 MPa $\sqrt{m}$. Defects which were taken into account appear in table 1.

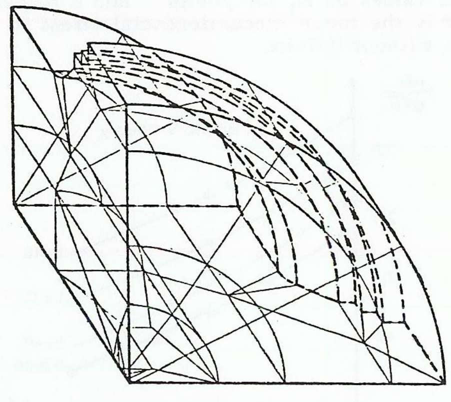


Fig.3 : 3D mesh

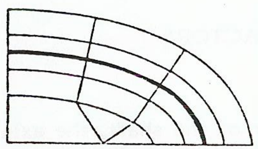


Fig.4 : mesh around the crack front

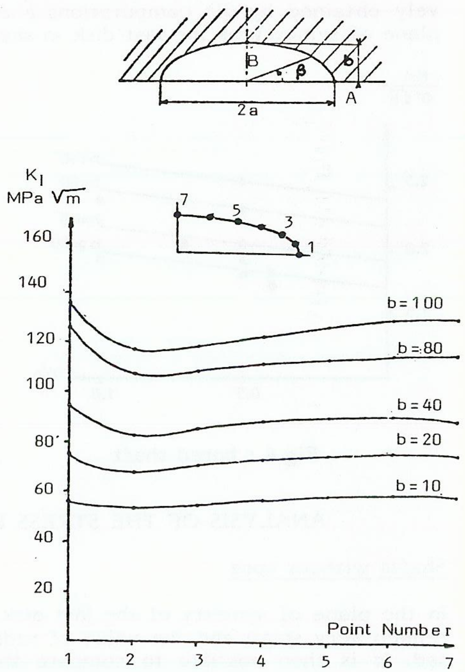


Fig.5 : K_I along the crack front

The stress intensity factors were determined by using the displacements obtained by 3D analyses. They were calculated for 7 points on one quarter of the ellipse. Table 1 gives the obtained K_I for 2 points A and B corresponding to the top of the crack length and crack depth respectively. Figure 5 shows the evolution of K_I with respect to the crack front length for b/a = 0.5.

TABLE 1 : K_I obtained by 3D analyses

b/a		Bored shaft (b) 100	80	40	20	10	Shaft without bore (a) 350	300	250	214	200	100	40	20
0.4	A	134	113											
	B	139	125											
0.5	A	134	126	95	76	59		79	73		63		30	
	B	130	115	89	76	62		131	117		108		43	
0.6	A						104	98	87					
	B						154	144	123					
0.7	A	151	138					111	100	89	86			
	B	112	100					144	128	123	118			
1.0	A	142			77		162	146			114	75	46	34
	B	96			61		174	155			117	80	47	34

Figures 6 and 7 show the evolution of $KA/\sigma\sqrt{b}$ and of $KB/\sigma\sqrt{b}$ with respect to b/a in a bored shaft ; KA and KB are the values of K_I for points A and B respectively obtained by 3D computations and σ is the mean circumferencial stress in the plane of symetry of the last disk in shafts without defect.

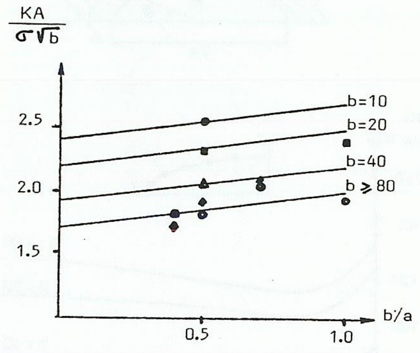


Fig.6 : bored shaft

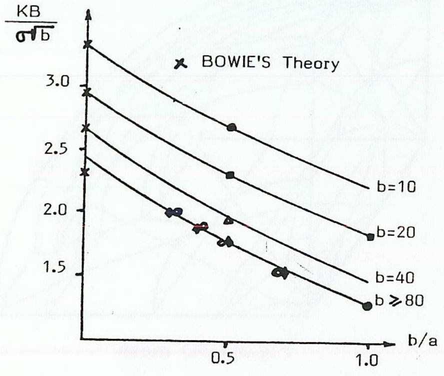


Fig.7 : bored shaft

ANALYSIS OF THE STRESS INTENSITY FACTORS

Shafts without bore

In the plane of symetry of the last disk in the center of the shaft, the axial stresses remain very small and the value of radial and circumferencial stresses are very closed. It is then possible to compare the values obtained by 3D analyses to values obtained analytically for an elliptical crack in an infinite medium and under uniform tractions (Aubry, Boissenot, Lange, 1977) :

$$\frac{K_I(\beta)}{\sigma\sqrt{b}} = \frac{\sqrt{\pi}}{\Phi_0}\left\{\sin^2\beta + \left(\frac{b}{a}\right)^2\cos^2\beta\right\}^{1/4} \quad (1)$$

with $\Phi_0 = \int_0^{\pi/2}\left(\cos^2\theta + \left(\frac{b}{a}\right)^2\sin^2\theta\right)^{1/2} d\theta$

the definition of β appears on figure 5. K_I obtained by 3D analyses and by relation (1) are only slightly different. The difference never exceeds 10 %. Figures 8 and 9 show the good agreement between those results.

