FINITE ELEMENT DETERMINATION OF TEARING MODULUS FOR APPLICATION TO INDUSTRIAL CASES

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ABSTRACTS

This paper presents a method to determine the Tearing Modulus without doing two complete computations.

KEYWORDS

Tearing modulus ; computations.

I. INTRODUCTION

During the life of a nuclear reactor, fractures can occur either in a brittle manner, in some accidental situation, or with large plastic deformations. First cases are now well handled and one needs mainly to review all the potential fractures, and to verify that the structure will not fail. For second cases PARIS [1] introduce the criteria of the tearing modulus.

The tearing modulus can be reached by two computations but cost will be too important. Here is presented a method to compute T at low cost.

II. THE TEARING MODULUS "T"

$$T = C \frac{dJ}{da}$$

T is proportional to the rate of variation of J, while the crack grows. On another hand, J can be interpreted as the potential energy release rate when the crack grows.

In elastic cases one can assume that to propagate a crack, a certain constant amount of energy is needed in front of it. Then, if during the propagation the value of this energy decreases, the crack stops, neglecting the effect of inertia on the propagation. But if during propagation the value of this energy stays even or increases, the crack propagates in an unstable way.

If plasticity occurs, it has been shown that for a given material, the value of the energy needed to propagate a crack essentially depends upon the increase of crack length which already occured. This implies the existence of the J-Δa curve for a given material (Δa is the increase of crack length).

This curve is controlling the crack growth for a large range of situations. Then, for a specific structure, if the actual value of Δa is known, it is possible to compare the value of T (structure) to the value of T (material), in order to predict the stability of the crack.

This stability criterion proposed by Paris is edited as:
if T struct. > T mat. : instability
if T struct. < T mat. : stability.

III. FINITE ELEMENT APPROACH

A/ Generality

The BILBO code is a finite element code for general purpose 3D analysis. The aim of one modulus of this code is fracture mechanics analysis. In order to compute the value of K or J it is already possible to use special brick element including K_I, K_{II}, K_{III} as unknowns in elastic cases, or to use the virtual crack extension method (Parks) in elastic or plastic analysis.

Several methods are possible in order to compute T among which the followings:
- calculate T from the algebraic expression of the derivative of J. From a numerical point of view this method might not be accurate because the expression uses derivative of stresses and strains, and they are not so well computed.
- Use two different meshes with two different cracks lengths, calculate the value of J for each one and approximate $\frac{dJ}{da}$ by the finite difference $\frac{\Delta J}{\Delta a}$. This method gives good results but, specially for non-linear 3D analysis, the cost is rapidly prohibitive because it needs two meshes, and two computation runs.
- The third method that we use considers that the two cracks lengths a and a + Δa are very similar and so the displacement fields U and U + ΔU must be very close. Then an iterative algorithm can be used in order to compute U + ΔU from U. This algorithm must converge rapidly.

B/ Computation of "T" for Elastic Analysis

Considering the equilibrium of the structure for a crack length a, we can write:

$$K_a U_a = F_a$$

Let us set :

$$K_{a+\Delta a} = K_a + \Delta K$$
$$U_{a+\Delta a} = U_a + \Delta U$$
$$F_a = F_{a+\Delta a}$$

Then the equilibrium for a crack length of a + Δa leads to:

$$K_{a+\Delta a} U_{a+\Delta a} = F_{a+\Delta a} = (K_a + \Delta K)\ (U_a + \Delta K)\ (U_a + \Delta U) = F_a = K_a U_a$$

One can obtain easily:

$$K_a \Delta U + \Delta K U_a + \Delta K \Delta U = 0.$$

Then, it is easy to imagine the following iterative scheme:

$$K_a \Delta U_n = - \Delta K U_a - \Delta K \Delta U_{n-1}$$

where:
- Δa is created by moving a set of nodes at the crack tip as in the virtual extension method.
- ΔK is obtained by computing the new stiffness matrix for all the elements modified by the change in length of the crack.
- $\Delta K U_a$ is a constant term.

To start the algorithm scheme, we set $\Delta U_{(o)} = 0$.

C/ Computation of "T" for Plastic Analysis

At a time t_o the equilibrium can be written as:

$$K_a U_a = F_a - F^P_a$$

where F^P_a = plastic forces.

Let us set:

$$F_a = F_{a+\Delta a}$$
$$F^P_{a+\Delta a} = F^P_a + \Delta F^P$$
$$K_{a+\Delta a} = K_a + \Delta K$$
$$U_{a+\Delta a} = U_a + \Delta U$$

in a very simple way we can obtain:

$$K_a \Delta U = - \Delta K U_a - \Delta K \Delta U - \Delta F^P$$

We can perform the same kind of iterative scheme to derive:

$$K_a \Delta U_n = - \Delta K U_a - \Delta K U_{n-1} - \Delta F^P_{n-1}$$

To do that is not equivalent with a computation of a crack of length a + Δa with loads varying from 0 to F_a. The iterative scheme leads to consider a crack of length a loaded up to F_a, and then to propagate the crack of a quantity Δa, letting stresses decrease behind the front crack.

From a practical point of view this algorithm is similar to the one of initial stresses used to resolve plastic analysis and then no programmation efforts are needed.

IV. VERIFICATION AND VALIDATION OF THE METHOD

In order to verify the code we have choosen a simple structure composed of a beam and a spring (Fig. 1).

An elastoplastic computation run has been carried out on this structure and we picked up one of the first step (nearly elastic) to compute T.

