NUMERICAL MODELLING OF DAMAGE AND FRACTURE IN CONCRETE

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ABSTRACT

An attempt is made to combine Damage Theory and Fracture Mechanics. The strain softening at the crack tip is realized by decreasing the elastic modulus in relation to the strain energy density locally absorbed, whereas both stable and unstable crack propagation are independently simulated. It appears clear that the two extreme situations of ultimate strength collapse at the ligament (small size) and brittle fracture (large size) are connected by a transition. The ability of describing such a transition shown by the Fictitious Crack Model and by the Strain Energy Density Theory are eventually discussed.

KEYWORDS

Fracture mechanics, Damage theory, Limit analysis, Dimensional analysis, Fictitious crack model, Strain energy density theory, Softening, Size effects, Concrete.

INTRODUCTION

The process of *damage* and *fracture* in concrete structures develops in subsequent stages.

1) At first, a damaged zone occurs and expands in the weakest point of the structure, where the material begins to soften. Such a zone may be considered isotropic and non-homogeneous, with a degraded elastic modulus dependent on the locally absorbed strain energy density.

2) The damaged zone, besides non-homogeneous, becomes anisotropic and the stresses relax prevailingly in one direction, which is that orthogonal to the developing crack.

3) A material discontinuity is eventually nucleated. At this stage, the anisotropy of the system can be considered as producted by the crack only, while the remaining part of the damaged zone may be considered as isotropic.

4) The crack propagates and, at the same time, the damaged zone expands. Obviously, they interact so that each of them is affected by the other failure mechanism.

The expansion of the damaged zone and the propagation of the crack have been described separately so far, respectively by *Damage Theory* and *Fracture Mechanics.* The former describes the mechanical damage of the material by decreasing the elastic modulus (Løland, 1980), whereas the latter considers the energy necessary to create a unit crack surface (Hillerborg, Modeer and Petersson, 1976). An attempt will be made to combine these two concepts and attention will be focused on the scale effects, which rise due to the co-existence of parameters with different physical dimensions. In fact, as has been recently shown by Carpinteri (1981, 1982-a, 1982-b), the usual size effects occurring in Material Strength can be explained with the combined application of Limit Analysis, Fracture Mechanics and Dimensional Analysis.

SOFTENING AND FRACTURE

Several authors have proposed softening constitutive laws connecting strain (or strain rate) with relaxed stress (or negative stress rate). Although these stress-strain laws are extremely convenient to be utilized as a computer input, they describe the progressive damage of concrete only on an average, and do not catch a phenomenon--crack formation—which is so greatly localized and anisotropic. In other words, they simulate the propagation of a microcracked zone and the fracturing process results to be smeared instead of being discrete.
The *Fictitious Crack Model* by Hillerborg, Modeer and Petersson (1976) assumes that the width w of the fracture zone in the tensile direction is originally equal to zero, whereas it is different from zero while it is actually developing. Stress at the softening stage, therefore, will be a function of such a width (Fig. 1-a).

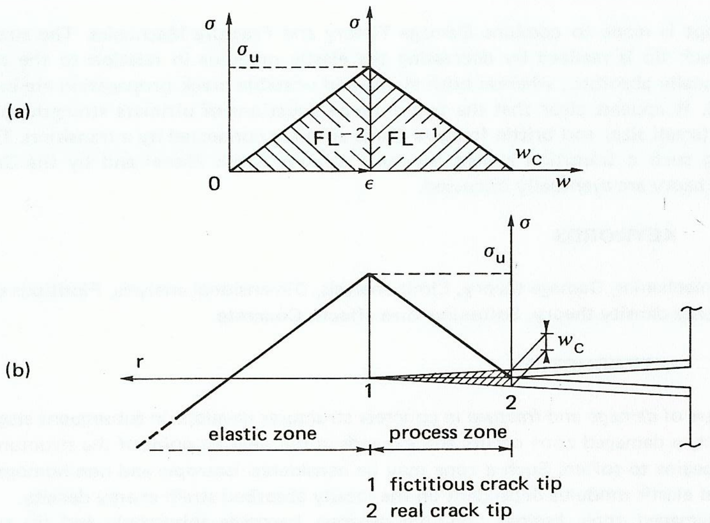

Fig. 1 - Fictitious Crack Model.

