NUMERICAL APPLICATIONS OF PATH INDEPENDENT INTEGRALS IN THE CASE OF THERMAL STRAINS, CREEP ANALYSIS AND MIXED MODE SITUATIONS

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ABSTRACT

Numerical determination of J-Integral by finite element path integration technique can be extended and applied to situations such as : thermal strains (area term), steady state creep (C^*-Integral evaluated from velocity fields), mixed modes in linear elastic fracture mechanics (J_I, J_{II}). Path independent quantities are calculated and can be used to correlate crack initiation or growth phenomena in these situations.

KEYWORDS

Fracture mechanics; finite element analysis; J-Integral; path independence; thermal strains; creep; C^*-Integral; mixed mode; (J_I, J_{II}) Integrals.

INTRODUCTION

This study is concerned with two-dimensional finite element determinations of J-Integral and extensions in various situations. After a mechanical analysis performed with INCA code (CASTEM system), a post processor is used to calculate line and area integrals. Three situations are considered : a longitudinally cracked tube subjected to thermal loading in the elastoplastic regime (J determination); a centre cracked plate made of a creeping material under uniform tension (C^* determination); a slant edge crack in a plate subjected to tension or bending in the elastic regime (J_I, J_{II} determination).

EVALUATION OF J-INTEGRAL IN THE PRESENCE OF A TEMPERATURE FIELD

If we consider the case of a 2D fracture mechanics analysis in the presence of a thermal strain field :

$$\epsilon^{th}_{ij} = \alpha\,(\Theta - \Theta_R)\;\delta_{ij}$$

$$\begin{cases} \alpha & : \text{dilatation coefficient} \\ \Theta & : \text{temperature field} \\ \Theta_R & : \text{mean temperature} \end{cases}$$

it is easy to see that the usual expression of J loses its fundamental path independence porperty, when evaluated with the mechanical strains ϵ^m_{ij} in the energy density term :

$$\begin{cases} J = \int_\Gamma (W^m\,dy - \vec{T}\cdot\dfrac{\partial \vec{u}}{\partial x}\,dl) \\ W^m = \int_o^{\epsilon^m} \sigma_{ij}\,d\epsilon^m_{ij} \end{cases} \qquad (1)$$

In order to restablish path independence it appears necessary to add an area integral to the previous line integral (Bui, 1978) :

$$J = J + \iint_A \alpha\,\sigma_{ii}\,\frac{\partial\Theta}{\partial x}\,dx\,dy \qquad (2)$$

A : surface surrounded by the contour

This expression is valid for a material which is temperature independent; if the properties (Young's modulus, stress-strain curve, dilatation coefficient) are strongly temperature dependent, relation (2) can be generalized in the following form :

$$J = J + \iint_A \left[\left[\alpha + (\Theta - \Theta_R)\frac{d\alpha}{d\Theta}\right]\sigma_{ii} - \frac{W^m_{el}}{E}\frac{dE}{d\Theta} - \int_o^{\epsilon}\frac{\partial\sigma}{\partial\Theta}\,d\epsilon\right]\frac{\partial\Theta}{\partial x}\,dA \qquad (3)$$

W^m_{el} : elastic part of mechanical strain energy density

Numerical application was conducted under temperature independent material assumptions, which already allows a large number of industrial applications. The finite element analysis of a longitudinally cracked tube ($R_i = 0.05$ m; $a/t = 0.5$; $t/R_i = 0.2$) was performed with isoparametric 6-noded triangular and 8-noded quadrangular elements in plane strain situation.
Material properties are :

$$\begin{cases} E = 200\ \text{GPa} \qquad \nu = 0.3 \qquad \alpha = 10^{-5}\ K^{-1} \\ \text{Stress-strain law } (\sigma > \sigma_o = 200\ \text{MPa}) \quad \epsilon/\epsilon_o = (\sigma/\sigma_o)^5 \end{cases}$$

