SUBCRITICAL GROWTH OF CRACKS WITH NON-SMALL PLASTIC ZONES IN VISCOELASTIC BODIES

A. A. Kaminsky

Institute of Mechanics of the Academy of Sciences of the Ukrainian SSR, Kiev, USSR

ABSTRACT

In the article, within the framework of the Leonov-Panasyuk-Dugdale model and of the γ_*-conception of G.P.Cherepanov the growth of microcracks in viscoelastic bodies is studied (when plastic zones at the edges are commensurable with their length) under cyclic loading, when fatigue and creep processes contribute commensurably to crack development.

With the assumption of the independence of action of fatigue and creep processes, the integro-differential equation is analyzed representing the kinetics of the microcrack growth in the viscoelastic medium. Numerical solution of this equation for a Maxwell body is presented.

KEYWORDS

Subcritical growth; crack; non-small plastic zone; viscoelastic body; fatique crack propagation; model Leonov-Panasyuk-Dugdale; equation crack propagation; the life a cracked plate.

The kinetics of the growth of fatigue macrocracks has been studied (Kaminsky, 1980; McCartney, 1978; Wnuk, 1971) in viscoelastic bodies in cases in which the conception of fine structure at crack tip is valid. However for microcracks with plastic zones at their edges, commensurable with their length, that approach can be considered as not founded and the introduction of coefficients of stress intensity is inappropriate.[1]

---

[1] Analogous situation could occur in the case of low-cycle fatigue.

In this article within the framework of the δ_K-model (Panasyuk, 1968), γ_*-theoretical conception of G.P.Cherepanov(1974) and of the kinetic theory of crack growth in viscoelastic bodies (Kaminsky, 1980), the process of microcrack growth has been studied in a viscoelastic body under cyclic load, when fatigue and creep processes are contributing commensurably to the crack growth.

It should be noted that in many studies of the fatigue crack growth the effect of material creep upon the kinetics of crack growth is usually neglected. However, in viscoelastic materials (in polymers, concretes, in metals at high temperatures) under the action of cyclic loading the defect growth occurs not only as a consequence of material fatigue but also as a result of the creep of material. The creep of material is very marked when the static component of loading greatly exceeds the cyclic one. In the following, mutual independence of the effects of the fatigue and creep processes is assumed. That assumption is valid for many viscoelastic materials (Rabotnov, 1966).

We shall analyze the case of slow crack growth, within the framework of the Leonov-Panasyuk-Dugdale model in the viscoelastic body under uniform load normal to crack lips (a case analogous to the Griffith problem). The intensity of external load changes in time according to the law

$$p = p_0 + \Delta p \sin wt , \qquad (1)$$

where p_0 , Δp , w are independent of time, and $\Delta p \ll p_0$. Since we have assumed that the fatigue and creep processes occur independently the crack growth rate can be represented in the form

$$\dot{l} = \dot{l}_{fat} + \dot{l}_{cr} , \qquad (2)$$

where $\dot{l}_{fat}$ -crack growth rate as a result of material fatigue, $\dot{l}_{cr}$ -crack growth rate consequent to material creep. We calculate the values contained in the righthand side of equation(2).

1. For determination of the term $\dot{l}_{fat}$, occurring as a result of fatigue phenomena in crack terminal zone (as a result of accumulation of fatigue damage) the solution of G.P.Cherepanov (1974) is uzed, based on γ_* -conception of fracture of viscoplastic bodies. According to that solution the relation between the stress p and the crack length l can be written in the form:

$$\frac{d\beta}{d\lambda} = \frac{1-2\lambda(\ln\cos\beta + \beta\,\mathrm{tg}\,\beta)}{\lambda^2(\beta\sec^2\beta - \mathrm{tg}\,\beta)} = f(\beta,\alpha), \qquad (3)$$

$$\lambda = \frac{2\sigma_s^2 l}{\pi E \gamma_*}; \qquad \beta = \frac{\pi p}{2\sigma_s} .$$

σ_s -yield limit under tension; E -modulus of elastisity; γ_* -specific fracture energy.

The equation (3) describes the crack growth in the period of one cycle. Integrating (3) from β_{min} to β_{max} and assuming λ constant during one cycle, the increment of crack length Δl during one cycle is obtained in the form

$$\Delta l = \int_{\beta_{min}}^{\beta_{max}} \frac{d\beta}{f(\beta,\lambda)} .$$

Proceeding now to continuons variables, we write the expression for crack growth rate in the form

$$\frac{dl}{dN} = \int_{\beta_{min}}^{\beta_{max}} \frac{d\beta}{f(\beta,\lambda)} = F(\beta_{max}, \beta_{min}, \lambda), \qquad (4)$$

N -number of loading cycles.

