DYNAMIC GROWTH OF CURVILINEAR RUPTURES AT VARIABLE SPEED

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ABSTRACT

It is a matter of fact that ruptures in rocks usually move along some curvilinear and/or branching surfaces. Hence, of special interest is the dynamic growth of rupture consisting of alternating open and shear mode elements along a sawtooth path. The purpose of this paperis to calculate dynamic field of elastic disturbances generated by complex ruptures propagating at variable speed along arbitrary curvilinear path. The solutions obtained can be utilized to solve the main problem of theoretical seismology, namely, to determine the location, orientation, trajectory and speed of fracture spreading inside the Earth basing upon analysis of dynamic displacement field. The paper may be of interest also for acoustic emission study. The two-dimensional model is considered for simplicity.

KEYWORDS

Dynamic propagation, curvilinear rupture, arbitrary variable speed, dislocation rupture, Riemann-Hilbert problem, self-similar solution, branching, limiting speed.

INTRODUCTION

The excessive passion for the force condition on rupture borders did not justify hopes in dynamical problems of seismology because the solutions at hand (Kostrov,1966, 1974) are too cumbersome for effective analysis. As it was shown earlier (Cherepanov and Afanas'ev, 1974, Bykovtsev, 1978,1979,1983 a, b, c; Bykovtsev and Cherepanov, 1980 a, b, c, 1981 a, b; Bykovtsev and Tavbaev, 1984; Cherepanov, 1979), the dislocation representation of ruptures is more convenient to use as compared to the force description. Below we apply the dislocation description of rupture. The dislocation rupture is defined as a cut at every point of which the magnitude and direction of the displacement discontinuity vector are given as functions of time and coordinates.

At first, we consider the following auxiliary problem. Let the dislocation rupture start from the origin of Cartesian rectangular coordinate system Oxyz at constant speed v along the positive x-axis at the initial time moment $t=0$ in a homogeneous isotropic elastic medium (μ - elastic shear modulus). Let Ux, Uy and Uz be the components of displacement vector along x, y and z axes, respectively, and σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{xz}, σ_{yz} the respective components of stress tensor. The problem is assumed to be plane, that is, Ux, Uy, Uz are functions of x and y only.

The basic equations of the dynamic theory of elasticity are in this case as follows:

$$U_x = U_x^p + U_x^s,\quad U_y = U_y^p + U_y^s,\quad U_z = U_z^s$$
$$\Delta U_\eta^p = \frac{1}{C_p^2}\frac{\partial^2 U_\eta^p}{\partial t^2},\ (\eta = x,y);\quad \Delta U_\eta^s = \frac{1}{C_s^2}\frac{\partial^2 U_\eta^s}{\partial t^2},\ (\eta = x,y,z) \tag{1}$$
$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},\quad \frac{\partial U_x^p}{\partial y} = \frac{\partial U_y^p}{\partial x},\quad \frac{\partial U_x^s}{\partial x} = -\frac{\partial U_y^s}{\partial y}$$

Here superscripts p and s correspond to longitudinal and transversal components of displacement, Cp and Cs are velocities of respective longitudinal and transversal waves ($Cp > Cs$).

Let the displacement discontinuity on rupture be the homogeneous function of zeroth dimension $f(r/t)$. In the general case this function may be represented as

$$\vec{f}(r/t) = f_1(r/t)\vec{i} + f_2(r/t)\vec{j} + f_3(r/t)\vec{k},\qquad (r = \sqrt{x^2+y^2}) \tag{2}$$

($\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors along x, y, z axes respectively).

Therefore, the general dislocation rupture may be treated as superposition of sliding, open and tearing modes. So, we have the following boundary problems:

sliding mode rupture (the problem is skew-symmetric with respect to x-axis)

$$U_x = \tfrac{1}{2} f_1(r/t),\ \sigma_{yy} = 0 \quad \text{for } y=0,\ 0<x<vt$$
$$U_x = 0,\ \sigma_{yy} = 0 \quad \text{for } y=0,\ x<0,\ x>vt \tag{3}$$

open mode rupture (the problem is symmetric with respect to x-axis)

$$U_y = \tfrac{1}{2} f_2(r/t),\ \sigma_{xy} = 0 \quad \text{for } y=0,\ 0<x<vt$$
$$U_y = 0,\ \sigma_{xy} = 0 \quad \text{for } y=0,\ x<0,\ x>vt \tag{4}$$

tearing mode rupture

$$U_z = \tfrac{1}{2} f_3(r/t) \quad \text{for } y=0,\ 0<x<vt$$
$$U_z = 0 \quad \text{for } y=0,\ x<0,\ x>vt \tag{5}$$

The problems posed are self-similar.

