EFFECTS OF MICROMECHANICAL MODELLING ON THE FRACTURE ANALYSIS OF THERMALLY LOADED FIBER REINFORCED COMPOSITES

F.-G. Buchholz and K. P. Herrmann

Institute of Mechanics, University of Paderborn, Pohlweg 47 - 49, D-4790 Paderborn, Federal Republic of Germany

ABSTRACT

In this paper three different micromechanical models of an unidirectionally fiber reinforced composite are investigated (single circular and hexagonal unit cell and hexagonal unit cell within an ensemble of seven hexagonal unit cells). It is shown that the self-stress energy arising in the unit cells due to uniform heating is rather independent of their geometric contours and the corresponding boundary conditions. Moreover, it is shown that the values of the energy release rate for extending interface cracks will be remarkably overestimated by the fracture analysis of the single unit cells instead of the ensemble.

KEYWORDS

Fiber reinforced composite material; micromechanical modelling; self stress energy; curved interface crack; energy release rate.

INTRODUCTION

If composite materials are subjected to thermal loads during service, thermal stresses arise on account of the different thermoelastic properties of the constituents of the compound. In superposition with stresses arising due to mechanical loads those thermal stresses can cause fracture of such a composite structure. Thereby the ultimate strength of some structural parts consisting of composite materials may be affected and that is why investigations of the fracture and failure behavior of mechanically and/or thermally loaded composite materials are important subjects in todays engineering science. In order to determine the mechanical behavior of composite materials in service different approaches have been used, for example mixture theories (Cooper, Pigott, 1977; Murakami, Maewal, Hegemier, 1979) or theories of failure mechanisms in combination with statistical aspects (Wu, 1979; Beaumont, Anstice, 1980). Referring to the micromechanical concept of composite materials (Rosen, Kulkami, McLaughlin, 1975) investigations often are restricted to the analysis of small representative sections of the compound (unit cells) (Sih and co-workers, 1973; Sendeckyj, 1974). The micromechanical aspect in thermal cracking of unidirectionally reinforced composites has been stressed recently in several papers by Buchholz, Herrmann, Strathmeier (1980), Braun, Herrmann (1981) and Herrmann, Braun (1983).

SELF-STRESS ENERGY AND CRACK SURFACE ENERGY IN SINGLE UNIT CELLS

First we consider an uncracked circular unit cell of thickness h with a circular fiber of radius r_f embedded in a concentric matrix of radius r_m (Fig. 1). Due to the differing thermoelastic material properties of fiber and matrix (Table 1) a non-uniform stress field will arise in this two-phase compound if it is subjected to a uniform temperature rise ΔT in service. The analysis of the stress and displacement fields of this uncracked unit cell results under the assumption of plane strain conditions in the following boundary-value problem of the stationary thermoelasticity (Herrmann, 1970).

$$\{\nabla^4 F(r) + \frac{E\alpha}{1-\nu}\nabla^2 T(r)\}_i = 0 \quad , \quad i = \{\begin{matrix} f & \text{fiber} \\ m & \text{matrix} \end{matrix} \tag{1}$$

where $F_i(r)$ denotes the AIRY stress functions for the fiber and matrix, respectively. Further, at the boundary of the matrix the boundary conditions

$$\sigma_{rr}(r_m) = \sigma_{r\phi}(r_m) = 0 \tag{2}$$

have to be fulfilled. Moreover the following continuity conditions

$$[\sigma_{rr}(r)]_{r=r_f} = [\sigma_{r\phi}(r)]_{r=r_f} = 0 \tag{3}$$

$$[u_r(r)]_{r=r_f} = [u_\phi(r)]_{r=r_f} = 0 \tag{4}$$

have to be valid at the fiber-matrix interface, with the definition of the jump relations

$$[\omega(r)]_{r=r_f} = \omega_f(r_f) - \omega_m(r_f) \tag{5}$$

A closed analytical solution of this boundary-value problem was given by Herrmann (1970) delivering the complete stress- and displacement fields within this two-phase compound as well as the self-stress energy

$$U = U_f + U_m \tag{6}$$

arising in a circular disc, with finite thickness h of the circular unit cell due to uniform heating. For plane strain conditions it has been shown by Buchholz (1982) that the self-stress energy U of such a disc can be separated into two independent terms

