MEASUREMENTS OF COD AND SHAPE OF MICROCRACK REGION NEAR CRACK TIP IN CONCRETE

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ABSTRACT

This paper has analyzed the curve of load-opening displacement of the notch edge of three-point bend beams according to experimental data. The critical load has been found to be 80% of the limit load. On the basis of the Griffith criterion and the principle of fracture mechanics, the boundary equation for the microcrack region near the crack tip for such brittle material as concrete has been derived.
Thereby, the relation between the length of microcrack of the crack tip, the fracture toughness, the tensile strength and the ratio of the minor and major axis of the elliptical crack has been established.

KEYWORDS

Concrete; mortar; biaxial strength; Griffith criterion; load-opening displacement curve.

INTRODUCTION

When using test specimens to study the fracture toughness K_{IC} of concrete, it is necessary to know the length of the crack and the load at the moment the crack becomes unstable. Formerly, attempts to measure the length of crack propagation prior to fracture by dyeing were made (Kaplan, 1961). This paper has analyzed the developing process of the microcrack region near the crack tip, using the load (P)—— opening displacement (V) curve of three-point bend beam specimens, so the critical load has been determined. On the basis of the Criffith criterion and the principle of fracture mechanics, the boundary equation for the microcrack region near the crack tip for such brittle material as concrete has been derived. Thereby, the relation between the length of microcrack of the crack tip, the fracture toughness, the tensile strength and the ratio of the minor and major axis of the elliptical crack has been established.

EXPERIMENT AND RESULTS

The cement used for the tests was #400 ordinary slag portland cement produced by the Liulihe Cement Plant, Hebei Province. Standard quartz sand and river sand were used in the mortar and concrete respectively. Two grades of river gravel were used. Their respective grain sizes were 5 - 20 and 20 - 40 mm, mixed in equal amounts. The mixes for the mortar was cement: water: sand =

1:0.45:2.35,and that for the concrete, cement: water: sand: gravel = 1:0.52:2.35·3.93.

The section dimensions of the three-point bend beams were 10 cm X B; B was the thickness of the specimens, which was severally 5, 10, 15, 20 and 25 cm. The 30^{o} notch with a height of 5 cm was adopted. The span of the beams was 45 and 40 cm for the mortar and concrete respectively. Steel moulds were used, and were removed in 48 hours after placing. Then the specimens were sent to a fog room to be cured. They were taken out of it just before the tests.

The tests were conducted on a 1-ton electronic testing machine. Its rate of loading was 1 mm/min. The load —— opening displacement curve (P — V curve) was recorded with a X — Y recorder. With the limit load P_{max} and the pre-formed crack length a equal to 5 cm, the fracture toughness averages K_{IC} = 479 $N/cm^{3/2}$ for 18 three-point bend beams of concrete and K_{IC} = 434 $N/cm^{3/2}$ for 15 similar beams of mortar were calculated.

The load-opening displacement curve,i.e., P — V curve can best reflect the changes of the crack tip under the action of load. It is not affected by the compressive deformation at the points of both loading and bearing. The end of the straight line section on the P — V curve is generally at the place where the load is 55 — 65% of the limit load P_{max}. After working out the statistics of P — V curve of 19 mortar and 19 concrete beams, the load at the end of the straight line section is found to be 0.59 P_{max} and 0.57 P_{max} respectively. To facilitate analysis, let us take 0.6 P_{max} as the load at the end of the straight line section on the P — V curve. In order to analyze the law of variation of BdV/dP with the change of P/P_{max}, we have examined the P — V curves of 19 mortar and 19 concrete beams. All the test points thus obtained are shown in figures 1 and 2. BdV/dP is the opening displacement of the notch edge of the three-point bend beam when the load is 1 N and width 1 mm, expressed in mm/N/mm as the unit. According to regression analysis, the relationship between BdV/dP and k for the mortar specimen is as follows:

$$B\frac{dV}{dP} = (1.166 + \frac{0.183}{1-k}) \times 10^{-3}, \qquad k \in [0.6,\ 1) \qquad (1)$$

$$B\frac{dV}{dP} = 1.625 \times 10^{-3}, \qquad k \in (0,\ 0.6]$$

correlation coefficient r = 0.79
For the concrete beam:

$$B\frac{dV}{dP} = (1.038 + \frac{0.387}{1-k}) \times 10^{-3}, \qquad k \in [0.6,1) \qquad (2)$$

$$B\frac{dV}{dP} = 2.008 \times 10^{-3}, \qquad k \in (0,\ 0.6]$$

correlation coefficient r = 0.86
where $k = P/P_{max}$, P_{max} being the limit load.