In the case considered

$$p = p_0(1+\gamma \sin wt), \qquad (5)$$

where $\gamma = \dfrac{\Delta p}{p_0}$.

Consequently

$$\beta = \frac{\pi p_0}{2\sigma_s}(1+\gamma \sin wt), \qquad (6)$$

$$\beta_{max} = p_0(1+\gamma),$$

$$\beta_{min} = p_0(1-\gamma).$$

Relation (3) is reduced to variables $x = l/l_*$, $\lambda = x/\beta_0^2$, where $\beta_0 = \pi p_0/2\sigma_s$, l_*-crack length in the precritical period. In that case we have

$$\frac{d\beta}{dx} = \frac{2\ln\sec\beta_0 - 2x(\ln\cos\beta + \beta\,\mathrm{tg}\,\beta)}{x^2(\beta\sec^2\beta - \mathrm{tg}\,\beta)} = f(\beta,x). \qquad (7)$$

Since $N = wt/2\pi$, taking account of (7) we express (5) in the form

$$\frac{dx}{dt} = \frac{w}{2\pi}\int_{\beta_0(1-\gamma)}^{\beta_0(1+\gamma)} \frac{d\beta}{f(\beta,x)} = \frac{w}{2\pi} F(\beta_0, \gamma, x) . \qquad (8)$$

2. Now the development of microcrack as a result of the creep of viscoelastic material is analyzed. The equation of the main period of the microcrack has the form (Kaminsky, 1980)

$$\delta_K = T_0\left\{\delta_0[\ell(t)] + \int_0^{q} R(q-\theta)\delta_0[\ell(\theta),\ell(t)]\,d\theta\right\}, \tag{9}$$

where δ_K -critical crack opening; $T_0\delta_0[\ell(t)]$-elastic crack opening at $x=\ell(t)$; $R(q-\theta)$-kernel of the integral operator of viscoelasticity; q -time period in which the crack tip moves at distance d.

We consider the case of low frequencies of cyclic loading W. Since

$$p(\tau) = p(t)(1+\gamma w(\tau-t)\cos wt), \tag{10}$$

in that case may be assumed $\gamma W(\tau-t) \ll 1$ and the following relation holds $p(\tau)\approx p(t)$.

Under the restrictions noted the equation of crack growth (9) for quasisteady crack growth rate will take the form (Kaminsky, 1980).

$$\frac{\ell_*}{\ell} = 1 + \Psi(\beta)\frac{\ell(t)}{\dot\ell(t)}\int_0^1 R\left(\frac{ms\ell}{\dot\ell}\right)\varphi(s)\,ds, \tag{11}$$

where

$$\varphi(s) = \frac{1}{4m}\left\{(2-ms)\ln\frac{2-ms-\rho^2+\rho\sqrt{m^2s^2-2ms+\rho^2}}{2-ms-\rho^2-\rho\sqrt{m^2s^2-2ms+\rho^2}} - ms\ln\frac{\rho^2-ms+\rho\sqrt{m^2s^2-2ms+\rho^2}}{\rho^2-ms-\rho\sqrt{m^2s^2-2ms+\rho^2}}\right\},$$

$$\Psi(\beta) = \frac{(1-\cos\beta)^2}{\ln\sec\beta}, \quad \ell_* = -\frac{\delta_K}{8T_0\sigma_s\ln\sec\beta},$$

$$\rho = \sin\beta, \qquad \beta = \frac{\pi p}{2\sigma_s}.$$

We shall analyze the Maxwell's material, in that case $R(t-\tau)=\lambda$ and equation (11) is presented in the form

$$\frac{\ell_*}{\ell} = 1 + \lambda\Psi(\beta)\xi(\beta)\frac{\ell}{\dot\ell}, \tag{12}$$

where

$$\xi(\beta) = \int_0^1 \varphi(s)\,ds. \tag{13}$$

The table 1 contains the values of $\xi(\beta)$ for different values of β.

