FRACTURE IN VISCOELASTIC SOLIDS

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ABSTRACT

In viscoelastic solids the energy approach and the model of fracture without cohesive forces is applied to the study of fracture processes due to crack propagation and debonding of a thin rigid inclusion. The adopted model is shown to describe fracture kinetics. The classical problems on stable and unstable cracks in a unbounded medium are considered. It is found that a fracture delay takes place in a certain range of loading. The fracture problem of a viscoelastic body reinforced by a rigid inclusion is treated. The kinetics of the debonding process is studied; it is important for the determination of the strength of composites.

KEYWORDS

Fracture; viscoelastic solids; energy approach; crack propagation; debonding process.

INTRODUCTION

Fracture of solids is a process developing in time. The dynamical process of global crack propagation in a fracturing solid very often starts only after a time on applying loads. Before that moment even under constant external load slow processes of fracture preparation are taking place. To describe them, it is necessery first of all to take into account viscous properties of the material. In the simplest case the material may be assumed to be linear viscoelastic. The first attempt to apply the Griffith's energy approach to the fracture of viscoelastic bodies was undertaken by Williams (1963) and directly to the crack propagation in viscoelastic bodies by Kostrov, Nikitin, Flitman (1969); Kostrov, Nikitin (1970). In these papers the expression for the power absorbed by a propagating crack in a continuum with arbitrary properties was obtained from the global energy balance. The fracture criterion for solids with arbitrary rheological properties was postulated by means of the derived equation. The application of this criterion to a linear viscoelastic body showed that for an ideal brittle fracture model without cohesive forces there was no crack kinetics. On the other hand for an unideal model of fracture (Leonov and Panasyuk, 1959) crack kinetics does exist and the classical Griffith's problem for this case was considered in detail (Kostrov, Nikitin and Flitman, 1970). Recently many papers on this subject are appearing (Christensen, 1979; McCartney, 1977; Popelar and Atkinson, 1980). The results of the particular papers and of some other papers presume that the simplified ideal brittle fracture model due to its oversimplicity cannot predict fracture kinetics and should be modified to involve more detailed description of a fracture process. The present paper shows that the ideal brittle fracture model describes the fracture kinetics in some degenerated way. The examples of crack propagation and the debonding of a rigid inclusion are considered.

BASIC EQUATIONS

Let us apply the energy approach to fracture. For a crack to move with a given velocity v a certain amount of the effective surface energy $\gamma_*(v)$ must be supplied into the crack tip. Then the energy absorbed by the propagating crack (Kostrov, Nikitin and Flitman, 1969) must be equal to the effective surface energy $\gamma_*(v)$ of the material:

$$\lim_{\varepsilon\to 0} \oint_{\ell_\varepsilon} \sigma_{ij}\dot{u}_i n_j \, d\ell = 2v\gamma_*(v) \tag{1}$$

where σ_{ij} is the stress tensor, u_i is the displacement vector, ℓ_ε is the contour of the cross section of the shell-body whose surface lies at the distant ε from the crack surface, n_j is the normal to the surface of the shell-body and the dot denotes the derivative with respect to time t. This criterion is applicable to arbitrary solids. In what follows this criterion will be applied to linear viscoelastic bodies. The constitutive equations for such bodies relate the isotropic $\sigma_{kk}, \varepsilon_{kk}$ and deviatoric s_{ij}, e_{ij} parts of the stress and strain tensors $\sigma_{ij}, \varepsilon_{ij}$ as follows:

$$\sigma_{kk} = 3K\{\varepsilon_{kk}\}_\tau^t \quad ; \quad s_{ij} = 2\mu\{e_{ij}\}_\tau^t \tag{2}$$

where $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$, $e_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon_{kk}/3$; K and μ being the linear integral operators of the type

$$\mu\{f(\tau)\}_\tau^t = \mu_0 f(t) - \int_0^t M(t-\tau)f(\tau)\,d\tau \tag{3}$$

