SOME GEOPHYSICAL PROBLEMS OF FRACTURE MECHANICS

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ABSTRACT

Basing on fracture mechanics concepts the theory of a number of well-known geophysical phenomena is considered. These are quasi--regular sets of polygonal breaks of "takyr" or "polygonal tundra" type in deserts or permafrost regions and closed landslips in clay sedimentary rocks. Some special geographical examples of these phenomena are indicated.

KEYWORDS

Fracture, crack, Earth, rock, theory, takyr, polygonal tundra, landslip.

INTRODUCTION

The development of cracks in rocks has formed the modern face of the Earth. There exist cracks of different kind in rocks sized from 10^{-8} m to 10^{7} m. The subjects of this paper are:

(i) quasiregular sets of ruptures on the Earth surface which give the landscape its specific appearance (takyrs in deserts, polygonal tundra in permafrost regions, regular sets of global faults discovered by space photography);

(ii) closed giant landslips.

Good understanding and theoretical description of this phenomena can be achieved only on the basis of fracture mechanics. The most detailed review of rock fracture from the fracture mechanics view-point is given by Rice, 1980. What follows is a continuation of recent studies by the authors (Cherepanov et al., 1973; Cherepanov, 1976, 1979, 1983; Bykovtsev and Cherepanov, 1980 a,b; Bykovtsev, 1983; Cherepanov and Bykovtsev, 1982,1984; Bykovtsev, Cherepanov and Ulomov, 1984).

The theory for polygonal ruptures and closed landslips treated below is of interest also for cracking of surface layers and films in structural metals due to thermal stresses, corrosion, shrinkage etc.

POLYGONAL RUPTURES ON THE EARTH SURFACE

In some régions of the Earth one can often meet with quasiregular rupture sets which yield the landscape its specific appearance. These are: takyrs in deserts, shrink cracks, frost shake cracks, polygonal tundra (Fig.1). The common cause of these phenomena is tensile stress in upper layers of soil. This stress appears due to non-uniform cooling or drying of rock.

From the viewpoint of stresses and strains the decrease in rock volume, ΔV, due to temperature drop or drying may be described by a relationship of the type $\Delta V = 3\beta\, TV$ where V is volume, T is temperature (or moisture), β is coefficient of temperature expansion (or shrinkage ratio). This analogy allows thermal modelling of shrinkage phenomenon. The volume decrease is a linear function of tensile stress on the surface which leads to surface cracking and formation of a complex geometrical structure of ruptures studied below in model approximation of periodic structure (Cherepanov and Bykovtsev, 1984 b).

We consider a regular periodic crack array whose front moves perpendicularly to the half-space boundary coinciding with the Earth surface (Fig.2). The crack front is in a plane located above the plane of zero stress (in which the change of temperature or moisture is equal to zero). Let this front coincide with the plane $X_1 = 0$ (in a coordinate system $Ox_1x_2x_3$ moving together with the crack front). We direct X-axis along the axis of one of the regular prisms P formed in the half-space by polygonal regular array of cracks. We introduce the following notation: L is the intersection of the rupture front with the prism P, S^- and S^+ are lateral surfaces of the prism P for $x_1>0$, $x_1<0$ respectively. The surface S^- coincides with a part of the crack boundaries, the surface S^+ being in the unfractured space.

The boundary conditions are of the following form:

$$x_1 \to \infty \quad \sigma_{11}=0,\ \sigma_{22}=p,\ \sigma_{33}=q\ (p>0,\ q>0),\ \sigma_{ij}=0\ (i\neq j) \qquad (1)$$

$$x_1 \to -\infty \quad \sigma_{ij} \to 0 \qquad (2)$$

$$(x_1 x_2 x_3) \in P \quad \sigma_{ij} n_j = 0 \quad (i,j=1,2,3) \qquad (3)$$

where n_j are components of the unit vector normal to the crack boundary, σ_{ij} are stress components. The quantities p, q in Eqn(1) are equal to corresponding tensile stresses in the upper layer of the earth appearing due to thermal decrease of volume (or shrinkage) before rupture, Eqn(3) means that the crack boundary is free of external loads. This problem is very complicated for a thorough solution. However, the technique of invariant Γ-Integrals enables us to extract useful relationships (Cherepanov, 1979).