TABLE 1 the Values of $\xi(\beta)$

β	$\xi(\beta)$
0.01	0.3333
0.1	0.3343
0.3	0.3425
0.5	0.3601

The equation (12) can be reduced to the form

$$\frac{d\ell}{dt} = g(\beta)\frac{\ell^2}{\ell_*-\ell}, \tag{14}$$

where

$$g(\beta) = \lambda\Psi(\beta)\xi(\beta).$$

Introducing dimensionless length $x=\ell/\ell_*$, we write the equation (14) in the form

$$\frac{dx}{dt} = g(\beta)\frac{x^2}{1-x}. \tag{15}$$

As is seen from the table 1, for $\beta < 0.5$ the value of the integral $\xi(\beta)$ changes very little and for simplicity may be assumed $\xi(\beta)\approx 1/3$. Taking account of that fact as well as of the relation (6), the equation (15) may be written in the form

$$\frac{dx}{dt} = \frac{\lambda}{3} \frac{[1-\cos\beta_0(1+\gamma\sin wt)]^2}{\ln\sec\beta_0(1+\gamma\sin wt)} \frac{x^2}{1-x} \cdot \qquad (I6)$$

3. Dividing two parts of the equation (2) by l_* and substituting the relation (8) for $\dot{x}_{fat}$ and the relation (I6) for $\dot{x}_{cr}$, we obtain the equation, describing the crack growth in viscoelastic material under cyclic loading

$$\frac{dx}{dt} = \frac{\lambda}{3} \frac{[1-\cos\beta_0(1+\gamma\sin wt)]}{\ln\sec[\beta_0(1+\gamma\sin wt)]} \frac{x^2}{1-x} + \frac{w}{2\pi} F(\beta_0,\gamma,x),$$

where

$$F(\beta_0,\gamma,x) = \int_{\beta_0(1-\gamma)}^{\beta_0(1+\gamma)} \frac{d\beta}{f(\beta,x)} \cdot \qquad (I7)$$

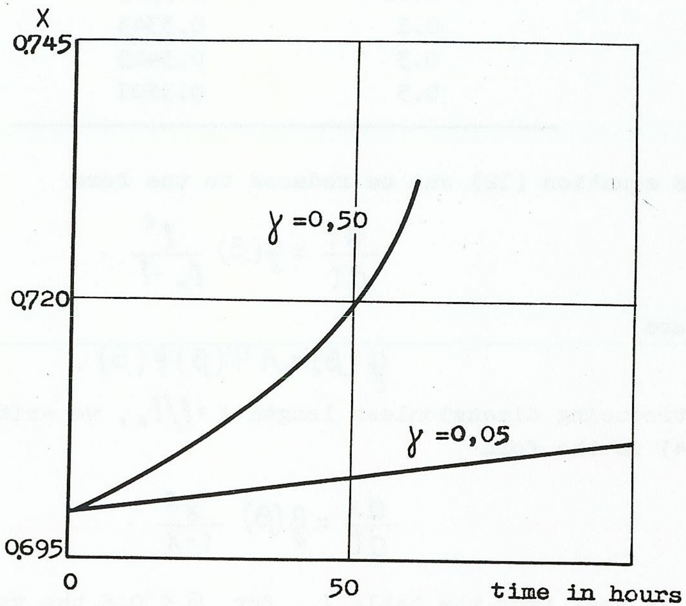

Fig. I. The relations between the dimensionless length and time.

In the figure the relations between the dimensionless length X and time are shown, calculated by numerical integration of the equation (I7) using the Runge-Kutta procedure. As follows from the character of the relation $X-t$, at $\beta_0 = 0.I$, $W = IH_z$ for different values of the loading parameter γ, the increase of cyclic load leads to an increase of the crack growth rate.

REFERENCES

Cherepanov, G.P. (I974). Mechanics of Brittle Fracture, Nauka, Moscow. pp. 640.
Kaminsky, A.A. (I980). Fracture Mechanics of Viscoelastic Bodies. Naukova Dumka, Kiev. pp. I60.
McCartney, L.N. (I978). Crack propagation in linear viscoelastic solids: some new results. Int. J. Fract., 6, 547-554.
Panasyuk, V. V. (I968). Limit Equilibrium State of Brittle Bodies with Cracks. Naukova Dumka, Kiev. pp. 246.
Rabotnov, Yu. N. (I966). Creep of Structures Elements. Nauka, Moscow. pp. 752.
Wnuk, M. P. (I97I). Subcritical growth of fracture (inelastic fatigue). Int. J. Fract. Mech., 7, 383-407.