Changing the abscissa axis as the unstable stage begins gives very important theoretical consequences. Dilatation, in fact, is a dimensionless quantity, while the width of the fracture zone is a quantity with the physical dimensions of a length. Size (or scale) effects in Fracture Mechanics can be explained by this transition. In fact, the area under a σ-ϵ curve represents a dissipated energy per unit volume, thus having the physical dimensions of a stress $[F]\,[L]^{-2}$. It is well-known that classical strength criteria, as Beltrami's Criterion limiting the strain energy density and Von Mises' Criterion limiting the distortional energy density, substantially set a limit

to an equivalent stress. On the other hand, the area under a σ-w curve represents a dissipated energy per unit area, thus having the dimensions of a surface energy $[F]\,[L]^{-1}$. Such physical dimensions are unusual in the field of traditional Solid Mechanics, whereas they reflect well-known concepts in traditional Thermodynamics. It is indeed the transition from a continuous to a discontinuous system which requires the introduction of such quantities. The area under the σ-w curve is the fracture energy (Fig. 1-a):

$$\mathscr{G}_F = \frac{1}{2}\,\sigma_u\,w_c \simeq \frac{K_C^2}{E}\,, \tag{1}$$

where E is the elastic modulus and K_C the critical value of the stress-intensity factor. $\mathscr{G}_F$ represents the energy necessary to have a unit crack growth.
Several authors have assumed a crack model similar to that by Hillerborg, Modeer and Petersson (1976). Gerstle, Ingraffea and Gergely (1982) generalized the Fictitious Crack Model to mixed mode crack problems and considered both normal and shear stress on the crack surface as functions of both the discontinuities in normal and tangential displacement. Bažant and Oh (1981) transformed the σ-w descending law into a σ-ϵ softening law; they simply divided the crack opening displacement (COD) w by the characteristic width of the crack band forming the fracture process zone. Wecharatana and Shah (1983) proved that the value w_c of the critical COD strongly affects the length of the process zone, whereas the latter is non-sensitive to the shape of the σ-w diagram. Visalvanich and Naaman (1982) proposed a generalized σ-w law for fiber reinforced mortar and plain concrete, which reproduces the experimental results very well.

DAMAGE AND FRACTURE

It is not yet clear whether the fracture toughness properties are included or not in the material constitutive law. This means that it is difficult to say if a unique σ-ϵ softening law does exist, or only a unique σ-w descending law, as asserted by Hillerborg. Recently, a model with a unique σ-ϵ descending law coupled with an independent fracture criterion was proposed by Carpinteri and Sih (1983). In such a model, the softening at the crack tip (Fig. 1-b) is realized by decreasing the elastic modulus in relation to the strain energy density locally absorbed, whereas both stable and unstable crack propagation are adequately and independently simulated.
The damage of the material at the crack tip and the crack growth increments are computed at each step on the basis of a uniaxial linear elastic-linear softening stress-strain relationship (Fig. 2),

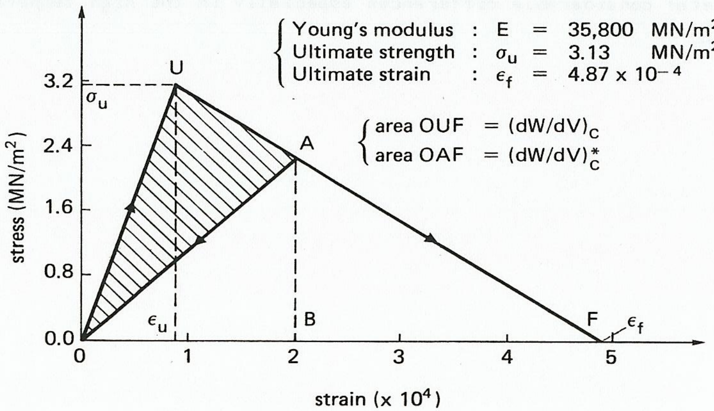

Fig. 2 - Strain softening constitutive law.