We consider the closed surface Σ in the space $x_1x_2x_3$: $\Sigma = S_E^- + S_E^+ + S_E + B^- + B^+$. Here B^- and B^+ are cross-sections of the infinite prism P for $x_1 \to -\infty$ and $x_1 \to +\infty$ respectively, S_E is a section of the surface obtained in the space as a track left by the circumference of radius E whose center is moving along the contour L (the plane in which the circumfe-

a)

b)

c)

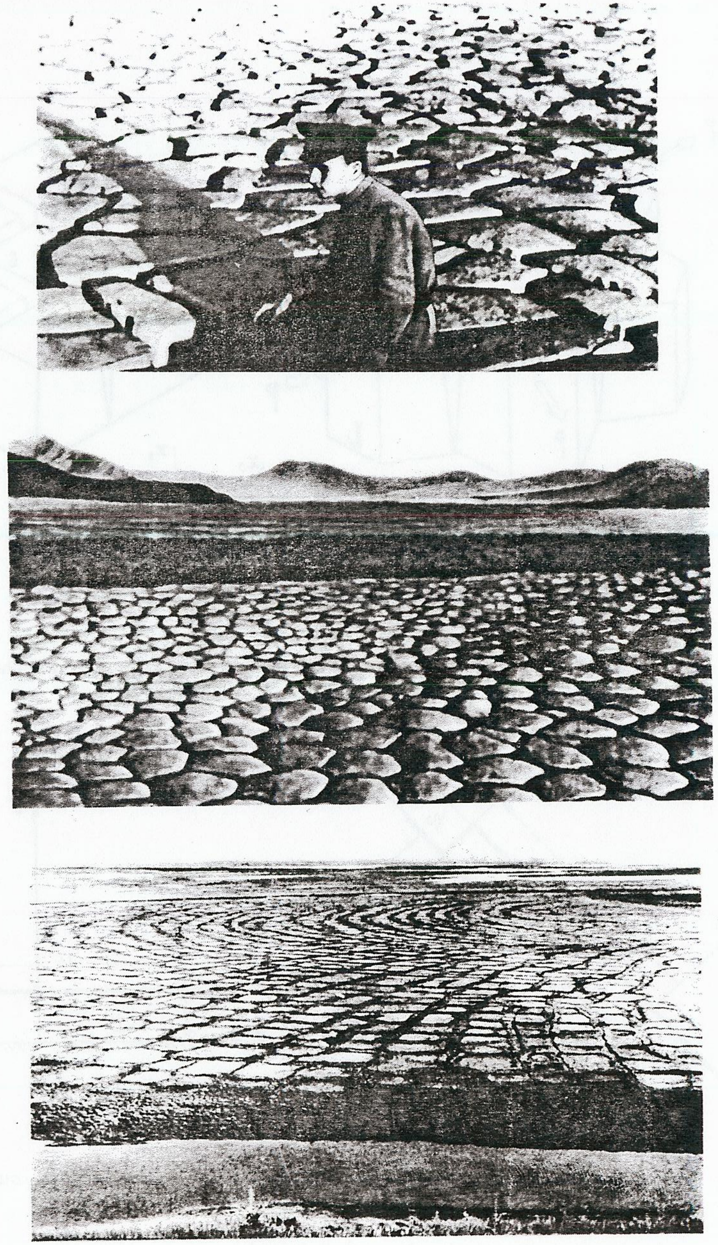

Fig. 1. Polygonal crack arrays on the Earth surface: a) takyr in Kizil-Kum desert, b) shrink cracks on Spitsbergen, c) polygonal tundra in Siberia.

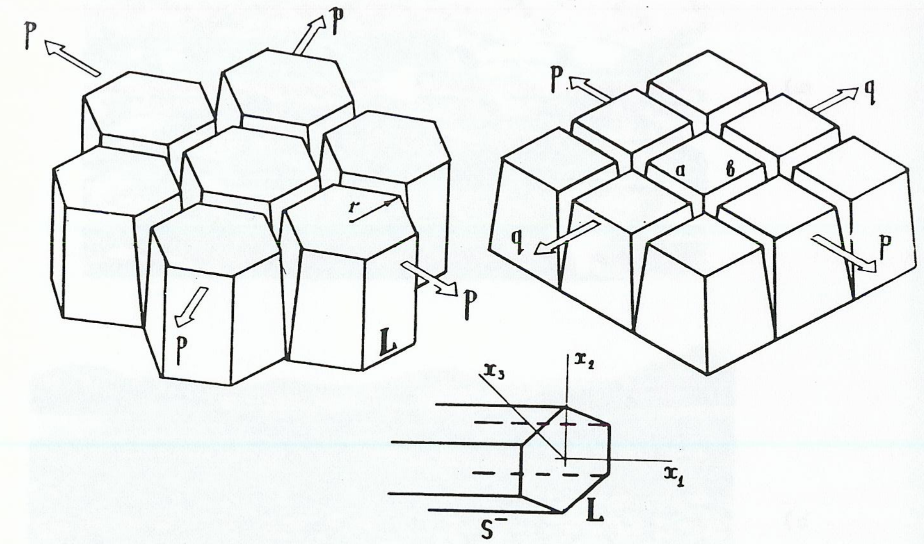

Fig. 2. A scheme of regular periodic crack array on the surface of a half-space.