$$U = U^z + U^{r,\phi} \tag{7}$$

The energy term U^z is due to the suppressed displacements in the axial z-direction of the circular compound cylinder and is given by

$$U^z = U^z_f + U^z_m \tag{8}$$

The energy term $U^{r,\phi}$ is due to the inhomogeneity of the compound within the r,ϕ-plane perpendicular to the z-axis of the unit cell and is given in (Buchholz, 1983) by

$$U^{r,\phi} = U^{r,\phi}_f + U^{r,\phi}_m \tag{9}$$

with the definitions

$$U^{r,\phi}_f = \frac{\pi}{2} h r_f^2 \{8C^2 \frac{(1+\nu_f)(1-2\nu_f)}{E_f}[(\frac{r_f}{r_m})^2 + (\frac{r_m}{r_f})^2 - 2]\} \tag{10}$$

$$U^{r,\phi}_m = \frac{\pi}{2} h(r_m^2 - r_f^2)\{8C^2 \frac{(1+\nu_m)}{E_m}[1+(1-2\nu_m)(\frac{r_f}{r_m})^2]\} \tag{11}$$

and

$$C = -\frac{2}{D}\{(1+\nu_f)\alpha_f \Delta T_f - (1+\nu_m)\alpha_m \Delta T_m\} \tag{12}$$

$$D = \frac{4}{E_f E_m}\{(1+\nu_f)(1-2\nu_f)E_m[\frac{r_f}{r_m} - \frac{r_m}{r_f}] - (1+\nu_m)E_f[\frac{r_m}{r_f} + (1-2\nu_m)\frac{r_f}{r_m}]\} \tag{13}$$

Referring to Buchholz (1983) the following relation holds

$$U^{r,\phi}(\phi_1 = 0) = \hat{U}^{r,\phi}(\phi_1 = 2\pi) = h \int_{\phi=0}^{\phi=\phi_1=2\pi} G(\phi) r_f \, d\phi \tag{14}$$

Equation (14) states that the energy $\hat{U}^{r,\phi}(\phi_1 = 2\pi)$ which the circular unit cell releases to an extending interface crack until complete separation of fiber and matrix (ϕ_1 crack angle, G (ϕ) energy release rate) is equal just to the energy term $U^{r,\phi}$ belonging to the uncracked unit cell. This is evident, as for a crack angle $\phi_1 = 2\pi$ fiber and matrix can expand unrestrictedly as separate homogeneous parts in the r,ϕ-plane, which means they are free of radial and transversal stresses σ_{rr} and $\sigma_{\phi\phi}$ and results in

$$U^{r,\phi}(\phi_1 = 2\pi) = 0 \tag{15}$$

Similar considerations lead to upper bounds for the self-stress energy withdrawable by any straight, circular or curved thermal crack within a single two-phase circular unit cell (Buchholz, 1983). Equation (14) also holds for single two-phase unit cells with any geometric contours, for example for the hexagonal unit cell (h-uc) of Fig. 1 too. But for this geometry to the authors knowledge no analytical expression is known for the self-stress energy U or $U^{r,\phi}$ due to a uniform temperature rise ΔT within the compound.

On the other hand for any single unit cell and any corresponding material section within a real composite structure the arising self-stress energy, due to a given temperature distribution, can be computed with very good accuracy (Jäcker, 1983) by the aid of the finite element method using the relation

$$\tilde{U}^{r,\phi} = \frac{1}{2} \underline{r}_e^T \underline{\underline{K}} \underline{r}_e \tag{16}$$

where $\underline{r}_e$ is the vector of the effective thermal displacements, $\underline{r}_e^T$ is the transposed vector and $\underline{\underline{K}}$ is the stiffness matrix of the corresponding finite element structure. Using the same finite element analysis, one can determine by various numerical procedures, the energy release rate G(ϕ) for any quasi-static extending crack which is of great interest from the fracture mechanical point of view.