according to which the stress may increase up to the ultimate strength (point U of Fig. 2), while the strain increases proportionally. Consequently, only the strain may increase, while the stress decreases linearly down to zero (point F of Fig. 2). If the loading is relaxed when the representative point is in A (Fig. 2), the unloading process is assumed to occur along the line A0, so that the new bi-linear constitutive relation will be the line 0AF. No permanent deformation is included in such a model, but only a degradation of the elastic modulus. While for a non-damaged material element the critical value $(dW/dV)_C$ of the strain energy density is equal to the area 0UF, for a damaged material element with representative point in A the degraded critical value $(dW/dV)_C^*$ is equal to the area 0AF. The above described model is extended to the three-dimensional field using the actual value of the absorbed strain energy density (dW/dV) as a measure of damage.
The Strain Energy Density Theory is applied to evaluate the crack growth increment at every loading step, as proposed by Sih (1973; 1974). It is based on the following fundamental hypotheses.
1) The strain energy density field can always be described by means of the following general relationship:

$$(dW/dV) = \frac{S}{r} , \tag{2}$$

where the strain energy density factor S is a function of the angular coordinate θ and, generally, of the radial coordinate r.
2) According to Beltrami's Criterion, all the elements in front of the crack tip, where a strain energy density higher than the critical value $(dW/dV)_C$ has been accumulated, are due to fail, so that the crack growth increment Δa can be expressed by the formula:

$$\Delta a = \frac{S}{(dW/dV)_C} , \tag{3}$$

where S is the actual value of the strain energy density factor.
3) When the crack growth increment is such that:

$$\Delta a = r_C = \frac{S_C}{(dW/dV)_C} , \tag{4}$$

the unstable crack propagation takes place.
S_C is supposed to be a material constant and represents the strength of the material against rapid and unstable crack propagation. It is connected with the critical value K_C of the stress-intensity factor by means of the relationship (Sih, 1973; 1974):

$$S_C \simeq \frac{K_C^2}{2\pi E} . \tag{5}$$

COMPETITION BETWEEN COLLAPSES OF A DIFFERENT NATURE

The three-point bending test of Fig. 3 is analyzed as structural geometry. If we apply Buckingham's Theorem for physical similitude and scale modeling and consider the energy $(dW/dV)_C$ and the specimen width b as fundamental quantities, it will be possible to define the dimensionless load:

$$\frac{P}{(dW/dV)_C\, b^2} = \Pi \left[\frac{\delta}{b} , \frac{E}{(dW/dV)_C} , \frac{\sigma_u}{(dW/dV)_C} , \nu , \frac{S_C}{(dW/dV)_C\, b} , \frac{\ell}{b} , \frac{t}{b} , \frac{a_0}{b} \right] , \tag{6}$$

where δ is the deflection.
When material properties and geometrical shape are kept constant, function (6) is simplified as follows:

$$\frac{P}{(dW/dV)_C\, b^2} = \Pi \left[\frac{\delta}{b} , \frac{S_C}{(dW/dV)_C\, b} \right] . \tag{7}$$

Thus, the argument S* appears in function Π, in addition to δ/b:

$$S^* = \frac{S_C}{(dW/dV)_C\, b} = \frac{r_C}{b} . \tag{8}$$

Argument S* is produced by the different physical dimensions of $(dW/dV)_C$ and S_C and describes the brittleness of the structure synthetically. Both material properties and structural size appear in it.

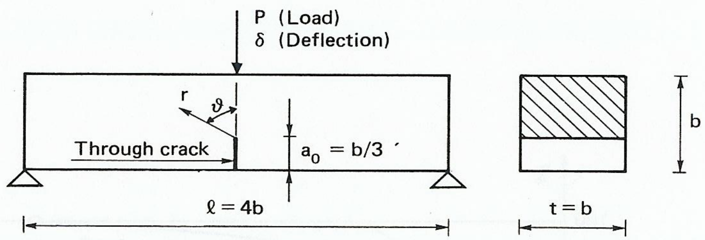

Fig. 3 - Three point bending test geometry.

The dimensionless P-δ diagrams are reported in Fig. 4, varying the structural scale. When the critical value S_C of the strain energy density factor is achieved, the curve drops down to zero. At that time, in fact, the unstable crack propagation takes place and the structure loses its loading capacity completely. By increasing the size the dimensionless maximum load decreases and the crack instability tends to anticipate the structural instability, which would always occur for $P_{max} = 332.90\ (dW/dV)_C\, b^2$. The points of crack instability in Fig. 4 are found by assuming a critical value S_C of the strain energy density factor equal to 7.85 N/m. By applying the relationship (5), it corresponds to a critical value K_C of the stress-intensity factor equal to 1.41 $MN/m^{3/2}$, which is a pertinent value for concrete (Carpinteri, 1981).

