RESIDUAL STIFFNESS PROPERTIES OF CRACKED COMPOSITE LAMINATES

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ABSTRACT

Cracks in composite laminates are characterized by a set of vectors, each representing an individual cracking mode. The vector components are taken as internal variables in the elastic strain energy function and the elastic constitutive equations are derived for the inplane loading condition. For low concentration of cracks in laminates, the residual stiffness properties are related to the initial elastic constants and the magnitude of the damage vectors. These equations are then used to predict stiffness reductions of composite laminates from the observed crack densities. Very good agreement with the experimental values is found.

KEYWORDS

Composite materials, stiffness changes, matrix cracking, crack density, damage mechanics.

INTRODUCTION

Damage in composite materials is generally of highly complex nature. However, extensive work done in the last decade on selected configurations of composite laminates using various non-destructive techniques has helped advance the understanding of damage (Hahn, 1979; Reifsnider, 1979; Stinchcomb and Reifsnider, 1979). Perhaps the best understood damage mode is the transverse cracking of off-axis plies in a laminate. Various investigations, e.g. Masters and Reifsnider (1982), have shown that, both under static loads and during initial stages of cyclic loading, multiple cracking of off-axis plies occurs in the form of matrix cracks growing parallel to fibres. The cracks increase in number with increasing load, or with increasing number of cycles of a cyclic load, and a state of saturation eventually ensues. The saturation state has some characteristic features, and has thus been called a characteristic damage state, CDS (Reifsnider and Talug, 1978). For instance, the saturation spacing of cracks depends on the laminate configuration (i.e. fiber orientation and lamina thickness and stacking sequence) but is independent of the loading history. The saturation crack spacing has been predicted for various laminates using a shear-lag model (Reifsnider, Henneke and Stinchcomb, 1979).

The reduction in stiffness of laminates undergoing transverse cracking in off-axis plies has been studied both experimentally and analytically (Highsmith and Reifsnider, 1982). The prediction of stiffness has been based on the one-dimensional shear-lag model (Reifsnider, Henneke and Stinchcomb, 1979), which determines the normal stress in the given direction in the cracked plies approximately. For a given crack spacing in a ply, then, the elastic modulus of the ply normal to the cracks is reduced by an amount corresponding to the reduction in the contribution of the cracked ply to the overall load carried by the laminate. The laminate analysis is then used to calculate the overall stiffness constants of the laminate using the degraded modulus of the cracked ply. The predictions agree generally with the experimental values, although occasional discrepancies are found. However, it is not clear to what extent the simplifying assumptions in the analysis affect predictions.

We shall here present a procedure for predicting stiffness reduction due to cracking using an entirely different approach. Rather than considering cracks explicitly in terms of their effects on stress redistributions, we shall look at cracks as microstructural entities such as dislocations and voids in metals. The presence of cracks will then be accounted for by introducing a set of internal state variables in the mechanical response functions. The approach along these lines has been developed for a general case of damage in composites (Talreja, 1983); we shall here specialize it for transverse cracking of off-axis plies in laminates. Specific equations relating the intensity of cracking and stiffness reductions will then be given for selected laminate configurations and predictions using these equations will be compared with experimentally observed values.

ELASTIC RESPONSE OF CRACKED LAMINATES

Consider a solid containing m sets of parallel planar cracks. Let us assign a vector $V^{(\alpha)}$ to each of the set of cracks, $\alpha = 1,2,\ldots m$. Let the orientation of a vector $V^{(\alpha)}$ be normal to the planes of the αth set of cracks and let its magnitude be given by

$$|V^{(\alpha)}|^2 = \eta_c \cdot \bar{\ell}_c \cdot \bar{w}_c \cdot f_c \ , \qquad (1)$$

where η_c is the crack number density, i.e. the number of cracks per unit surface area, $\bar{\ell}_c$ and $\bar{w}_c$ are the average length and the average width of the cracks, respectively, and f_c is a factor that depends on the crack shape and on the constraint to crack opening provided by the surrounding material.