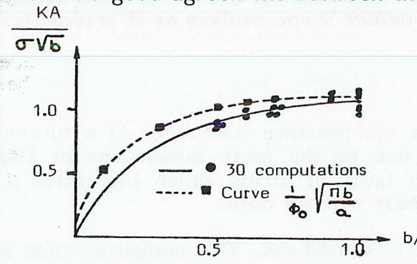


Fig.8 : plain shaft

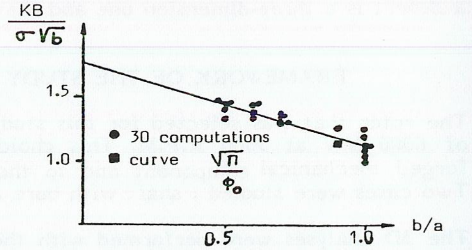


Fig.9 : plain shaft

Shafts with bore

Two 2D semi-analytical formulas may be considered for surface cracks in that case.

Semi-elliptical surface crack in an infinite plate under traction. (Newman, 1977). When the thickness is very large compared to the dimension b of the crack, K_I may be obtained by :

$$\frac{K_I\,\Phi_0}{\sigma\sqrt{\pi b}} = \left(1.096 - 0.087\,\frac{b}{a}\right)\left\{\sin^2\beta + \left(\frac{b}{a}\right)^2\cos^2\beta\right\}^{1/4}\left\{1.+0.1\,(1.-\sin\beta)^2\right\} \quad (2)$$

KA and KB may be obtained by writing $\beta = \pi/2$ and $\beta = 0$ in relation (2). Table 2 shows the comparison between the value of KA and KB obtained by 3D analyses and by relation (2).

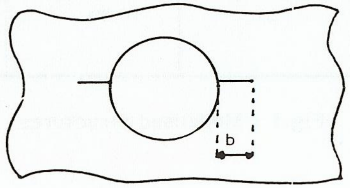
TABLE 2 : $KB/\sigma\sqrt{b}$ and $KA/\sigma\sqrt{b}$ obtained by 2D and 3D methods

b/a	$KB/\sigma\sqrt{b}$ 2D	3D	$KA/\sigma\sqrt{b}$ 2D	3D
0.4	1.63	1.90	1.14	1.72 to 1.83
0.5	1.54	1.75 to 2.67	1.20	1.83 to 2.54
0.7	1.37	1.52	1.26	2.06 to 2.10
1.0	1.14	1.31 to 1.86	1.25	1.94 to 2.35

2D results are always lower than 3D results. The maximum value of the error on KI ranges from 14 % for b/a = 0.4 to 50 % for b/a = 1. Furthermore KA = KB is obtained for b/a ≃ 0.8 by relation (2) and for b/a = 0.5 by 3D analyses. Finally 3D analyses show that $K/\sigma\sqrt{b}$ is not independant of b (Fig. 6 and 7) which is not the case in relation (2).

Crack(s) eminating from a hole in an infinite plate (Bowie, 1956). Corresponding formulas obviously can't take into account the ratio b/a of the ellipse. These formulas may be written as :

$$K_I/\sigma\sqrt{b} = \sqrt{\pi} \times f(b/R) \quad (3)$$



where R is the radius of the bore and f(b/R) is tabulated.

Anyhow, it may be considered that relation (3) give the limiting value of $KI/\sigma\sqrt{b}$ when "a" is very large ($b/a \simeq 0$). Furthermore, these relations are taking into account a variation of $KI/\sigma\sqrt{b}$ with respect to b, the same phenomenon beeing noticed in 3D analyses.

Results obtained by using relations (4) are always over the 3D results ; the difference increasing with b/a. The maximum value of the error 50 % for b/a = 1. On the other hand relations (4) may be used to complete 3D results for small b/a.

Practical consequence. This study shows that for shaft without bore the use of the 2D method is validated to calculate the value of KI. On the other hand for bored shafts 3D methods are needed.

CONSEQUENCE ON THE DESIGN OF SHAFTS

Using 3D results, the fatigue behaviour of shafts with and without bore are compared. For a given initial crack, the critical size of the defect and the number of cycle leading to the ruin of the structure are determinated for both cases.