The subscript "0" denotes the instantaneous moduli and compliances while the subscript "∞" denotes the long-term moduli and compliances. For example

$$\mu_\infty = \mu_0 - \int_0^t M(\tau)\,d\tau \tag{4}$$

BEHAVIOR OF CRACKS IN VISCOELASTIC SOLIDS

Consider the classical Griffith's problem: an unbounded plane contains a cut of length $2\ell_0$, located along the axis x_1 with the origin at the center of the crack. The axis x_2 is normal to the axis x_1. Assume that for $t<0$ there is no stress and strain in the body, and at $t=+0$ the stress $\sigma_{22}=\sigma$ is instantaneously applied at infinity and then maintained constant. It is evident that under this condition the crack can only increase its length. In this case the correspondence principle (Graham, 1968) can be used. Omitting the details of the solution of this well-known problem let us write down the main terms of the asymptotic solution for stress σ_{22} and the displacement u_2 near, say, the right tip of the crack. It is these terms that make a contribution to the criterion of fracture:

$$\sigma_{22} = \sigma \ell^{1/2}(t)\,\mathrm{Re}[2(z-\ell(t))]^{-1/2}$$
$$u_2 = \sigma\Omega\{\ell^{1/2}(\tau)\,\mathrm{Im}[2(z-\ell(\tau))]^{1/2}\}_\tau^t \tag{5}$$

Here $z = x_1 + ix_2$, and Ω is the linear integral operator of the plane problem of the theory of elasticity; this operator is expressed in terms of the operators μ and ν for plane strain and plane stress states, as $\Omega = \mu^{-1}(1-\nu)$, $\Omega = \mu^{-1}(1+\nu)$, respectively. Using (5), the criterion of crack growth (1) may be written after some transformation as follows:

$$\frac{\pi\sigma^2\ell\Omega_0}{2}\left[1 + \frac{1}{\pi}\int_0^t \frac{\Omega(t-\tau)v(\tau)\ell^{1/2}(\tau)}{\Omega_0 v(t)\,\ell^{1/2}(t)}\,\varphi(t,\tau)\,d\tau\right] = 2\gamma_* \tag{6}$$

where $\varphi(t,\tau) = \lim_{\varepsilon\to 0}\oint_{\ell_\varepsilon} \mathrm{Re}[z-\ell(t)]^{-1/2}\,\mathrm{Im}[z-\ell(\tau)]^{-1/2}\,d\ell$

It can be easily shown that the function $\varphi(t,\tau)$ is nonzero and equal to π only when $\ell(t)=\ell(\tau)$. Thus $\varphi(t,\tau)$ is equal to π for the crack at rest and equal to zero for a moving one. As a result the left hand side of (6) increases with time for a motionless crack and would presserve its initial value for a moving crack. Being aware of the properties of the function $\varphi(t,\tau)$, it is easy to understand the behavior of the crack for different loads.

Assume that the load satisfies the following inequality:

$$\sigma > (4\gamma_*/\pi\ell_0\Omega_0)^{1/2} \equiv \sigma_0 \tag{7}$$

From (6) it is clear that static equilibrium is not possible and for $t>0$ the crack propagates dynamically and can never come to rest since the greater the crack length the more the crack is overdriven.

If $\sigma<\sigma_0$, the crack is initially at rest. But in this case $\ell=\ell_0$ and the left hand side of (6) increases, although the stress intensity factor remains constant. It can happen that

$$\sigma<(4\gamma_*/\pi\ell_0\Omega_\infty)^{1/2}\equiv\sigma_\infty \tag{8}$$

Then the crack will be at rest for all times. If, however, $\sigma_\infty<\sigma<\sigma_0$ there exists the time $t=t_r$ at which the equation (6) is valid and then the crack is overdriven and run-away instability starts. The time of fracture delay t_r satisfies the following equation, which results from (6)

$$\int_0^t \Omega(\tau)d\tau=\Omega_0\left(\frac{\sigma_0^2}{\sigma^2}-1\right) \tag{9}$$