Fig. 1. illustrates the case of two sets of parallel cracks in a solid and shows the two vectors $V^{(1)}$ and $V^{(2)}$ that represent the two cracking modes. The magnitudes of vectors can vary over the solid to account for inhomogeneous cracking modes.

Consider now cracked composite laminates with following restrictions.

1. Thickness is small,
2. loading is in the plane of laminates, and
3. each cracking mode consists of transverse cracking in the plies only.

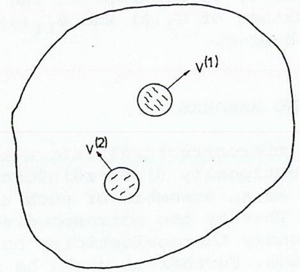

Fig. 1. A solid containing two sets of parallel cracks.

We thus do not consider other cracking modes such as delamination, fiber breakage and interfacial debonding.

The transverse cracking will now be represented by vectors $V^{(\alpha)}$, each vector oriented normal to fibres in a cracked ply. Let $V_p^{(\alpha)}$ be the vector components in a Cartesian coordinate system placed in the midplane of the laminate. Considering the vector components as internal variables representing the micro(crack)structure, we write the elastic strain energy function for the cracked laminate as

$$W = W\left(e_p \, , V_q^{(\alpha)}\right) \qquad (2)$$

where e_p are the strain tensor components in the Voigt notation with $p = 1,2$ and 6.

For simplicity, let us consider one vector V representing a single cracking mode in a laminate. Expressing the strain energy function as a polynomial P, we have

$$W = P(e_1 \, , e_2 \, , e_6 \, , V_1 \, , V_2) \qquad (3)$$

The form of the polynomial P will be restricted by the symmetry properties of a laminate. The restrictions are accounted for by taking the polynomial P as a function of the integrity bases (invariants) associated with the given symmetry. For a polynomial, which is a function of a symmetric second-order tensor and a vector, the integrity bases associated with orthotropic symmetry have been given by Smith, Smith and Rivlin (1963). Using these, we have the following invariants for our case.

$$I_1 = e_1 \, , \; I_2 = e_2 \, , \; I_3 = e_6^2$$
$$I_4 = V_1^2 \, , \; I_5 = V_2^2 \, , \; I_6 = V_1 V_2 e_6 \qquad (4)$$

The most general polynomial P restricted to quadratic terms in strain components and quadratic terms in vector components is now given by

$$\begin{aligned} W = {} & k_1 I_1^2 + k_2 I_1 I_2 + k_3 I_1^2 I_4 + k_4 I_1^2 I_5 + k_5 I_1 I_6 + k_6 I_2^2 + k_7 I_2^2 I_4 \\ & + k_8 I_2^2 I_5 + k_9 I_2 I_6 + k_{10} I_3 + k_{11} I_3 I_4 + k_{12} I_3 I_5 + k_{13} I_1 I_2 I_4 \\ & + k_{14} I_1 I_2 I_5 \\ & + P_0 + P_1(e_p \, , V_q) + P_2(V_p) \ , \end{aligned} \qquad (5)$$

where P_0 is a constant and P_1 and P_2 are linear and quadratic functions of their variables, respectively.

Using (4) in (5) we get,

$$W = k_1e_1^2 + k_2e_1e_2 + k_3e_1^2V_1^2 + k_4e_1^2V_2^2 + k_5e_1e_6V_1V_2 + k_6e_2^2 + k_7e_2^2V_1^2 + k_8e_2^2V_2^2 + k_9e_2e_6V_1V_2 + k_{10}e_6^2 + k_{11}e_6^2V_1^2 + k_{12}e_6^2V_2^2 + k_{13}e_1e_2V_1^2 + k_{14}e_1e_2V_2^2 + P_0 + P_1 + P_2 \quad (6)$$