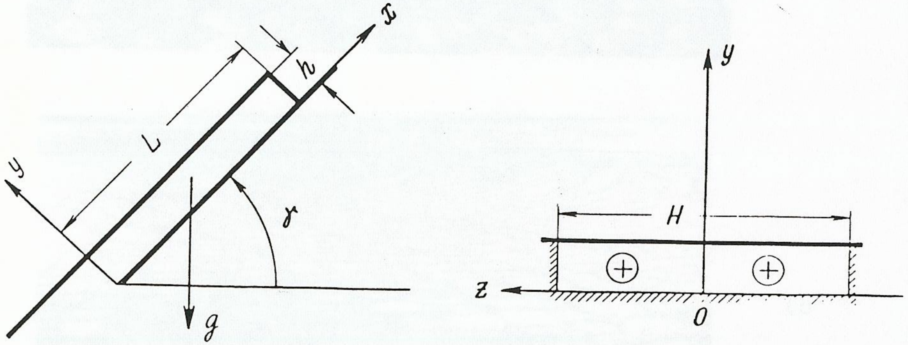

Fig. 3. A scheme of a closed landslip on the Earth surface.

rence is located is perpendicular to the contour L at any moment of motion), S_E^- and S_E^+ are respective surfaces S^- and S^+, without the part cut off by the surface S_E. The value of E is assumed small as compared to the linear dimension of the cell. The following equation holds (Cherepanov, 1979)

$$\int_{\Sigma}(\rho U n_1 - \sigma_{ij} n_j u_{i,1})\, d\Sigma = 0 \qquad (i,j=1,2,3) \tag{4}$$

Here ρ is density, U is specific elastic potential, u_i are displacement components, n_i are components of a unit vector normal to surface Σ.

We calculate successively integrals in Eqn(4). Since $n_1=0$ on S_E^- and S_E^+, $\sigma_{i1}^-=0$ on S_E^- and moreover, $\sigma_{12}n_2+\sigma_{13}n_3=0$ on S_E^+ and $u_{2,1}=u_{3,1}=0$ on S_E^+ due to symmetry conditions the following equations are valid

$$\int_{S_E^-}(\ldots)\, d\Sigma = \int_{S_E^+}(\ldots)\, d\Sigma = 0 \tag{5}$$

The following equation also holds (Cherepanov, 1979)

$$-\oint_L \Gamma\, dl = 2\int_{S_E}(\rho U n_1 - \sigma_{ij} n_j u_{i,1})\, d\Sigma \tag{6}$$

Here Γ is x_1-component of the vector of the energy flux density on the contour L.

According to Eqn (3), we have

$$\int_{B^-} U n_1\, d\Sigma = 0, \quad \int_{B^+} U n_1\, d\Sigma = S U_0 \tag{7}$$

Here S is the area of an array cell inside the contour L, U_0 is the value of U for $x_1 \longrightarrow +\infty$ which is equal to

$$U_0 = (p^2+q^2-2\nu pq)/2\rho E \tag{8}$$

In accordance with Eqns (5)-(7), Eqn (4) takes the following shape:

$$2\rho S U_0 = \oint_L \Gamma\, dl \tag{9}$$

$$2\rho S U_0 = \Gamma_c P \tag{10}$$

where P is perimeter of the contour L. Let us consider some special types of arrays.

Hexagonal array. It is natural to suppose that the crack array should be regular for uniform tensile stress field (when $p=q$). From the principle of minimum of energy dissipation (Cherepanov, 1979) it follows that the crack array should be hexagonal for the case of uniform extension.

In this case $S=\frac{3}{2}\sqrt{3}r^2$, $P=6r$, where r is the radius of the circle circumscribed around the regular hexagon.

Hence, using (8) and (10), we have dependence of array parameter on operating stress

$$z = 2E\Gamma_c / p^2(1-\nu)\sqrt{3} \qquad (p=q) \tag{11}$$

Rectangular array (quadrangular net). For non-uniform extension of rock when $p \neq q$, at first, parallel cracks develop along the maximal stress, then (perhaps in another season) the other system of parallel cracks perpendicular to the old one appears. This is typical, e.g., in the vicinity of a river bend because of different conditions of deforming the river bank and oxbow. Frost shake crack arrays appear usually in the same manner. Let $p > q$. Then, at first, parallel cracks form at some distance between them in plane strain conditions with respect to x_3 under one stress p . In this case we have (Cherepanov,1979)

$$a p^2 = E\Gamma_c / (1+\nu^2) \tag{12}$$

Thus, array parameter a depends very simply on stress p .