Temperature field is given by a logarithmic radial distribution ($R_i < r < R_o$)

$$\Theta(r) = \Theta_o \frac{\text{Log}\ (r/R_i)}{\text{Log}\ (R_o/R_i)}$$

The numerical evaluation of relations (1)(2) is carried out in the following way : quantities such as stresses or displacement gradients are determined at each nodal point of the contour; line integral is evaluated by using linear interpolation between two consecutive nodes and area term is evaluated by Gauss point technique. A linear elastic calculation was carried out for $\alpha\Theta_o = 0.015$. Results thus obtained are plotted on Fig. 1 where the importance of the surface integral is clearly seen (contour radius corresponds to a circle of the same area). The selected value can be chosen as : J = 410 kNm^{-1}. In the termoplastic regime analysis was conducted in seven regular temperature steps from $\Theta_o = 30$ K to $\Theta_o = 210$ K; incremental plasticity equations based on Von Mises criterion and normality rule are solved by using a two level iteration scheme of initial stress type. Numerical results are given in Table 1 and plotted on Fig. 2 for the last step; one can notice that path independence is well maintained. The contribution of the area term can reach 30% of the total value for the largest contour at the last step; the influence of plasticity on J-Integral is moderate in that case (see ratio J/J_{el} in Table 1).

Table 1 Evaluation of J in thermoplasticity

Θ_o (K)	J (kNm^{-1})	J/J_{el}
30	0.16	1.00
60	0.67	1.01
90	1.61	1.09
120	3.27	1.25
150	5.41	1.32
180	8.19	1.39
210	11.4	1.42

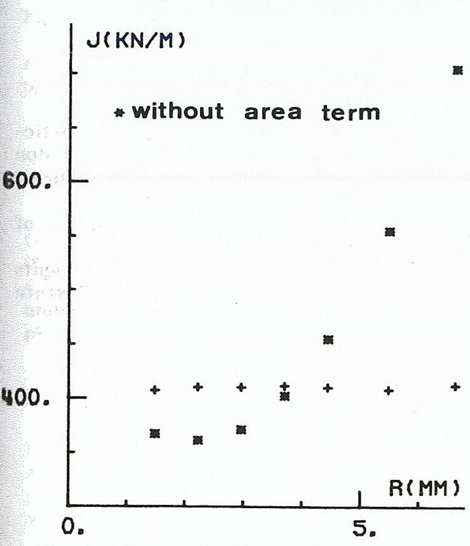

Fig. 1 Thermoelastic results

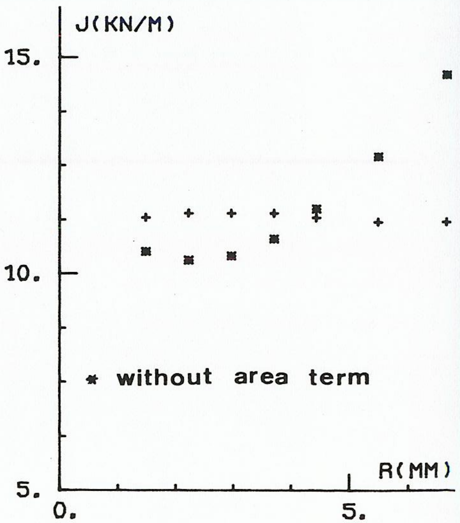

Fig. 2 Thermoplastic results (Θ_o = 210 K)

This small study shows the possible use of a criterion based on J to predict crack initiation in the presence of thermal strains from a finite element point of view. It can be applied for instance in the important case of a flawed structure subjected to a thermal shock and material toughness J_{1C}(tip) corresponding to the temperature around the crack tip must be used.

NUMERICAL EVALUATION OF C*-INTEGRAL FOR A CREEPING MATERIAL

For a creeping material J can no longer be considered as a crack tip field parameter because it loses its fundamental path independence property. Begley and Landes (1976) introduced the C*-Integral as a crack tip field velocity parameter :

$$\begin{cases} C^* = \int_{\Gamma} (W^* \, dy - \vec{T} \cdot \frac{\partial \dot{\vec{u}}}{\partial x} \, dl) \\ W^* = \int_o^{\dot{\varepsilon}} \sigma \, d\dot{\varepsilon} \end{cases} \quad (4)$$

In the usual case of a Norton law $\dot{\varepsilon} = A \sigma^N$, W^* reduces to :

$$W^* = \frac{N}{N+1} A \sigma^{N+1} \quad (5)$$

Dang-Van and Mudry (1981) demonstrated path independence of C*-Integral in the case of steady state creep ($\dot{\sigma}_{ij} = o$) for a non-moving crack.
Numerical application was done on a centre cracked plate (crack length 2a = 0.1 m; height 2H = 0.5 m; width 2w = 0.2 m) under uniform tension σ_{ap} (see mesh on Fig. 3).
Material properties are :

$$\begin{cases} E = 200 \text{ GPa} \qquad \nu = 0.3 \\ \text{Stress strain law} \quad \varepsilon/\varepsilon_o = (\sigma/\sigma_o)^{13} \qquad \sigma_o = 400 \text{ MPa} \\ \text{Creep law} \quad \dot{\varepsilon} = A\sigma^N \qquad N = 8 \qquad A = 10^{-24} h^{-1} MPa^{-8} \end{cases}$$