THE FUNDAMENTAL SOLUTIONS

We utilize the general approach (Cherepanov, 1979) to these problems which allows us to reduce the self-similar problems to some Riemann-Hilbert problems of the theory of analytic functions of the complex variable.

If the functions LUx, LUy, LUz are homogeneous, then we introduce the following notation:

$$U_x^\circ = LU_x,\ U_y^\circ = LU_y,\ U_z^\circ = LU_z,\ \sigma_{xx}^\circ = L\sigma_{xx},$$
$$\sigma_{yy}^\circ = L\sigma_{yy},\ \sigma_{xy}^\circ = L\sigma_{xy},\ \sigma_{xz}^\circ = L\sigma_{xz},\ \sigma_{yz}^\circ = L\sigma_{yz} \tag{6}$$

where L is the linear differential operator of the type

$$L = \frac{\partial^{m+n}}{\partial x^m \partial t^n}$$

The general representation of the solutions are of the following form: for the problems symmetrical with respect to x-axis (Cherepanov, 1979)

$$U_x^\circ = \mathrm{Re}[U_p(Z_p) + U_s(Z_s)],\quad U_y^\circ = \mathrm{Re}[V_p(Z_p) - V_s(Z_s)],\quad U_z^\circ = 0$$
$$\sigma_{xx}^\circ = \frac{\mu}{C_s^{-2}}\mathrm{Re}\left\{\frac{[C_s^{-2} - 2(C_p^{-2} - Z_p^2)](C_p^{-2} - 2Z_p^2)}{C_p^{-2} - Z_p^2} W_2'(Z_p)\frac{\partial Z_p}{\partial y} - 4Z_p^2 W_2'(Z_s)\frac{\partial Z_s}{\partial y}\right\} \tag{7}$$
$$\sigma_{yy}^\circ = \frac{\mu}{C_s^{-2}}\mathrm{Re}\left\{\frac{(C_s^{-2} - 2Z_p^2)^2}{C_p^{-2} - Z_p^2} W_2'(Z_p)\frac{\partial Z_p}{\partial y} + 4Z_s^2 W_2'(Z_s)\frac{\partial Z_s}{\partial y}\right\}$$
$$\sigma_{xy}^\circ = \frac{2\mu}{C_s^{-2}}\mathrm{Re}\left\{(C_s^{-2} - 2Z_p^2) W_2'(Z_p)\frac{\partial Z_p}{\partial x} - (C_s^{-2} - 2Z_s^2) W_2'(Z_s)\frac{\partial Z_s}{\partial x}\right\}$$
$$U_p'(Z) = \frac{Z(C_s^{-2} - 2Z^2)}{C_s^{-2}\sqrt{C_p^{-2} - Z^2}} W_2'(Z),\qquad V_p'(Z) = \frac{C_s^{-2} - 2Z^2}{C_s^{-2}} W_2'(Z)$$
$$U_s'(Z) = -\frac{2Z\sqrt{C_s^{-2} - Z}}{C_s^{-2}} W_2'(Z),\qquad V_s'(Z) = \frac{2Z^2}{C_s^{-2}} W_2'(Z)$$

for the problems skew-symmetrical with respect to x-axis (Bykovtsev and Cherepanov, 1980 b)

$$U_x^\circ = \mathrm{Re}[U_p(Z_p) + U_s(Z_s)],\quad U_y^\circ = \mathrm{Re}[V_p(Z_p) + V_s(Z_s)],\quad U_z^\circ = 0$$
$$\sigma_{xx}^\circ = \frac{2\mu}{C_s^{-2}}\mathrm{Re}\left\{[C_s^{-2} - 2(C_p^{-2} - Z_p^2)] W_1'(Z_p)\frac{\partial Z_p}{\partial x} + (C_s^{-2} - 2Z_s^2) W_1'(Z_s)\frac{\partial Z_s}{\partial x}\right\} \tag{8}$$
$$\sigma_{yy}^\circ = \frac{2\mu}{C_s^{-2}}\mathrm{Re}\left\{(C_s^{-2} - 2Z_p^{-2}) W_1'(Z_p)\frac{\partial Z_p}{\partial x} - (C_s^{-2} - 2Z_s^2) W_1'(Z_s)\frac{\partial Z_s}{\partial x}\right\}$$
$$\sigma_{xy}^\circ = \frac{\mu}{C_s^{-2}}\mathrm{Re}\left\{4Z_p\sqrt{C_p^{-2} - Z_p^2}\, W_1'(Z_p)\frac{\partial Z_p}{\partial x} + \frac{(C_s^{-2} - 2Z_s^2)^2}{Z_s\sqrt{C_s^{-2} - Z_s^2}} W_1'(Z_s)\frac{\partial Z_s}{\partial x}\right\}$$
$$U_p'(Z) = \frac{2Z^2}{C_s^{-2}} W_1'(Z),\qquad V_p'(Z) = \frac{2Z\sqrt{C_p^{-2} - Z^2}}{C_s^{-2}} W_1'(Z)$$
$$U_s'(Z) = \frac{C_s^{-2} - 2Z^2}{C_s^{-2}} W_1'(Z),\qquad V_s'(Z) = -\frac{Z(C_s^{-2} - 2Z^2)}{C_s^{-2}\sqrt{C_s^{-2} - Z^2}} W_1'(Z)$$