Material properties

A lot of metallurgical research have been made on this subject (Coulon, 1977). The material properties of the steel used for steam turbines are well known. AA has determined experimentaly the curve K_{IC} with respect to the temperature and the parameters Co and m of the PARIS's law, for a NiCrMoVa steel (2 to 3.5% Ni) which is used in LP rotors.

The value of K_{IC} (110 MPa $\sqrt{m}$) which has been taken into account in this study corresponds to a F.A.T.T. of + 10°C and a minimum temperature of 0°C.

The values of Co and m of the PARIS's law (relation 4) are Co = 2.59 10^{-11} and m = 2.5.

$$\frac{da}{dN} = C_0 \, (\Delta K)^m \qquad (4)$$

with a : depth of the defect
and N : number of cycles

Fatigue crack propagation

For a rotor turbine a cycle may be represented by a run up followed by a run down. The stress is the centerpart of the shaft varies from zero to a maximum value which corresponds to the rated speed. The stress that will be taken into account as the reference stress is the mean circumferencial stress in the plane of symetry of the last disk : σ = 232 MPa (This same stress has been used for calculating K_I). Furthermore it is supposed that this mean stress is the same for bored and plains shafts.

ΔK used in the PARIS's law is then equal to the rated speed K_I determined by the 3D analyses, these values depend upon the dimensions and the shape of the crack.

In order to determine the evolution of a given defect, the following assumptions are made :

- the crack remains (semi-) elliptic
- points A and B are moving independantly one from each other, the values that are taken into account for KA and KB depend upon b/a and b during the preceeding cycle thus leading to an evolutive shape of the defect
- the critical defect is reached when KA or KB is equal to the K_{IC} of the material.

Under these assumptions, it is possible to determine the evolution of a given defect and the number of cycle leading to the ruin of the structure. Figures 10 and 11 (with b/a ranging from 0.1 to 1.0) show off that the final shape of a defect is independant from its initial shape, but depends upon the type of shaft (bored or not). For a bored shaft, b/a tends to 0.6 and for a plain one b/a tends to 1.0. These shapes are corresponding to a constant value of K_I along the crack front. Figure 12 shows the evolution of b with respect to the number of cycle for an initial b/a equal to 0.3 in a bored shaft and in a plain one. It is visible on this figure that the number of cycles leading to the failure of the structure is far more important for plain shafts than for bored ones.

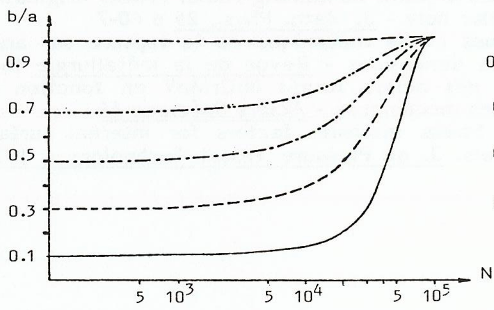


Fig. 10 : plain shaft

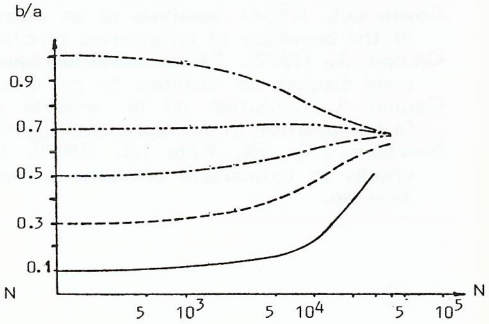


Fig.11 : bored shaft

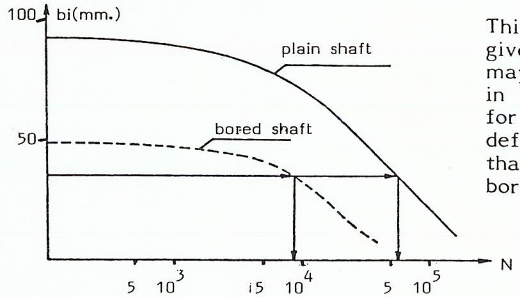


Fig.12 : evolution of b with respect to the number of cycle

This study clearly shows off that for a given number of cycles, initial defects may be bigger in plain shafts than in bored one. Figure 12 shows that for a life time of 20 000 cycles, initial defects in plain shaft must be smaller than 60 mm, and initial defects in bored ones must be smaller than 20 mm.

CONCLUSION

It is vital that rotors keep their integrity throughout the service life of the station. The few examples in the world, where such integrity has not been maintained for various reasons, have highlighted the importance of the problem.

The numerical analysis methods now available associated with the developments of fracture mechanics have enabled to obtain a better knowledge of the behaviour of the rotor vis-à-vis brittle fracture. The studies conducted by AA and Cetim have given a good idea of the validity of the simplified calculation methods currently used and have shown that in some cases a 3D is necessary.

Concerning rotor design, the interest of avoiding boring clearly appears even for large size integral rotors, taking into account the progress achieved in the production of rotor forgings and the better behaviour of now bored rotors from the point of view of fatigue.

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