Imposing the condition,

$W = 0$ for $e_p = 0$ and $V_q = 0$, we get,

$$P_0 = 0 \quad (7)$$

and further,

$$\sigma_p = \frac{\partial W}{\partial e_p} = 0 \text{ for } e_p = 0 \text{ gives}$$

$$P_1 = 0 \quad (8)$$

Applying the stress formula we now obtain,

$$\sigma_p = C_{pq}e_q = \left(C^0_{pq} + C^1_{pq}\right)e_q \quad (9)$$

where

$$C^0_{pq} = \begin{bmatrix} 2k_1 & k_2 & 0 \\ k_2 & 2k_6 & 0 \\ 0 & 0 & 2k_{10} \end{bmatrix} \quad (10)$$

and

$$C^1_{pq} = \begin{bmatrix} (2k_3V_1^2 + 2k_4V_2^2) & (k_{13}V_1^2 + k_{14}V_2^2) & k_5V_1V_2 \\ (k_{13}V_1^2 + k_{14}V_2^2) & (2k_7V_1^2 + 2k_8V_2^2) & k_9V_1V_2 \\ k_5V_1V_2 & k_9V_1V_2 & (2k_{11}V_1^2 + 2k_{12}V_2^2) \end{bmatrix} \quad (11)$$

C^0_{pq} is the orthotropic stiffness matrix of the uncracked laminate and C^1_{pq} corresponds to the contribution due to cracks. Eq. (11) shows that cracking will, in general, remove the initial orthotropic symmetry in a laminate.

RESIDUAL STIFFNESS PROPERTIES

The stiffness components of a cracked laminate with transverse cracking in one ply are given by

$$C_{pq} = C^0_{pq} + C^1_{pq} \quad (12)$$

It can be shown that for cracking in m plies the stiffness components are given by (Talreja, 1983)

$$C_{pq} = C^0_{pq} + C^\alpha_{pq} \quad (13)$$

where $\alpha = 1,2,3,\ldots m$.

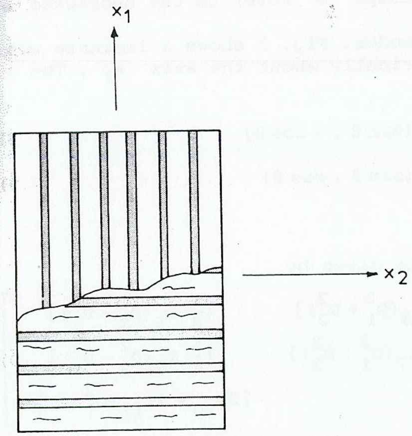

Fig. 2. A cross-ply laminate with cracks in the 90° ply.

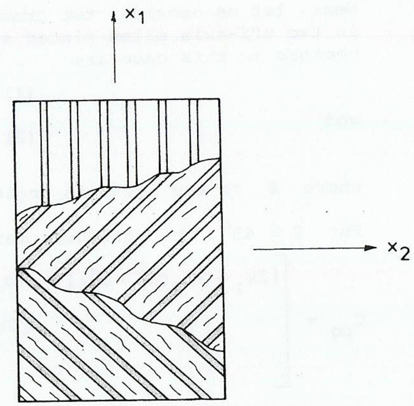

Fig. 3. Cracks in two plies symmetrically placed about the 0° ply in a laminate.

Let us first consider one cracking mode. Fig. 2 shows a cross-ply laminate which has cracks in the 90° plies. The damage vector for this case is given by

$$V = D(1\ ,\ 0) \quad (14)$$

Substituting this in Eq. (11) and using Eq. (12), we obtain

$$C_{pq} = \begin{bmatrix} (2k_1 + 2k_3D^2) & (k_2 + k_{13}D^2) & 0 \\ (k_2 + k_{13}D^2) & (2k_6 + 2k_7D^2) & 0 \\ 0 & 0 & (2k_{10} + 2k_{11}D^2) \end{bmatrix} \quad (15)$$

The orthotropic symmetry in the stiffness coefficients is thus retained for this cracking mode.