SINGLE CIRCULAR AND HEXAGONAL UNIT CELLS (c-uc and h-uc)

The following investigations refer to the three different micromechanical models of a unidirectionally fiber reinforced composite material as shown in Fig. 1. The main differences between these models are the different geometries (circular- and hexagonal unit cells, c-uc and h-uc) and the different boundary conditions acting on their contours. Thereby the single c-uc and h-uc have vanishing normal stresses and no displacement restrictions on the free matrix surfaces. The hexagonal unit cell within the compound (h-uc-c) has normal stresses acting on the contour lines between the inner and outer unit cells and compatibility conditions which have to be fulfilled there in addition. All three unit cells have the same fiber volume fraction of $V_f/V=30\%$ which corresponds to a radius fraction of $r_f/r_m=5.47/10$ for the c-uc. Furthermore, the same material combination of SiC-fiber and Aℓ-matrix and the same temperature rises of $\Delta T_i=100^oC$ are considered ($T_i=T_o+\Delta T_i$, i=f,m; T_o - temperature of the unstressed initial state; the thermoelastic material parameters are given in Table 1).

TABLE 1 Thermoelastic Material Parameters of the Fiber-Matrix Compound

Notation		Fiber SiC	Matrix Aℓ
Young's modulus	$E[Nmm^{-2}]$	449,300.	69,650.
Poisson's ratio	$\nu[1]$	0.266	0.339
Lin. coeff. of thermal expansion	$\alpha[K^{-1}]$	$4.5\cdot10^{-6}$	$2.4\cdot10^{-5}$

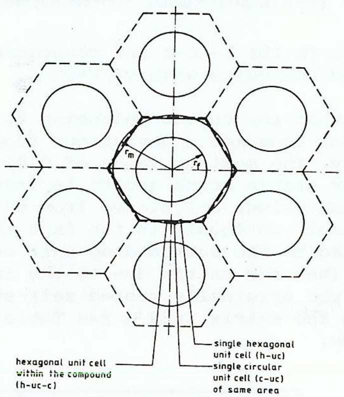

Fig. 1. Cross sections of three different unit cells

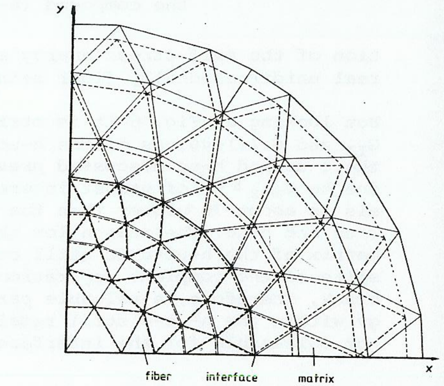

Fig. 2. Displacement field of a single circular unit cell (--- undeformed, —— deformed)

Figure 2 shows the finite element mesh of the c-uc and the numerically calculated[1] displacement field of the uncracked two-phase compound. It is evident that the displacement field of the deformed structure is geometrically similar to the undeformed one because of the axial symmetry of the c-uc. In Fig. 3 radial and transversal stresses σ_{rri} and $\sigma_{\phi\phi i}$, i=f,m of the c-uc are plotted versus a radial line of 0 degree (standing for all radial directions). Within the fiber both stresses turn out to be constant (tension) and corresponding to the analytical continuity and boundary conditions (Eqs. (2) and (3)) σ_{rrm} decreases monotonically to zero on the free surface ($r=r_m$). In contrast $\sigma_{\phi\phi}$ jumps at the interface from tension to compression and does not vanish for $r=r_m$. By comparing these graphs with the given analytical

[1] performed at the University of Paderborn Computer Center on a PRIME 750 using 6-node linear strain elements TRIMP 6 of the ASKA finite element computer programme.

reference solutions of the c-uc (Herrmann, 1970) it can be concluded that the used finite element discretisation is fully sufficient, although it is not very fine with respect to the bigger h-uc-c model and the computer facilities.

In Fig. 4 the corresponding displacement field of the h-uc is given and a detailed analysis shows that the outer contour of the deformed structure is no more straight but slightly curved. It seems evident that the radial tension stresses acting nearly uniformly on the fiber-matrix interface will lead to greater deformations on the narrower sides of the hexagonal matrix than in the thicker and stiffer edges. Figure 5 shows the corresponding stress curves of the h-uc and the reference solutions of the c-uc. The stresses on the 0 degree line (to one of the edges) are slightly higher near to the fiber-matrix interface and within the matrix compared with the stresses σ_{rri} and $\sigma_{\phi\phi i}$, i=f,m of the c-uc. The opposite result holds along the 30 degrees line to the side contour of the matrix. The main difference is given by the vanishing stress $\sigma_{\phi\phi m}$ along the 0 degree line into the edge of the hexagonal matrix. This means that the edges of the single h-uc are free of stresses and strains which will decrease the amount of self-stress energy arising in this model.