There is a stable propagation process of the crack from the moment the concrete is subjected to load to its fracture. In this process, the crack develops with the increases in load, and cracking ceases when no extra load is applied. As loading reaches the critical stage, the crack will fracture rapidly, and the structural elements will lose their bearing capacity. According to this analysis, BdV/dP increases slowly during the stable propagation of

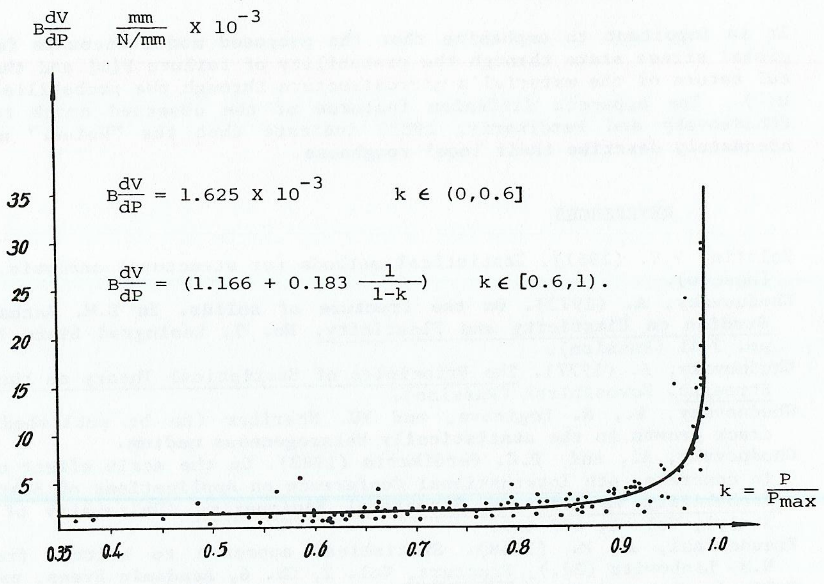

Fig. 1. The $B\frac{dV}{dP} \sim \frac{P}{P_{max}}$ curve for mortar specimens

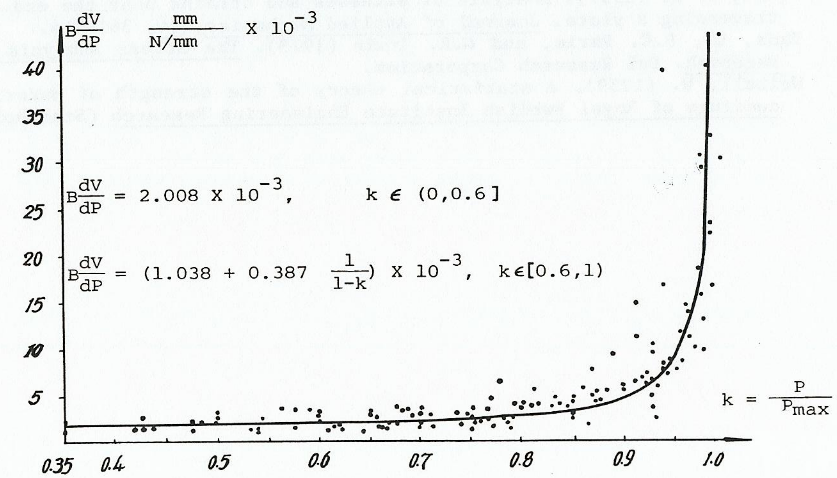

Fig. 2. The $B\frac{dV}{dP} \sim \frac{P}{P_{max}}$ curve for concrete specimens

the crack: and when the propagation of the crack becomes unstable, BdV/dP will increase sharply. Now let us analyze the law of variation of the slope of the curves. Let y = BdV/dP, then for the concrete beam we obtain:

$$\frac{dy}{dk} = \frac{0.387 \times 10^{-3}}{(1-k)^2}, \qquad k \in [0.6,\ 1) \tag{3}$$

and for the mortar beam:

$$\frac{dy}{dk} = \frac{0.183 \times 10^{-3}}{(1-k)^2}, \qquad k \in [0.6,\ 1) \tag{4}$$

The variation curve dy/dk —— k of the concrete beam is shown in figure 3 from which it can be seen that BdV/dP increases slowly before $P < 0.8\ P_{max}$, and sharply when $P \geq 0.8\ P_{max}$. This indicates that $0.8\ P_{max}$ might be the critical load of the concrete beam.