Few remarks have to be pointed out :

- the value of T found is in good agreement with quasi-analytical solution given by TADA [3].
- The value of J obtained by the virtual extension method is not really constant when the set of moving nodes changes.
- The value of T computed with $\frac{J_{a+\Delta a} - J_a}{\Delta a}$ is extremely stable in front of the moving nodes if $J_{a+\Delta a}$ and J_a are computed with the same set of moving nodes.

V. CONCLUSION

The proposed method for T Finite Element computation has two main advantages :

- it seems that the computed value of T is stable eventhough (Table 1), the value for J is not completely stable.
- the computational cost associated with the method is low in elastic analysis. In plastic analysis, it is equivalent to one load step, which is reasonable.

Up to now, the obtained values for T compares well with semi-analytical solutions ; more validation work is needed.

The proposed method has a great potential interest, in order to analyse industrial situations where crack stability is critical.

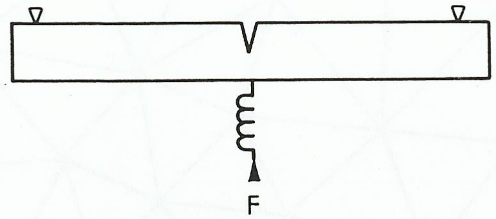
REFERENCES

[1] Paris, Tada, Zahoor, Ernst. The theory of instability of the Tearing Mode of elastic plastic crack growth. ASTM-STP 668.

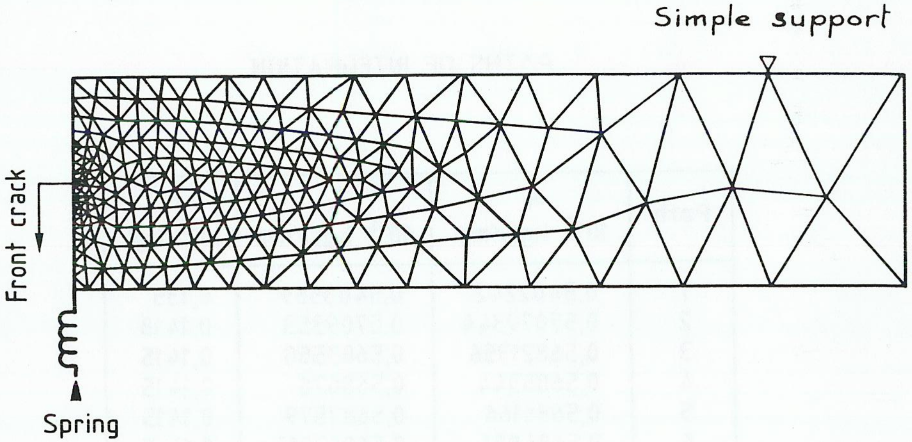
[2] Parks. (1977). The virtual crack extension method for non-linear material behaviour. Computer Methods in Applied Mechanics and Engineering, Vol. 12, 353-354.
Parks. (December 1974). A stiffness derivative finite element technique for determination of crack tip stress intensity factors. International Journal of Fracture, Vol. 10, n°4.

[3] Tada, Paris, Irwin. (1973). The stress analysis of cracks handbook. Mel Research Corp. Hettertown, PA.

Fig. 1. Cracked beam used for the computations.

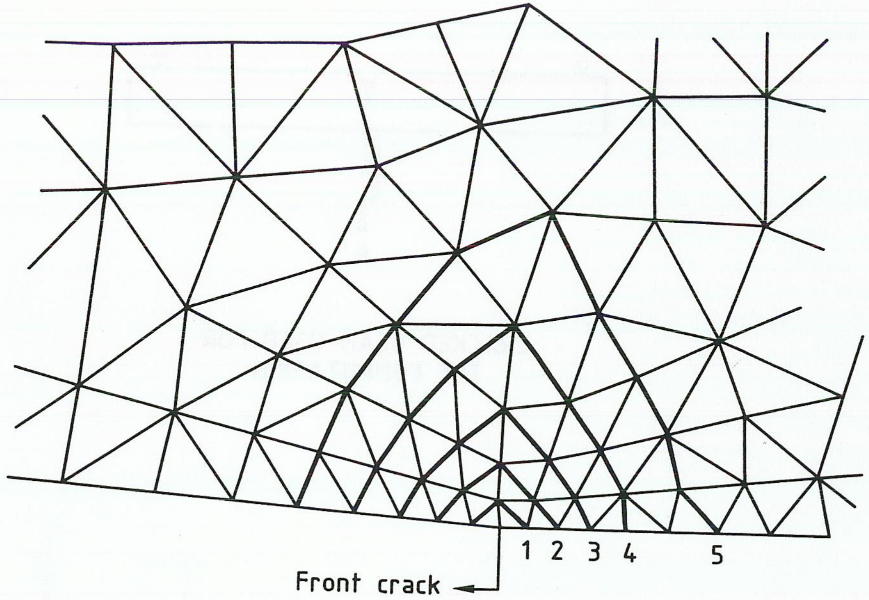


CRACKED BEAM USED FOR
THE COMPUTATION



MESH OF HALF STRUCTURE

Figure 1



PATHS OF INTEGRATION

Path	J_1 10^{-6} kgf/mm	J_2 10^{-6} kgf/mm	T 10^{-6} kgf
1	0,5402242	0,5403589	0,135
2	0,57079344	0,5709353	0,1418
3	0,56821356	0,5683550	0,1415
4	0,5685344	0,568676	0,1415
5	0,5686166	0,5687579	0,1415
6	0,5684884	0,56863015	0,1417
7	0,5679457	0,56808736	0,1417
8	0,5695503	0,56969188	0,1416
9	0,5693338	0,56947512	0,1413

RESULTS FOR DIFFERENT PATHS

Figure 2