Similarly, we determine the other parameter of rectangular array ignoring the interaction of different crack systems

$$b q^2 = E\Gamma_c / (1+\nu^2) \tag{13}$$

Eqns (12) and (13) lead to a simple rule: the ratio of different sides of a rectangle in a quadrangular net equals the reciprocal ratio of corresponding squared stresses

$$a/b = q^2/p^2 \tag{14}$$

For permafrost rocks and ice we take the following values of constants: $E = 10^3$ kg/cm^2 (Romanovsky, 1977), $\Gamma_c = 1.5 \times 10^{-4}$ kg/cm (Cherepanov, 1979). In the case of hegagonal array we have, according to (11): p = 0.01 kg/cm^2 for r = 25 M, p = 0.05 kg/cm^2 for r = 1M.

CLOSED LANDSLIPS

As closed landslips we shall refer those whose front has not yet appeared on the earth surface. These landslips are most dangerous and unpredictable. The well-known giant Atchinsky landslip near the town of Angren in the Uzbek SSR seems to belong to this type. The plane of the landslip is at the depth of 40-120 M in the field of clay rocks, the area of the landslip is 4.1 x 2.7 km^2. The volume and mass of the landslip are 5 x 10^8 and 10^{12} kg respectively. The landslip plane inclination to the horizontal plane is 6°. The displacement of the landslip is approximately constant along the slope and into the depth, it amounted 12 ± 2 M during the period from 1974 to 1981.

The landslip body may be represented as a thin rectangular parallelepiped $0 < x < L$, $0 < y < h$, $|z| < \frac{1}{2}H$, while $h \ll L \sim H$ (Fig.3). The slip surface consists of three planes: $y = 0$, $z = \pm\frac{1}{2}H$. The landslip front along $x = y = 0$ and $z = \pm\frac{1}{2}H$ is imbedded in rock. The back face of the landslip for $x = L$

may be considered free because of low resistance of rock to extension. Deformation of the landslip may be assumed as plane strain with respect to z except for the vicinity of the faces $z = \pm\frac{1}{2}H$. Since $L \sim H \gg h$, we have $\partial\sigma_x/\partial x \ll \partial\tau_{xy}/\partial y$ and $\partial\tau_{xy}/\partial x \ll \partial\sigma_y/\partial y$. Hence, the equilibrium equations are as follows

$$\partial\tau_{xy}/\partial y = -\rho g \sin\gamma, \quad \partial\sigma_y/\partial y = -\rho g \cos\gamma \tag{15}$$

Here σ_x , σ_y , τ_{xy} are stresses, ρ is rock density, γ is inclination angle to the horizontal plane.

We consider rock as a laminated body, the boundaries of the layers being perpendicular to y-axis (Fig.3). Eqns (15) show the stresses σ_y and τ_{xy} to be piecewise linear functions of y. For equilibrium limiting state we have the following equation:

$$\tau_{xy} = k + \mu\sigma_y \qquad y = 0 \tag{16}$$

This equation enables us to obtain the dependence of critical value of h on ρ , γ , μ , k , and, hence, on rock moisture content because of linear relationship between k , μ and moisture content. E.g., for homogeneous landslip we have

$$\rho g h(\sin\gamma - \mu\cos\gamma) = k \tag{17}$$

The landslip velocity v is governed by rock creep in a thin layer between opposite banks of the slip surface $v = \Delta\tau_{xy}/\eta = \Delta\rho g h \sin\gamma/\eta$. Here η is viscosity coefficient, Δ is the thickness of viscous layer of the order of 1 M.

Besides, for closed landslips the following equation holds (Cherepanov, 1979):

$$\Gamma = 2\rho g h u \sin\gamma \qquad \left(u = \int_0^t v(t)\,dt\right) \tag{18}$$

Here Γ is the density of energy flux on the landslip front, u is the displacement of the landslip.

According to the general theory of motion of the displacement discontinuity surfaces in solids, there exists a certain critical value of Γ characterizing the beginning of motion of the surface which is equal to the energy dissipation Γ_c per unit of the new-formed, slip surface. If $\Gamma < \Gamma_c$, then the landslip front is fixed and the rock near the frontal face is plastically flowing. This is the case realized presently in the Atchinsky landslip. After Γ becomes equal to Γ_c the slip front will begin to propagate and involve new mass of rock to the landslip.

The additional load on the surface in the region of plastic flow in the vicinity of the landslip front has double effect. On one hand, the resistance to plastic flow of rock on this region increases; on the other hand, the value of Γ and, hence, the danger of landslip motion grows.

CONCLUSION

The fracture mechanics approach is shown to be of practical importance in the analysis of many interesting geophysical phenomena.

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