for the antiplane problems

$$U_x^\circ = U_y^\circ = 0,\quad U_z^\circ = \mathrm{Re}\, W_3(Z_s),\quad \sigma_{xz}^\circ = \mu\,\mathrm{Re}\, W_3'(Z_s)\frac{\partial Z_s}{\partial x},\quad \sigma_{yz}^\circ = \mu\,\mathrm{Re}\, W_3'(Z_s)\frac{\partial Z_s}{\partial y} \tag{9}$$

where $Z_k=(tx-iy\sqrt{t^2-C_k^{-2}(x^2+y^2)})(x^2+y^2)^{-1}, \quad (k=p,s)$

The boundary problem (1)-(5) may be formulated in terms of the representations (7)-(9) as the following Dirichlet problem:

$$Re\,W_j(Z)=\tfrac{1}{2}f_j\left(\tfrac{1}{Z}\right) \quad \text{for } Im\,Z=0,\ Re\,Z>v^{-1} \qquad (10)$$
$$Re\,W_j(Z)=0 \quad \text{for } Im\,Z=0,\ Re\,Z<v^{-1}$$

The solution of this problem is provided by the Schwartz integral

$$W_j\left(\tfrac{1}{Z}\right)=\frac{1}{2\pi i}\int_0^v \frac{f_j(t)\,dt}{t-Z}+iC_0 \qquad (11)$$

From here, we can find stresses and displacements by means of formulae (7)-(9) taking in account that L=1 because displacements are homogeneous functions.

We consider the case when $\vec{f}\left(\frac{z}{t}\right)=\vec{B}(b_1,b_2,b_3)$ =const in more detail. For this case the solution of the problem (10) is of the form

$$W_j(Z)=-\frac{ib_j}{2\pi}\ln(1-vZ), \quad (j=1,2,3) \qquad (12)$$

Formulae (12) provide fundamental solutions which allow, on the basis of the superposition principle, to construct the solution of the problem for arbitrary system of ruptures propagating at variable speeds along arbitrary trajectories (Bykovtsev, 1978, 1979, 1983a, b, c; Bykovtsev and Cherepanov, 1980a, b).

ANALYSIS OF THE SOLUTION

Analysis of stresses near a moving edge of rupture shows that, after a certain critical velocity v_* is achieved, there appear two symmetrical maxima for the stress $\sigma_{z\varphi}$ (sliding mode) and for the stress $\sigma_{\varphi\varphi}$ (open mode). This implies that the linear propagation of rupture appears to be impossible for $v>v_*$, that is, rupture path either kinks or branches. The equation for this critical velocity is identical for both sliding and open modes (Bykovtsev and Cherepanov, 1980 c, 1981 b). It has the form (Bykovtsev and Cherepanov, 1980 c)

$$4\sqrt{1-\alpha^2\beta^2}\,\sqrt{1-\alpha^2}-(\alpha^2-2)^2-2(2-\alpha^2)\alpha^4=0, \quad v_*=\alpha C_s$$

Here are some values of the root, α, of this equation in terms of Poisson's ratio ν :

α	0.46	0.485	0.515	0.55	0.59	0.64
ν	0	0.1	0.2	0.3	0.4	0.5

It is seen that the critical velocity of linear propagation for dislocation ruptures is approximately ten percent less than for cracks (Cherepanov, 1979).

The plots of stress coefficients K^{I}_{zz}, $K^{I}_{\varphi\varphi}$, $K^{I}_{z\varphi}$ and K^{II}_{zz}, $K^{II}_{\varphi\varphi}$, $K^{II}_{z\varphi}$ for open and sliding modes respectively are given in Fig.1. In the case of open mode rupture for $v>v_*$ the stress $\sigma_{z\varphi}$ appears to grow considerably and the stress $\sigma_{\varphi\varphi}$ to have two symmetrical maxima. This should result in the appearance of two symmetrical sliding mode branches. In the case of sliding mode rupture there exists extension in the half-plane on one side of the rupture.