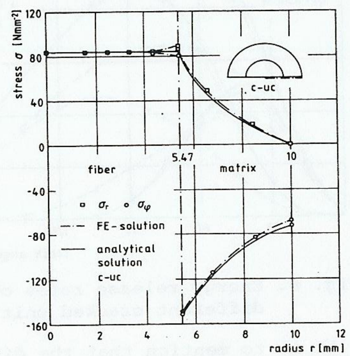

Fig. 3. Stresses within the single circular unit cell

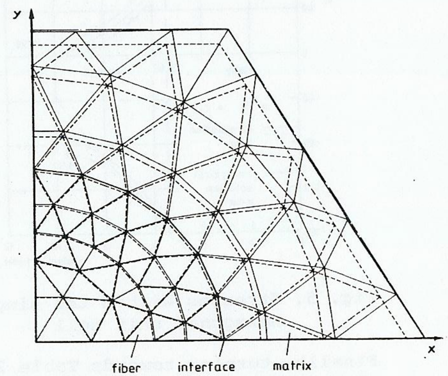

Fig. 4. Displacement field of a hexagonal unit cell (--- undeformed, —— deformed)

In Table 2 the calculated self-stress energies of the three models are given (Jäcker, 1983). Thereby, with 1.6% difference the numerically computed self-stress energy $\tilde{U}^{r,\phi}$ (Eq. (16)) of the c-uc is in very good agreement with the analytically calculated value $U^{r,\phi}$, Eqs. (9)-(13). As suggested above the value $\tilde{U}^{r,\phi}$ for the h-uc is slightly lower (1.5%) compared with the $\tilde{U}^{r,\phi}$-value of the c-uc.

TABLE 2 Self-Stress Energy and Crack Surface Energy of Different Unit Cells

U[Nmm]	c-uc		h-uc		h-uc-c
$U^{r,\phi}$	21.125				
	1.6 %				
$\tilde{U}^{r,\phi}$	20.778	1.5 %	20.467	1.4 %	20.757
	7.8 %		0.1 %		
$\hat{U}^{r,\phi}$	19.475	1.7 %	19.142	33.1 %	12.805
			34.2 %		

Now we put our attention to the thermally loaded curved interface cracks and their energy release rates in dependence on crack length, which is of great importance concerning the analysis of thermal fracture behavior of composite materials. Looking at Fig. 6 it can be seen that the energy release rates of the interface cracks extending quasistatically in the fiber-matrix interface of the c-uc and the h-uc nearly give the same graphs with increasing crack angle ϕ which means that both models lead to the same thermal fracture behavior. With respect to the corresponding values of $\tilde{U}^{r,\phi}$ and the fact that both cracks withdraw the whole self-stress energy of the single unit cells this result is evident. Further Fig. 6 shows another interesting result concerning the thermal fracture behavior. It can be seen that a starting thermal interface crack opens up under predominating mode I conditions until crack angles up to nearly 30 degrees but afterwards the G_{II}-values increase to a distinct higher value at $\phi \approx 90$ degrees, whereas $G_I(\phi)$ goes to zero for crack angles $\phi > 90$ degrees.