If the stress at infinity grows monotonically with time, the time of fracture is obtained from the equation

$$\frac{\sigma_0^2}{\sigma(t_r)}=\sigma(t_r)+\frac{1}{\Omega_0}\int_0^{t_r}\Omega(t_r-\tau)\sigma(\tau)d\tau \tag{10}$$

It is interesting to consider the effects of viscosity on the behavior of a crack, which is stable in an elastic material. The example of such a crack is the same as considered above but the crack in question is loaded by a pair of concentrated forces P, which act at the center of the crack in a direction normal to the crack surface and opposite to each other starting from $t=+0$. In this case the asymptotic solutions in the vicinity of the crack tip for the pertinent values have the forms

$$\sigma_{22}=P\ell^{1/2}\,\mathrm{Re}[2(z-\ell(t))]^{-1/2}\;;\;u_2=P\Omega\{\ell^{1/2}(\tau)\,\mathrm{Im}[2(z-\ell(t))]^{1/2}\}_\tau^t \tag{11}$$

Taking into account (11) and using the condition of crack growth (6), we have

$$\frac{\pi P^2\Omega_0}{2\ell}\left[1+\frac{1}{\pi}\int_0^t\frac{\Omega(t-\tau)v(\tau)\ell(t)}{\Omega_0 v(t)\ell(\tau)}\varphi(t,\tau)d\tau\right]=2\gamma_* \tag{12}$$

Let us first assume that the load is not too great so that

$$P<(4\gamma_*\ell_0/\pi\Omega_\infty)^{1/2}\equiv P_\infty \tag{13}$$

Then although the left hand side of (12) increases with time, it never reaches its critical value $2\gamma_*$ and the crack will be at rest for all times.

Now let us suppose the load is large enough so that

$$P>(4\gamma_*\ell_0/\pi\Omega_0)^{1/2}\equiv P_0 \tag{14}$$

Then the left hand side of (12) immediately exceeds the value $2\gamma_*$ and the crack length jumps to the value $\ell(+0)=\pi\Omega_0P^2/4\gamma_*$. For time $t>0$ the crack cannot be at rest, since when $\ell=\ell(+0)$ the left hand side of (12) has its limiting value and increases due to the viscoelastic properties of the material. Further it is natural to suggest that the crack starts to move slowly, so that at time $t=+0$ or at some later moment it has nonzero velocity. According to (12) this suggestion leads to the contradictory conclusion that ℓ = constant. The situation is rather peculiar: the crack can neither be at rest nor move. What is the way out? The only possibility is to assume that the crack is progressing in a jumplike manner. Being at rest it is overdriven to some extent, then it jumps dynamically to some other length, probably overshooting its equilibrium state, and then it comes to rest until it is again overdriven and so on. The amount of overdriving or overshooting cannot be found from the analysis (Molchanov and Nikitin 1973). The equilibrium crack length around which the crack jumps can be easily found from (12)

$$\ell(t)=\frac{\pi P^2}{4\gamma_*}\left[\Omega_0+\int_0^t\Omega(\tau)d\tau\right] \tag{15}$$

Similar reasoning shows that the behavior of a crack for the loads over the range $P_\infty<P<P_0$ is analogous to the behavior described above, except that it starts without the initial jump.