This should lead to the formation of open mode branch in this range. Hence, the propagation of the main rupture is accompanied with intermitting formation of open and sliding mode branches.

The theoretical seismograms (time history of displacements) calculated by the help of superposition of fundamental solutions for two cases of typical kinking ruptures are given in Figs 2 and 3.

It is seen from Figs 2 and 3 that the peaks on theoretical seismograms correspond to the moments of formation or stop of the branches. In some cases the sign-alternating seismograms may be also obtained (Bykovtsev and Cherepanov, 1980 a,b).

The signal duration in the direction of motion is less than in the opposite direction, i.e. frequences in the direction of motion are higher than in the opposite direction. This is a visual demonstration of Doppler effect. This peculiarity of wave field can be used to find the plane and direction of the main rupture propagation.

It is of interest that in the case of dislocation description the displacement field far from the original rupture practically coincides with that for the crack with force conditions on its borders (Bykovtsev, 1978, 1979; Bykovtsev and Cherepanov, 1980 b).

CONCLUSION

The dislocation description of ruptures used above is of practical importance for the study of the dynamic propagation of curvilinear ruptures at variable speed.

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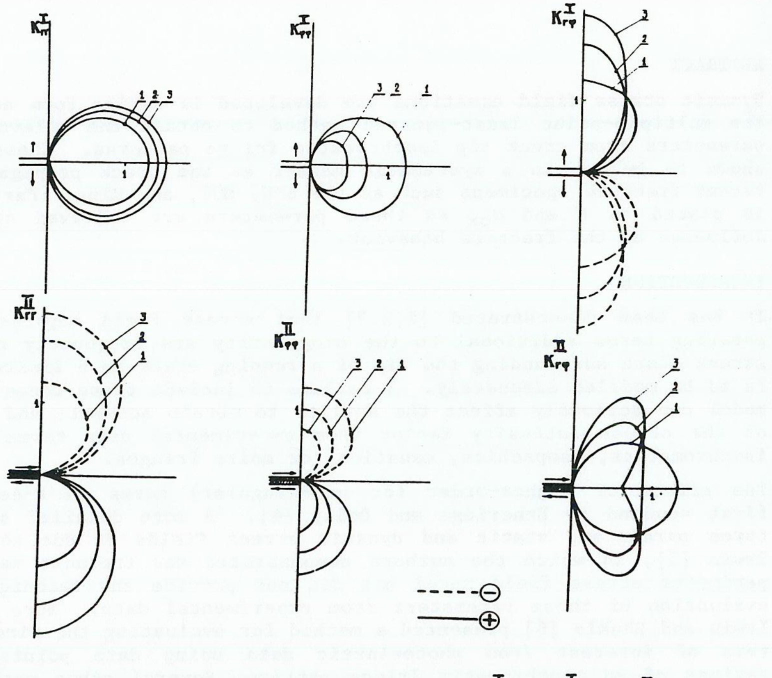

Fig. 1. A plot of stress coefficients K^{I}_{rr}, $K^{I}_{\varphi\varphi}$, $K^{I}_{r\varphi}$ (open mode) and K^{II}_{rr}, $K^{II}_{\varphi\varphi}$, $K^{II}_{r\varphi}$ (sliding mode) for the following rupture velocities:
1 - v = 0.1 c_s, 2 - v = 0.7c_s, 3 - v = 0.8 c_s