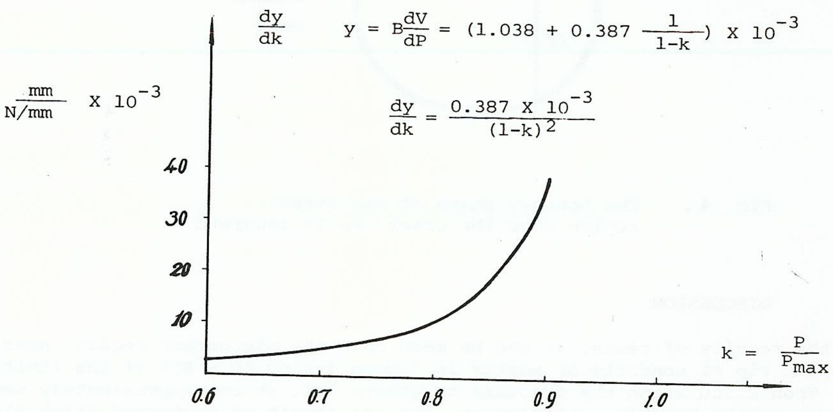

Fig. 3. The principle of variation of $B\frac{dV}{dP}$ with $k = P/P_{max}$ in concrete

There may be two reasons why the P-V curve starts to bend after $0.6\ P_{max}$. Firstly, the bond crack between aggregates and cement paste propagations with the increase of load, or new microcracks are produced, thereby, lowering the stiffness of the beam. Secondly, the microcrack region might be produced at the crack tip in concrete just as the plastic deformation zone is at the crack tip in metal materials. As the notch is deeper, the effect of the first factor might be smaller. The second factor is the primary one. So in later analyses, the effect of the first factor will be neglected, and it is believed that the bending of the P-V curve is primarily caused by the generation of the microcrack region at the crack tip.

To sum up, the P-V curve may be divided into three sections: when $P < 0.6\ P_{max}$, BdV/dP is a constant, the microcrack region near the crack tip is very small, and so can be neglected; when $0.6\ P_{max} \leq P < 0.8\ P_{max}$, BdV/dP stably increases according to the law of linearity, and the microcrack region enlarges slowly; and when $P \geq 0.8\ P_{max}$, BdV/dP increases rapidly, and so $P \approx 0.8\ P_{max}$ can be regarded as the critical load.

SHAPE OF MICROCRACK REGION NEAR THE CRACK TIP IN CONCRETE AND BOUNDARY EQUATION

The idea of solving this problem is: first to find out the stress at the crack tip at the moment of crack propagation and to establish successively the relationship between the stress and the macro-tensile strength of concrete, and then to establish the relationship between the stress and the stress of the crack tip calculated with fracture mechanics. Thus the boundary equation of the microcrack region will be obtained.

Take a large concrete slab subjected to uniform tensile stress σ_y. On it there is a symmetric and penetrating crack mode I. Now let us take such a crack as an elliptical hole whose major axis is 2a, and minor axis 2b. From theory of elasticity, we know that the tangential normal stress σ_β of the elliptical hole periphery is:

$$\sigma_\beta = \frac{\sigma_y(\alpha^2 \cos^2\beta + 2\alpha\cos^2\beta - \sin^2\beta)}{\alpha^2\cos^2\beta + \sin^2\beta} \tag{5}$$

where $\alpha = b/a$, and β is the eccentric angle of the hole. The maximum tensile stress should occur near the end of the major axis, and so it can be taken that this stress occurs at $\beta = 0$. Suppose the uniaxial tensile strength of concrete is R_t. Then the slab will break when $\sigma_y = R_t$. Substitute $\beta = 0$ and $\sigma_y = R_t$ into formula (5), and considering that the crack is very flat, the maximum tensile stress $\sigma_{\beta max}$ of the crack tip at the time of destruction is:

$$\sigma_{\beta\,max} = \frac{(2+\alpha)}{\alpha} R_t \approx \frac{2R_t}{\alpha} \tag{6}$$

The results of most biaxial strength tests of concrete show that the tensile strength in the biaxial tensile zone is roughly equal to the uniaxial tensile strength (Tasuji, 1976). Therefore, the strength criterion in the tensile zone is $\sigma_{max} = \sigma_{\beta\,max}$, i.e., it corresponds with the Griffith criterion.

