COMPRESSIVE FRACTURE OF GLASSY BRITTLE MATERIALS

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ABSTRACT

Pre-cracked blocks of PMMA (Perspex) and casting resin (Araldite) were compressed under a flat platen until fracture occurred. Three distinct failure modes were observed depending on the depth of the pre-crack and the width of the platen. The axial splitting failure mode is explained on the basis of the energy balance concept of fracture. The role played by the depth of the pre-crack and the width of the platen in controlling the failure mode transition from gross yielding to compression splitting and to buckling is highlighted.

KEYWORDS

Compression fracture; polymers; two-strut model; axial splitting; stress intensity factor.

INTRODUCTION

Recent papers by Kendall (1978) and Karihaloo (1979) questioned the use of compressive strength as the fundamental property of glassy brittle materials and sought to replace it with a more sensible parameter based on the Griffith (1920) energy criterion of cracking. It was argued that the idea of a compressive strength, i.e. constant critical stress for failure, was only useful when applied to failure by yielding. On the other hand, since brittle materials behaved in a bewildering variety of ways when compressed (Obert 1972) the notion of compressive strength became less than useful because compression cracking did not always take place at the same stress. There was thus a clear need for a better definition of fracture strength under compression which, unlike compressive strength, would not change however much the gross mode of failure altered. It was also noted that the compressive strength is sensitive to the arrangement of the compressing platens. Under wide platens the failure invariably occurs by cone formation, whereas if the platen is deliberately reduced in size, to become a punch, axial splitting dominates (Knight, Swain & Chaudri 1977).

Compression splitting has been employed in the past in a variety of forms for testing the strength of brittle materials. The disk or Brazilian test is an example of this, as is the indentation test. Apart from the difficulty of interpreting compression splitting observations in terms of strength, these tests are troublesome from the experimental standpoint because failure is invariably rapid and catastrophic. An obvious reason for catastrophic failure is the lack of a suitable initial flaw from which a crack might easily grow.

In the experiments reported here, sharp starting cracks were introduced into the test specimens so that slow compression splitting could then be studied, without having to wait for the nucleation of a crack due to plastic indentation (Hagan 1981). The compression split of the pre-cracked block of elastic materials is explained by a simple mechanical model. Two other modes of failure were also observed, namely gross yielding under the compressing platen, and elastic instability and buckling of the specimens. It is argued that compression splitting of glassy brittle materials can be advantageously used to study their fracture behaviour under compression.

SPECIMEN PREPARATION AND TESTING PROCEDURE

The specimen geometry is shown in Fig. 1. The specimens of glassy material, Perspex, were machined into a block from a sheet of rectangular section.

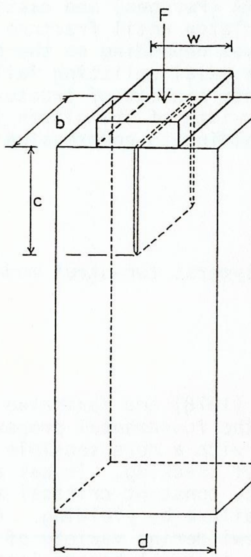

Fig. 1. Geometry of test specimens

Epoxy resin specimens were prepared by mixing Araldite resin and hardener (LC 177) in the ratio of 5:1 by weight. The specimens were cold cured at room temperature and stripped from the moulds after twenty-four hours.

The initiator cracks were cut into the specimens of both materials by a sharp circular saw. Propagation of this initiated flaw was induced by loading a flat steel platen onto the upper surface of the sample, taking care to make contact over the whole area of compression. The compressive force was applied concentrically and gradually at a nominal rate of 0.6 kN/sec until the specimen failed in one mode or another. For each pre-crack depth and platen width a minimum of three specimens were tested. The elastic modulus and yield stress of each of the materials were measured in three-point bending and compression on unnotched specimens.

TEST RESULTS AND DISCUSSION

Figure 2 shows the compressive force (F) at failure as a function of the pre-crack (c) and the platen width (w) for Perspex. These results were obtained

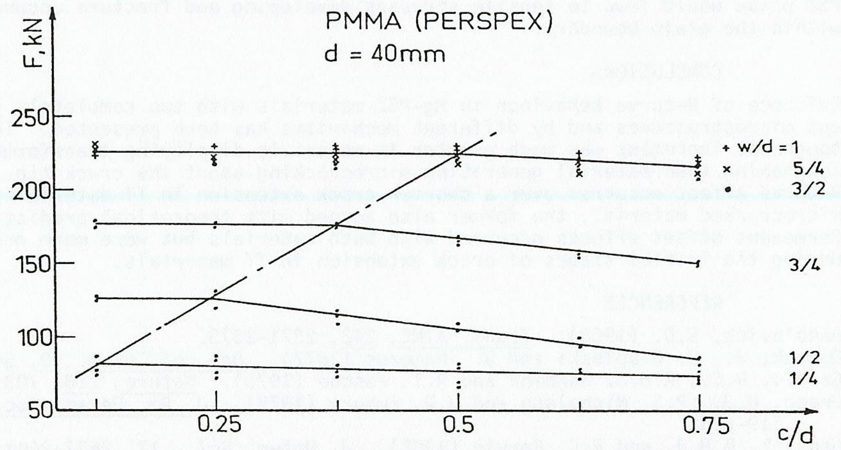

Fig. 2. Variation of compressive failure force with pre-crack (c) and platen width (w).