DEBONDING OF A VISCOELASTIC MEDIUM FROM A THIN RIGID INCLUSION

Many composites consist of a viscoelastic matrix reinforced by rather rigid laminates. Debonding is a typical mode of failure in such materials. Therefore let us consider the model plane problem of a infinitesimal thin absolutely rigid linear inclusion with the length $2\ell_0$ in the unbounded viscoelastic plane. At the time $t=+0$ at infinity the homogeneous stress state $\sigma_{11}=p$, $\sigma_{12}=\sigma_{22}=0$ is created and then maintained constant. The processes of deformation and debonding of the inclusion due to stress concentration are studied. Owing to symmetry only the upper half-plane may be considered. The boundary conditions along $x_2=0$ have the form:

$$u_2=0,\ |x_1|<\infty;\ u_1=0,\ |x_1|<\ell(t)\ ;\ \sigma_{12}=0,\ |x_1|>\ell(t) \tag{16}$$

where $\ell(t)$ is the length at the time t of the intact segment of the matrix-inclusion interface. It is assumed that there is no friction along the debonded part. The solution of the plane problem (16) in the case of elastic behavior of the matrix can be easily found. However the correspondence principle is not valid for this case. To obtain the solution of the viscoelastic problem let us act as follows. Instead of the second condition (16) consider the condition

$$\sigma_{12} = f(x_1,t), \quad |x_1| < \ell(t), \; x_2 = 0 \tag{17}$$

where the function $f(x_1,t)$ is temporally regarded as known. The elastic solution of this problem can be easily obtained and moreover the correspondence principle is valid for this case. This solution has the following form for $\partial u_1(x_1,0,t)/\partial x_1$:

$$\frac{\partial u_1(x_1,0,t)}{\partial x_1} = \frac{1}{8}\mu^{-1}(æ+1)\{p\}_\tau^t - \frac{1}{\pi}\mu^{-1}(æ+1)^{-1}æ\left\{\int_{-\ell(\tau)}^{\ell(\tau)} \frac{f(\zeta,\tau)}{\zeta - x_1}d\zeta\right\}_\tau^t \tag{18}$$

For the initial problem the displacement $u_1(x_1,0,t)$ is equal to zero when $|x_1| < \ell(t)$. From here we have the following integral equation for the determination of the unknown function $f(x_1,t)$

$$\int_{-\ell(t)}^{\ell(t)} \frac{f(\zeta,t)}{\zeta - x_1} d\zeta = \pi p F(t), \quad |x_1| < \ell(t), \; x_2 = 0, \; t > 0 \tag{19}$$

where $F(t) = æ^{-1}(æ+1)^2\{1\}_\tau^t/8$, $æ(\cdot) = (3-\nu)(1+\nu)^{-1}\{\cdot\}_\tau^t$. Solving the integral equation of the Cauchy tipe and taking into account the third condition (16) we derive

$$f(x_1,t) = p\frac{F(t)x_1}{\sqrt{\ell^2(t) - x_1^2}} \tag{20}$$

It is worth noting that application of the correspondence principle to the initial problem (16) would lead to a different result. Using (20) we can write the asymptotic expressions near $z = \ell(t)$ for σ_{12} and $\partial u_1/\partial x_1$, required for the further analysis of the debonding process:

$$\sigma_{12}(x_1,x_2,t) = \frac{1}{\sqrt{2}} p \operatorname{Im}[(z-\bar{z})(æ+1)^{-1}\{F(\tau)\ell^{1/2}(\tau)(z-\ell(\tau))^{-3/2}\}_\tau^t - F(t)\ell^{1/2}(t)(z-\ell(t))^{-1/2}]$$

$$\frac{\partial u_1(x_1,x_2,t)}{\partial x_1} = \frac{1}{\sqrt{2}} p \operatorname{Re}[\tfrac{1}{2}\mu^{-1}(æ+1)^{-1}\{(z-\bar{z})F(\tau)\ell^{1/2}(\tau)(z-\ell(\tau))^{-3/2} - 2æ\{F(\xi)\ell^{1/2}(\xi)(z-\ell(\xi))^{1/2}\}_\xi^\tau\}_\tau^t] \tag{21}$$