Finite element analysis in plane strain situation was divided in two phases : elastic-plastic loading during which stress is gradually increased from σ_{ap} = 20 MPa to σ_{ap} = 260 MPa; creep phase where load is kept constant (σ_{ap} = 260 MPa) during three hours.
In relation (4) displacement velocity $\dot{u}$ is evaluated by finite difference of displacement fields for two consecutive time steps : $\dot{u} = \Delta u / \Delta t$.
The following results have been obtained : in the early stage of creep C* presents a strong path dependence but this dependence tends to vanish when a steady state is reached (see Fig. 4) and C* becomes constant (C* = 29 KN m^{-1} h^{-1}).

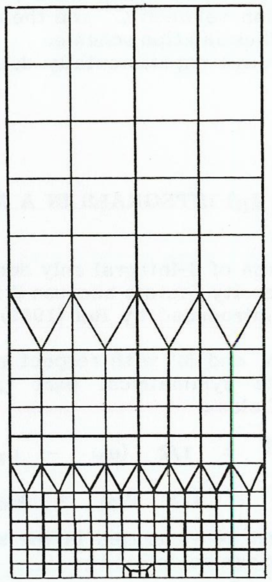

Fig. 3 Quarter of a centre cracked plate

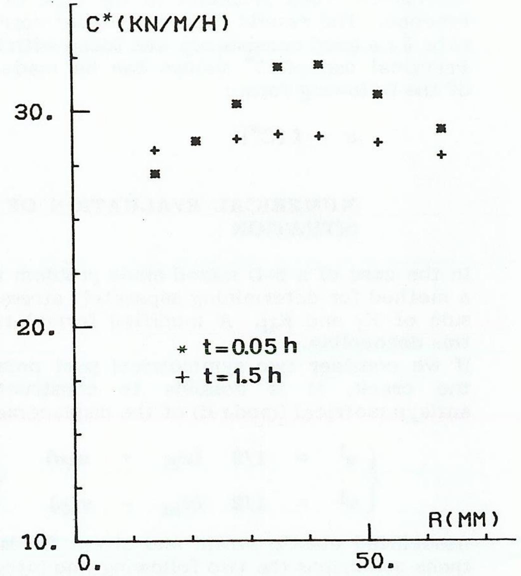

Fig. 4 C* path independence

A simple correlation is used by experimentalists to estimate C* from crack mouth opening rate measurements in the case of a CCP specimen (Taira and others, 1979) :

$$C^* = \frac{N-1}{N+1} \sigma_{net} \dot{\delta} \quad (6)$$

σ_{net} : averaged stress in the uncracked ligament

Our finite element results lead to $\dot{\delta}$ = 0.074 mm h^{-1} which, by application of relation (6) gives C* = 30 kN m^{-1} h^{-1}. This value is very close to the path integration result.
C*-Integral can also be evaluated with EPRI handbook estimation technique (Kumar and others, 1981) through the following relationship deduced by analogy with fully plastic J formula.

$$C^* = Aa\,(1 - \frac{a}{w})\, h_1 \left(\frac{\sigma_{net}}{2\sqrt{3}}\right)^{N+1} \quad (7)$$

In our case h_1 = 1.68 and the application of relation (7) leads to C* = 32 kN m^{-1} h^{-1}. As the EPRI estimation procedure cannot take into account effects of plasticity prior to creep phase, we consider that there is a good agreement between that value and our finite element results.

This study shows the numerical applicability of C^*-Integral concept to fracture mechanics creep problems in the case of a stationary crack when steady state is reached. The results support a linear correlation between C^* and the crack opening rate $\dot{\delta}$; a good consistency was found with EPRI estimation scheme.
Practical use of C^* values can be made through experimentally determined laws of the following form :

$$\dot{a} = f(C^*)$$

NUMERICAL EVALUATION OF (J_I, J_{II}) INTEGRALS IN A MIXED MODE SITUATION

In the case of a 2-D mixed mode problem the use of J-Integral only does not provide a method for determining separately stress intensity factors because J is a quadratic sum of K_I and K_{II}. A modified formulation introduced by Bui (1982) now permits this decoupling.
If we consider two symmetrical part points M and M' with respect to the axis of the crack, it is possible to construct the symmetrical part (mode I) and antisymmetrical (mode II) of the displacement fields :

$$\begin{cases} u^I = 1/2 \quad (u_M + u_{M'}) \\ v^I = 1/2 \quad (v_M - v_{M'}) \end{cases} \qquad \begin{cases} u^{II} = 1/2 \quad (u_M - u_{M'}) \\ v^{II} = 1/2 \quad (v_M + v_{M'}) \end{cases} \qquad (8)$$