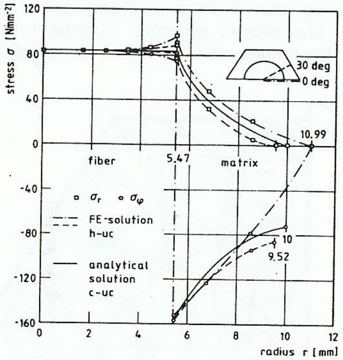

Fig. 5. Stresses within the single hexagonal unit cell

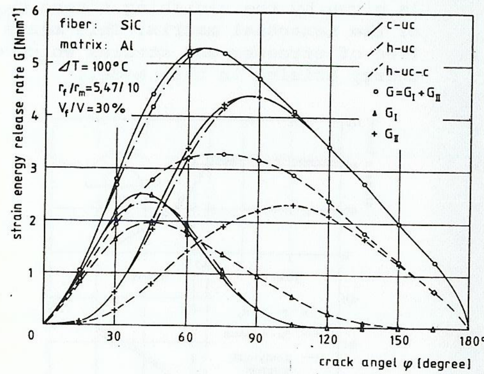

Fig. 6. Energy release rates of three different cracked unit cells

Finally, turning towards Table 2 again, it is to mention that the difference of 7.8 % between the values of $U^{r,\phi}$ and $\hat{U}^{r,\phi}$ for the c-uc is caused by the coarse finite element mesh in combination with the numerical procedure of the modified crack closure integral (Rybicki, Kanninen, 1977) by which the strain energy release rates G (ϕ) have been calculated.

HEXAGONAL UNIT CELL WITHIN THE COMPOUND (h-uc-c)

In Fig. 7 a hexagonal unit cell within a compound of seven cells can be seen before and after the uniform temperature rise of $\Delta T=100°C$. The contour lines between the inner and outer unit cells of the deformed h-uc-c structure show no deviations from straight lines as it has been noticed in Fig. 4 for the single h-uc. This is a result of the compatible displacement fields of the inner and outer unit cells and has further the effect that neither the side lines nor the edges of the h-uc-c are free from stresses and strains, respectively. Especially, this can be noticed by the graphs belonging to the σ_{rrm} stresses and plotted against the radial lines of 0 and 30 degrees in Fig. 8 and 9, respectively. Consequently, the arising self-stress energy $\tilde{U}^{r,\phi}$ in the h-uc-c is higher than in the single h-uc, where the small amount of just 1.4 % was a surprising result. Furthermore, it can be stated that the analytical expression for $U^{r,\phi}$ of the c-uc delivers a very good approxima-

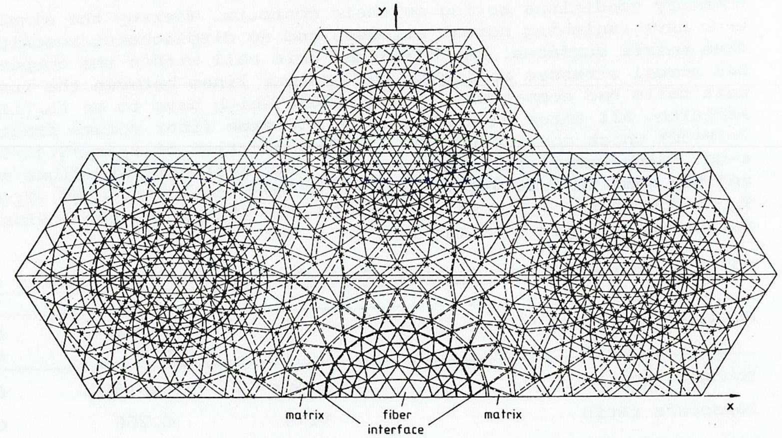

Fig. 7. Displacement field of a hexagonal unit cell within the compound (h-uc-c) (--- undeformed, ——deformed)

tion of the self-stress energy arising in the h-uc-c and consequently in a real unidirectionally fiber reinforced composite with $V_f/V=30$ %.

But looking at Fig. 6 it is striking that the curves belonging to the G_I, G_{II} and G (ϕ)-values of the h-uc-c show remarkable deviations from those of the c-uc and h-uc discussed previously. The maximum value of G(ϕ) which is due to $G_{max} \leq G_C$ of utmost interest for stable crack growth in fracture analysis is about 36 % lower than the maximum values calculated from the single c-uc and h-uc. The reason for this result obviously is the fact that the matrix of the h-uc-c is still connected to the surrounding unit cells in spite of the complete separation of fiber and matrix due to the interface crack. Therefore, a valuable part of the originally stored self-stress energy within the h-uc-c still remains in the matrix ($\approx$34 %, see Table 2) and is not releasable for the interface crack.