Substituting (21) into (1) and applying the energy criterion of fracture to the debonding process, we have

$$\frac{1}{2}\pi p^2 \ell(t) F^2(t) R_0 \left[1 + \frac{1}{\pi}\int_0^t \frac{R(t-\tau)F(\tau)\ell^{1/2}(\tau)}{R_0 F(t)\ell^{1/2}(t)} \varphi(t,\tau)d\tau\right] = \gamma_* \tag{22}$$

where now γ_* is the debonding energy per unit surface, the function $\varphi(t,\tau)$ is the same as in (6) and $R\{\cdot\}_\tau^t = \mu^{-1}(æ+1)^{-1}æ\{\cdot\}_\tau^t$. Suppose that the applied stress is rather large so that

$$p > (2\gamma_*/\pi\ell_0 R_0 F_0^2)^{1/2} \equiv p_0 \tag{23}$$

where $F_0 = F(+0)$. Then the right hand side of (22) exceeds the limiting value from the very beginning. Therefore instantaneous debonding occurs, so that the size of the intact part immediately takes the value $\ell_1 = 2\gamma_*/\pi p^2 R_0 F_0^2$. The further development of the debonding process strictly depends on the behavior of the function $F(t)$. If $F(t)$ increases monotonically the integral term in (22) vanishes and the debonding is described by the equation

$$\ell(t) = 2\gamma_*/\pi p^2 R_0 F^2(t) \tag{24}$$

When F decreases analysis of the equation shows that ℓ can neither decrease nor remain constant. Now as in the previous case for the stable crack one should suggest that the debonding process takes place in a jumplike manner. The crack motion law can be deduced from (22)

$$\ell(t) = 2\gamma_*/\pi p^2 R_0 F^2(t)\left[1 + \int_0^t \frac{R(t-\tau)F(\tau)}{R_0 F(t)}d\tau\right] \tag{25}$$

If the material properties are such that the integral in (25) exists when $t \to \infty$, the final dimension of the intact part will be $\ell_\infty = 2\gamma_*/\pi p^2 F_\infty R\{F(\tau)\}_\tau^\infty$.

The load may turn out to be

$$p < (2\gamma_*/\pi\ell_0 F_\infty R\{F(\tau)\}_\tau^\infty)^{1/2} \equiv p_\infty \tag{26}$$

Then equation (22) is not true and the debonding process does not occur. For loads over the range $p_\infty < p < p_0$ equation (22) is true starting from some moment t_r . After that, depending on the behavior of the function $F(t)$, either smooth or jumplike debonding process described above starts. The debonding delay time is found from the equation

$$\frac{F_0}{F(t_r)}\left[\frac{p_0^2}{p^2} - \frac{F^2(t_r)}{F_0^2}\right] = \int_0^{t_r} \frac{R(t_r-\tau)F(\tau)}{R_0 F_0} d\tau \tag{27}$$

deduced from (22).

As an example consider the linear viscoelastic medium which is elastically compressible and possesses standard solid properties under shear. The function $F(t)$ for the particular solid is expressed as

$$F(t) = \frac{2(2\omega+1)^2}{(\omega+2)(7\omega+2)} + \frac{3}{2}\omega(\beta-1)\left[\frac{e^{-\frac{(\omega+2)t}{(\omega\beta+2)\tau_\sigma}}}{(\omega+2)(\omega\beta+2)} - \frac{e^{-\frac{(7\omega+2)t}{(7\omega\beta+2)\tau_\sigma}}}{(7\omega+2)(7\omega\beta+2)}\right] \quad (28)$$

where $\beta=\tau_\varepsilon/\tau_\sigma$, $\omega=2\mu_0/3K_0$ and τ_σ is the stress relaxation time, τ_ε is the strain relaxation time. This function decreases reaching the minimum value and then increases approaching the value $F(\infty)=2(2\omega+1)^2/(\omega+2)(7\omega+2)$. In spite of the function $F(t)$ the smooth debonding process does not take place since the inequality $F(t)<F_0$ is valid.

Thus the simplest fracture model without cohesive forces permits us to analyse the kinetics of fracture processes due to crack propagation or debonding in a linear viscoelastic medium.

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