According to linearly elastic fracture mechanics, the stress at the tip of mode I crack is:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (7)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

the major principal stress σ_1 is:

$$\sigma_I = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right); \quad (8)$$

According to the Griffith criterion, when σ_1 reaches σ_β max, the crack tip will start to crack and then form a microcrack region. Let $\sigma_1 = \sigma_\beta$ max. After simplification, we have obtained the boundary equation for the microcrack region:

$$r_x = \frac{K_{IC}^2 \alpha^2}{8\pi R_t^2}\left[\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right)\right]^2 \quad (9)$$

the length of the microcrack on the X axis is:

$$r_x = \frac{K_{IC}^2 \alpha^2}{8\pi R_t^2} \quad (10)$$

From expression (9), the shape of the boundary of the microcrack region can be drawn (see Fig. 4).

As to the fracture of the metal specimen under the state of plane stress, when stress relaxation after the yield at the crack tip is not considered, the boundary equation of the yield zone found with the Tresca criterion is:

$$r = \frac{K_{IC}^2}{2\pi\sigma_s^2}\left[\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right)\right]^2 \quad (11)$$

The length of the yield zone on the X axis is:

$$r_o = \frac{K_{IC}^2}{2\pi\sigma_s^2} \quad (12)$$

Compare found experssions (9) with (11) and (10) with (12). It can be seen that the shape of the microcrack region near the crack tip in such brittle materials as concrete is just the same as that of the plastic zone at the crack tip in metal materials under the state of plane stress derived from the Tresca yield criterion, but r_x is not equal to r_o.

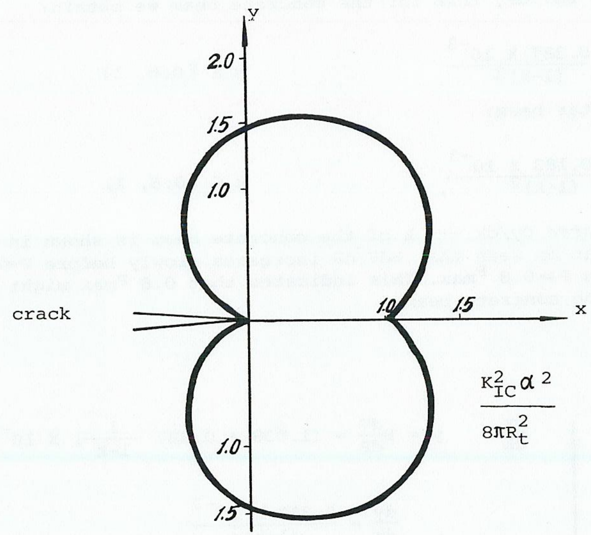

Fig. 4 . The bounday shape of microcrack region near the crack tip in concrete

DISCUSSION

From the results of tests, it can be seen that the microcrack region near the crack tip of concrete or mortar is formed during 60 - 80% of the limit load. When calculating the fracture toughness K_{IC}, we can approximately use 0.8 P_{max} as the load for calculation, and the length of preformed crack plus r_x as the crack length for calculation. The shape of the microcrack region near the crack tip of such brittle materials as concrete is given by formula (10). In order to calculate r_x, it is necessary to know the fracture toughness K_{IC}, the tensile strength R_t of concrete and the ratio α of the minor and major axis of the elliptical crack in concrete. But it is difficult to get the value of α. Therefore, although formula (10) has established the relation between r_x, K_{IC}, R_t and α, it is difficult to be used to calculate r_x. In future, on the basis of the research above, numerical analysis methods and experimental results should be used to determine the dimensions of the microcrack region near the crack tip.

CONCLUSION

1. Based on the analysis of the load-opening displacement curve, it can be concluded: when $P < 0.6\, P_{max}$, the microcrack region is very small and can be neglected; when $0.6\, P_{max} \leq P < 0.8\, P_{max}$, the microcrack region propagates slowly, and its effect is already not able to be overlooked; when $P \geq 0.8\, P_{max}$, the microcrack region develops sharply, causing test specimens to lose

their bearing capacity. When $P = 0.8\ P_{max}$, the load is critical, and the crack length at this instant should be taken as the crack length for calculation. Because the number of specimens was limited, in future more experimental data should be used to prove the conclusion.

2. Utilizing the Griffith criterion and the principle of modern fracture mechanics, and taking into account the results of biaxial tensile tests, the boundary equation for calculating the microcrack region near the crack tip in concrete has been derived. Its boundary shape is the same as that of the plastic deformation zone derived according to Tresca yield criterion near the crack tip of the metal specimen under the state of plane stress. But the size of the microcrack region of concrete is dependent on the fracture toughness K_{IC}, the tensile strength R_t and α of concrete.

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