The residual elastic moduli can now be calculated by using the relations:

$$E_1 = \frac{C_{11}C_{22} - C_{12}^2}{C_{22}} \ , \quad E_2 = \frac{C_{11}C_{22} - C_{12}^2}{C_{11}}$$

$$\nu_{12} = \frac{C_{12}}{C_{22}} \quad \text{and} \quad C_{12} = C_{66} \quad (16)$$

From Eqs. (15) and (16) we obtain,

$$E_1 = E_1^0 + 2D^2[k_3 + k_7(\nu^0_{12})^2 - k_{13}\nu^0_{12}]$$

$$E_2 = E_2^0 + 2D^2[k_7 + k_3(\nu^0_{21})^2 - k_{13}\nu^0_{21}] \quad (17)$$

$$\nu_{12} = \nu^0_{12} + D^2\left[\frac{1 - \nu^0_{12}\nu^0_{21}}{E^0_2}\right](k_{13} - 2k_7\nu^0_{12}) \quad \text{and} \quad G_{12} = G^0_{12} + 2D^2k_{11}$$

In Eqs. (17) the moduli with superscript 0 refer to the uncracked laminate.

Next, let us consider two cracking modes. Fig. 3 shows a laminate with cracks in two off-axis plies placed symmetrically about the axis x_1. The damage vectors in this case are

$$v^{(1)} = D_1(\sin\theta \, , -\cos\theta) \tag{18}$$

and

$$v^{(2)} = D_2(\sin\theta \, , \cos\theta) \tag{19}$$

where θ is the off-axis angle.

For $\theta = 45^{\circ}$ the stiffness matrix is given by

$$C_{pq} = \begin{bmatrix} [2k_1 + 2k_3(D_1^2 + D_2^2)] & [k_2 + k_{13}(D_1^2 + D_2^2)] & [\frac{1}{4} k_5(D_2^2 - D_1^2)] \\ & [2k_6 + 2k_7(D_1^2 + D_2^2)] & [\frac{1}{4} k_9(D_2^2 - D_1^2)] \\ & & [2k_{10} + 2k_{11}(D_1^2 + D_2^2)] \end{bmatrix} \tag{20}$$

where only half of the symmetric matrix is shown.

It is seen that, for $D_1 = D_2$, the orthotropic symmetry is retained. However, we shall assume that, even for $D_1 \neq D_2$, the orthotropic symmetry will be retained approximately. Equivalently, we shall assume that the constants k_5 and k_9, which represent the interactions between the normal and the shear strains in a cracked laminate, are small.

The residual elastic moduli for this case are given by

$$E_1 = E_1^0 + 2(D_1^2 + D_2^2)[k_3 + k_7(\nu_{12}^0)^2 - k_{13}\nu_{12}^0]$$

$$E_2 = E_2^0 + 2(D_1^2 + D_2^2)[k_7 + k_3(\nu_{21}^0)^2 - k_{13}\nu_{21}^0]$$

$$\nu_{12} = \nu_{12}^0 + (D_1^2 + D_2^2)\left[\frac{1 - \nu_{12}^0\nu_{21}^0}{E_2^0}\right](k_{13} - 2k_7\nu_{12}^0) \tag{21}$$

and

$$G_{12} = G_{12}^0 + 2(D_1^2 + D_2^2)k_{11}$$

PREDICTION OF STIFFNESS REDUCTIONS

Highsmith and Reifsnider (1982) have reported stiffness reductions and crack densities for the following 4 laminates of glass/epoxy.

Laminate 1: $(0\,,90_3)_s$, Laminate 2: $(90_3\,,0)_s$

Laminate 3: $(0\,,90)_s$, Laminate 4: $(0\,,\pm 45)_s$

We shall use the data for laminate 1 to calculate the unknown constants in the stiffness reduction equations (17) and use these to predict the stiffness reductions for the remaining laminates. Since the measurement of the shear modulus G_{12} is suspect due to the method used (Highsmith and Reifsnider, 1982), we shall not concern ourselves with predicting this property. Also, since the change in the shear modulus is independent of the changes in the other moduli for the cracking modes in the laminates considered, it should suffice to treat the other moduli for the sake of demonstrating the prediction capability of the method.