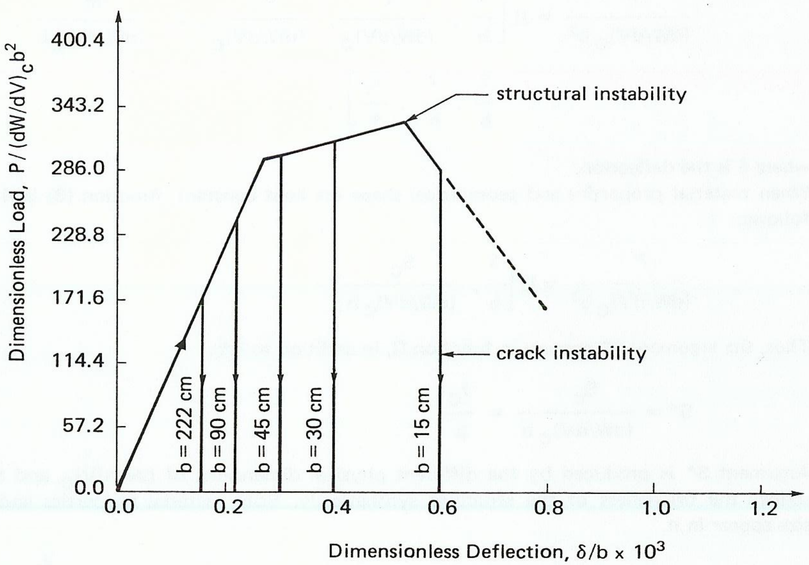

Fig. 4 - Dimensionless load-deflection response.

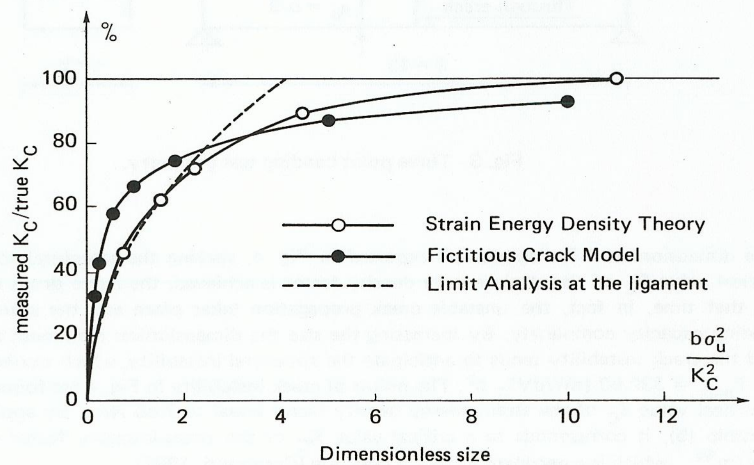

Fig. 5 - Transition from ultimate strength instability to crack instability.

The ratio between measured K_C and true K_C (*) is reported in Fig. 5 as a function of the dimensionless specimen size $b\sigma_u^2/K_C^2$ (**). From Fig. 5 it appears clear that the two extreme situations of ultimate strength collapse at the ligament (small size) and brittle fracture (large size) are connected by a transition, which can be described by both Fictitious Crack Model (FCM) and Strain Energy Density Theory (SEDT). It is worth noting that the FCM prediction overestimates the limit load for small size while it tends asymptotically to the true value of K_C—without ever achieving it—for large size. On the other hand, the SEDT predicts the limit load for small size and the true value of K_C for large size exactly.

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(*) The measured K_C is the critical value of the stress-intensity factor obtained at the maximum load bearable by the specimen, whereas the true K_C is the critical value of the stress-intensity factor considered as a constant material property.

(**) The dimensionless specimen size $b\sigma_u^2/K_C^2$ is equal to $1/s^2$, b/ℓ_{CH} or $1/\pi S^*$, where respectively s is the brittleness number defined by Carpinteri (1982), ℓ_{CH} is the characteristic length defined by Hillerborg, Modeer and Petersson (1976) and S^* is the dimensionless number defined by Carpinteri and Sih (1983).