from specimens about 100 mm long and of square section (b = d = 40 mm). Further tests were performed on narrower Perspex specimens (d = 18 mm) which permitted a study of larger c/d ratios. The results of these additional tests are shown separately in Fig. 3.

In order to get a clearer picture of the three different observed modes of failure, Fig. 4 shows the average force values (as read off from Fig. 2-3) as a function of w/d for various pre-crack depths (c/d). In region I, i.e. when $w/d \leq \frac{1}{2}$, the predominant mode of failure was axial splitting for values of pre-crack depth $c/d \leq 3/2$. It was noticed that the compression split travelled neatly down the centre of the specimen, eventually splitting it into halves. There was noticeable buckling of the specimens with large pre-cracks ($c/d > 3/2$).

In region II ($\frac{1}{2} < w/d \leq 1$), the mode of failure was a combination of gross yielding and axial splitting. Compression splitting occurred only after substantial yielding under the platen. Indeed, no splitting was observed for small pre-cracks ($c/d < \frac{1}{2}$). In this region any attempt to use elastic theory would lead to very erroneous results (Kells and Mills 1982). When wider platens were used ($w/d > 1$), the specimens failed either by yielding under the platen ($c/d \leq \frac{1}{2}$) or by elastic instability and buckling ($c/d > \frac{1}{2}$).

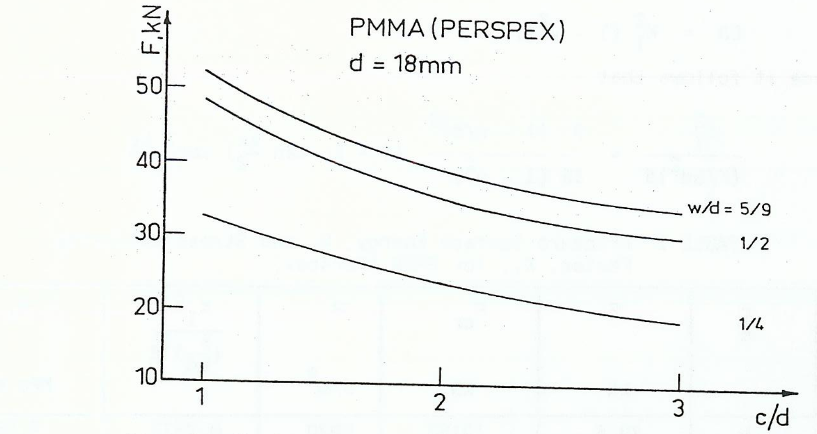

Fig. 3. Variation of compressive failure force with large pre-crack (c) and platen width (w).

These results fall into region III in Fig. 4.

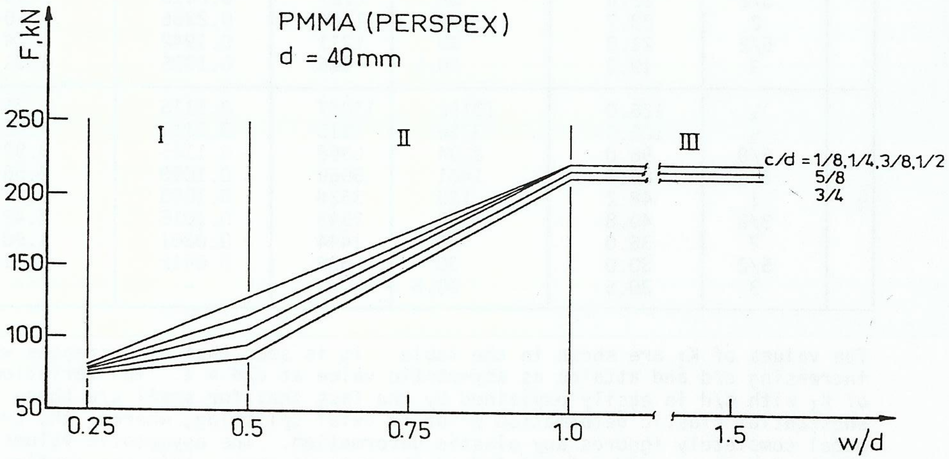

Fig. 4. Compressive force for various observed failure modes

For lack of space, no results have been reported here for Araldite. These will be discussed during the presentation of the paper. Experiments on Polymer specimens showed that for narrow platens ($w/d \leq \frac{1}{2}$) and sufficiently long initiating cracks ($1 < c/d < 3/2$) it was possible to propagate flaws steadily and controllably along the mid-plane of the specimens and to study this process until fracture. The polymer materials behave more or less elastically in this range with very little plastic deformation, particularly for longer pre-cracks. A model is proposed below to describe how an existing fissure of length c may be propagated by a force F applied compressively through the platen.