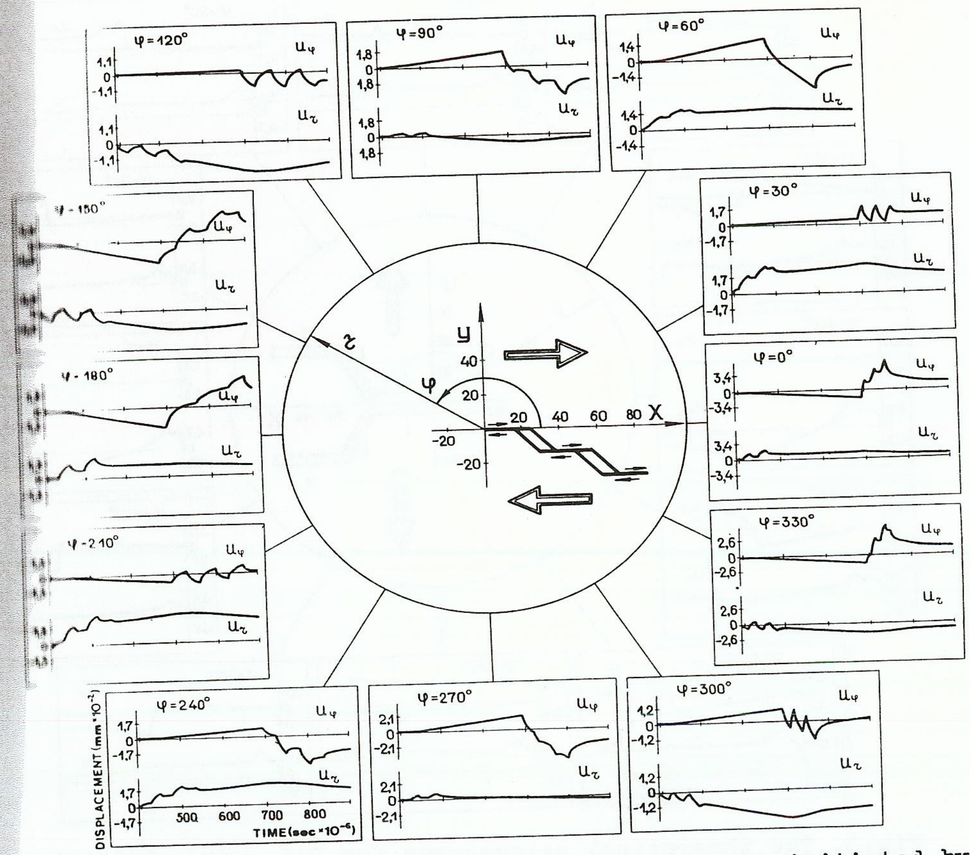

Fig.2. The theoretical seismograms for P+S - wave initiated by sliding mode dislocation rupture.

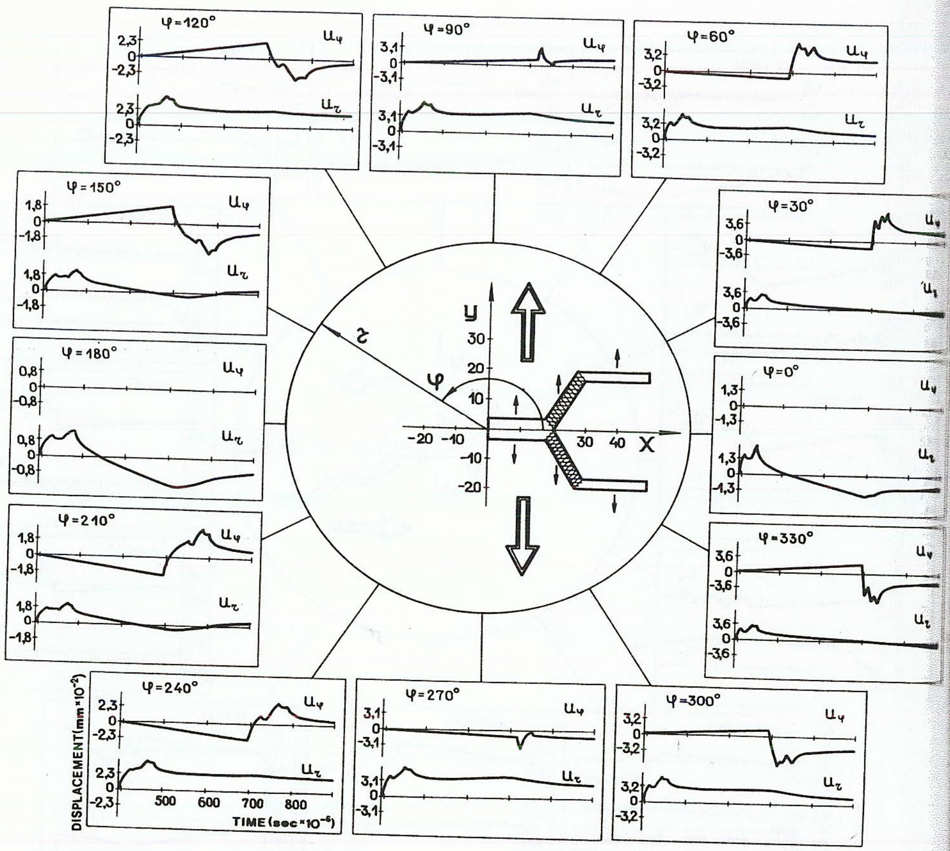

Fig.3. The theoretical seismograms for P+S - wave initiated by open mode dislocation rupture.