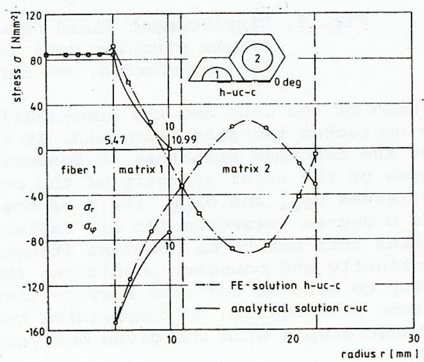

Fig. 8. Stresses within the h-uc-c along 0 degree line

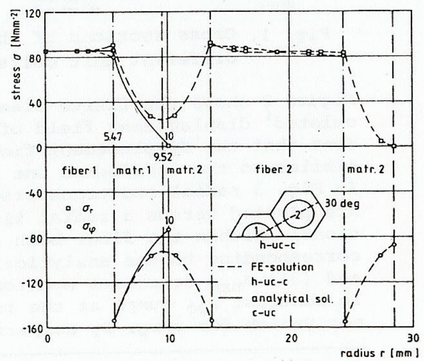

Fig. 9. Stresses within the h-uc-c along 30 degrees line

Moreover, Fig. 6 shows, in contrast to the results of the single unit cells, that the maximum values of $G_I(\phi)$ and $G_{II}(\phi)$ turn out to be of about the same magnitude for the h-uc-c.

CONCLUDING REMARKS

In order to obtain microstructural informations about the thermal fracture behavior of unidirectionally fiber reinforced composites different shaped unit cells as well as an ensemble of such unit cells under thermal loading were investigated. Thereby the corresponding mixed boundary-value problems of the plane stationary thermoelasticity have been solved by applying the finite element method. Further it could be shown that the self-stress energy stored in the considered three types of unit cells (c-uc, h-uc, h-uc-c) is rather independent of the geometric contour of the unit cells as well as of the corresponding boundary conditions, respectively. Finally, it was pointed out that the fracture analysis of single unit cells with one dominating interface crack leads to remarkably too conservative results concerning the maximum values of the energy release rates.

REFERENCES

Beaumont, P. W. R. and P. D. Anstice (1980). J. Material Science, 15, 2619-2635.

Braun, H. and K. Herrmann (1981). In D. Francois (Ed.), Advances in Fracture Mechanics, Vol. 1, Pergamon Press, London, 485-493.

Buchholz, F.-G., K. Herrmann and U. Strathmeier (1980). DVM-Bericht, 12. Sitzung Arbeitskreis Bruchvorgänge, Freiburg, 215-222.

Buchholz, F.-G. (1982). Lecture presented at GAMM - Wiss. Jahrestagung, Budapest.

Buchholz, F.-G. (1983). DVM-Bericht, 15. Sitzung Arbeitskreis Bruchvorgänge, Darmstadt, 31-40.

Cooper, G. A. and M. R. Pigott (1977). In D. M. R. Taplin (Ed.), Fracture, Vol. 1, University of Waterloo Press, 557-605.

Herrmann, K. (1970). In K. Schröder (Ed.), Beiträge zur Spannungs- und Dehnungsanalyse, VI, Akademie Verlag, Berlin, 21-39.

Herrmann, K. and H. Braun (1983). Eng. Fract. Mech., 18, 975-996.

Jäcker, K. P. (1983), Studienarbeit, Institute of Mechanics, University of Paderborn.

Murakami, H., A. Maewal and G. A. Hegemier (1979). Int. J. Solids and Structures, 15, 325-357.

Rosen, B. W., S. V. Kulkami and P. V. McLaughlin (1975). In C. T. Herakovich (Ed.), Inelastic Behavior of Composite Materials, ASME, AMD-13, New York, 17-72.

Rybicki, E. F. and M. F. Kanninen (1977). Eng. Fract. Mech., 9, 931-938.

Sendeckyj, G. P. (1974). Int. J. Fracture, 10, 45-52.

Sih, G. C. and co-workers (1973). In J. J. Witney (Ed.), Analysis of Test Methods for High Modulus Fibers and Composites, ASTM, STP-521, Philadelphia, 98-132.

Wu, E. M. (1979). In G. C. Sih and V. Tamuzs (Eds.), Fracture of Composite Materials, Sijthoff & Noordhoff, Alphen aan den Rijn, 63-76.