Associated elastic strain and stress fields must be separated in the same way. In these conditions the two following line integrals can be defined :

$$J^I = \int_\Gamma (W(\sigma^I)\, dy - \vec{T^I} \cdot \frac{\partial \vec{u^I}}{\partial x}\, dl) \qquad J^{II} = \int_\Gamma (W(\sigma^{II})\, dy - \vec{T^{II}} \cdot \frac{\partial \vec{u^{II}}}{\partial x}\, dl) \qquad (9)$$

These integrals are shown to be path independent and uniquely related to the corresponding stress intensity factors :

$$J^I = \frac{K_I^2}{E'} \qquad J^{II} = \frac{K_{II}^2}{E'} \qquad J = J^I + J^{II} \qquad (10)$$

Elastic finite element analysis was carried out on a plate with a slant edge crack in plane strain situation (see mesh and geometry on Fig. 5).
Material properties are :

$$E = 200 \text{ GPa} \qquad \nu = 0.3$$

Two types of loading are considered : uniform tension $\sigma = 100$ MPa and bending moment M produced by a linearly varying stress of magnitude $\sigma_b = 6\ M/b^2 = 100$ MPa.
Line integrals (J_I, J_{II}) are evaluated on six circular contours surrounding the crack tip; results concerning tension loading are plotted on Fig. 6. Numerical values are given in Table 2 as the well as corresponding shepe factors :

$$F_I = \frac{K_I}{\sigma\sqrt{\pi a}} \qquad F_{II} = \frac{K_{II}}{\sigma\sqrt{\pi a}}$$

These results are in good agreement with those published in the compendium of stress intensity factors (Rooke and Cartwright, 1976).

Table 2 Mixed mode results

	J_I (kNm^{-1})	J_{II} (kNm^{-1})	F_I	F_{II}
Tension	0.980	0.229	1.17	0.57
Bending	0.425	0.072	0.77	0.32

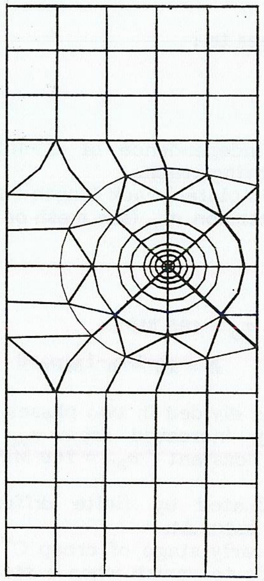

Fig. 5 Slant edge crack in a plate

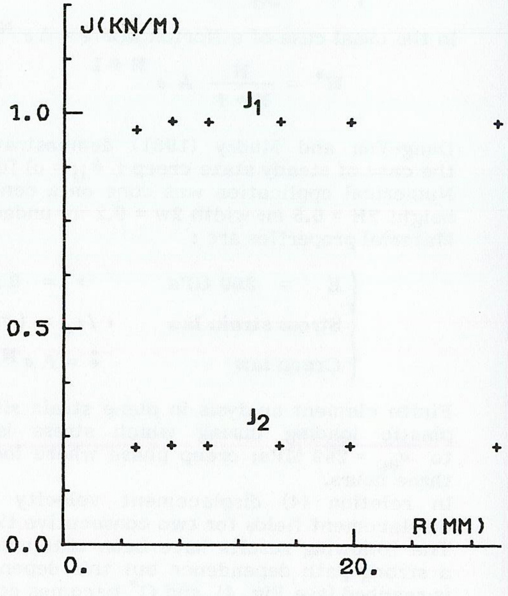

Fig. 6 Results for tension loading

This major interest of the previous method for determining K_I and K_{II} is to take advantage of path independence property of J_I and J_{II}, which does not necessitate a very refined mesh in the vicinity of the crack tip. Moreover it is easy to implement in an already existing J-Integral evaluation code because it uses the same integration routines; it can also be generalized to thermal problems.
It is often important to be able to estimate K_I and K_{II} separately in many practical cases such as directional criteria, mixed mode fatigue crack growth,...

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