The basic data for a single ply and the initial elastic moduli of the laminates, calculated by laminate analysis, are shown in Table 1.

The squares of the magnitudes of damage vectors, D^2, are calculated using the definition given by Eq. (1) for the cracking modes in all four laminates, and are given in Table 2. Also shown in this table are the observed values of the number of cracks per unit length of specimen. The following procedure is used in calculating D^2.

The average crack length, $\bar{\ell}_c$ is taken as the width of a specimen W for cracks in 90° plies, since the observation shows that the cracks span the entire distance from edge to edge of a specimen soon after initiation (Highsmith and Reifsnider, 1982). For the 45° plies the crack length is taken as $W/\cos 45^{\circ}$. The average width of cracks, $\bar{w}_c$ is equated to the thickness of the plies that contain the cracks, e.g. for laminate 1, $\bar{w}_c = 6t_0$, where t_0 is the single ply thickness. The crack number density, n_c is equated to the observed number of cracks per unit length of specimen divided by the width W.

The factor f_c is assumed to be a function of the crack width to the specimen thickness ratio and is taken as

$$f_c = \frac{nt_0}{t} \, , \tag{22}$$

where n is the number of plies containing a transverse crack and t is the thickness of the specimen (laminate). Due to the lack of information regarding the nature of the constraint on crack opening and sliding, any further refinement of this factor is not possible at present. We have further assumed that for cracks ending in the free surface, e.g. in $(90_3\,,0)_s$ laminate, the constraint is essentially absent, i.e. $f_c = 1$.

Table 3 shows the three constants found by using the data for laminate 1, and the predictions of stiffness reductions using these for the other laminates. The observed values, as reported in Highsmith and Reifsnider (1982), are also shown. The accuracy of the predictions may be characterized as very good.

Table 1. Elastic moduli for glass/epoxy lamina and laminates. Single ply thickness = 0.203 mm .

Moduli	E_1, GPa	E_2, GPa	ν_{12}	G_{12}, GPa
Lamina	41.7	13.0	0.300	3.4
Laminate 1	20.3	34.75	0.112	-
Laminate 2	20.3	34.75	0.112	-
Laminate 3	27.57	27.57	0.142	-
Laminate 4	21.79	13.93	0.561	-

Table 2. Magnitudes of the damage vectors.

	$n_c \cdot W$ mm^{-1}	$\bar{\ell}_c$ mm	$\bar{w}_c$ mm	f_c	D^2
Laminate 1. Cracks in 6 90° plies	0.75	W	1.218	0.75	0.6851
Laminate 2. Cracks in 3 90° plies	0.51	W	0.609	1.0	0.3106
Laminate 3. Cracks in 2 90° plies	1.90	W	0.406	0.50	0.3857
Laminate 4. Cracks in +45° ply	1.27	1.414W	0.203	0.17	0.0609
Cracks in −45° ply	0.85	1.414W	0.203	0.17	0.0406

Table 3. Predicted and observed stiffness reduction of cracked laminates.

Laminate	ΔE_1, %		$\Delta\nu_{12}$, %		ΔE_2, %	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
2	38.8	38.1	71.0	67.4	0.0	0.5
3	17.1	17.2	49.8	41.4	7.3	0.4
4	9.4	8.3	8.1	7.5	5.3	0.1

$k_3 = -6.713$ GPa , $k_7 = -0.762$ GPa and $k_{13} = -4.467$ GPa . Data for Laminate 1: $\Delta E_1 = 42.0\%$, $\Delta\nu_{12} = 74.3\%$ and $\Delta E_2 = 0.6\%$.

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