AXIAL SPLITTING MODEL

In the region I delineated above, the platen was sufficiently narrow ($w/d < \frac{1}{2}$) so that the influence of uncertain end conditions under the platen on the failure mode may be disregarded. The models proposed earlier by Kendall (1978), Karihaloo (1979), and Kells and Mills (1982) predicted that the compressive force causing axial splitting was independent of the pre-crack length, so that splitting should proceed at constant speed under a steady load. However, experimental evidence points to the contrary. It was observed that the cracks accelerated at constant load, exactly as Griffith cracks do under tension. It is believed that this drawback of the earlier models can be removed simply be recognising the following. The crack effectively divides the specimen into two short struts (Fig. 5(a)) which bend under compression to open up the central gap. However, as opposed to the earlier models where each strut was actually treated as a cantilever, subjected to a constant bending moment, it is proposed to take into account the variation of bending moment along the length of the strut.

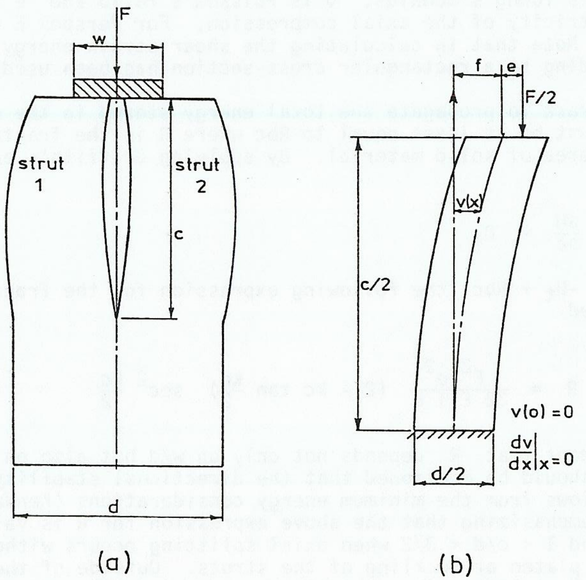

Fig. 5. Proposed two-strut model for axial splitting

The total energy U_t, i.e. the sum of strain energy in the elastic struts, potential energy in the load and surface energy of the crack is computed and then the Griffith energy conservation principle is applied as the crack grows an infinitesimal amount dc. The strain energy stored in each of the two struts is due to bending and shear. The shear component cannot be ignored since the crack is less than twice as long as the specimen width. Each of the two struts is regarded as simply supported and is subjected to an eccentric axial compression F/2 (Fig. 5(b)). It should be remarked that the crack faces did not touch each other for $c/d \leq 3/2$ and $w/d \leq \frac{1}{2}$, so that the closure force introduced by Kells and Mills (1982) does not arise. The crack faces did however come into contact under the platen for $c/d > 3/2$ and $w/d > \frac{1}{2}$, but these values are outside the range (region I) for which the present model is being proposed. Without going into detail, it can be shown that the bending strain energy stored in the struts U_b, the shear strain energy U_s, and the potential energy in the load U_f are given by:

$$U_b = \frac{F^2 B^2}{8 E I k} (kc + \sin kc),$$

$$U_s = \frac{0.3 F^2 B^2 k(1 + \nu)}{A E} (kc - \sin kc), \qquad (1)$$

$$U_f = -\frac{F k B^2}{2} (kc - \sin kc),$$

where $I = bd^3/96$, $A = bd$, $k^2 = F/2EI$ and $B = e/\cos(kc/2)$.

Here E is Young's modulus, ν is Poisson's ratio and $e = d(1 - w/d)/4$ is the eccentricity of the axial compression. For Perspex $E = 2.5$ GPa and $\nu = 0.4$. Note that in calculating the shear strain energy the shape factor corresponding to a rectangular cross-section has been used.

For the crack to propagate the total energy stored in the struts $U_t = U_b + U_s + U_f$ must be at least equal to Rbc where R is the fracture surface energy per unit area of solid material. By applying Griffith's energy balance equation

$$\frac{\partial U}{\partial c} = 0, \qquad (2)$$

where $U = -U_t + Rbc$, the following expression for the fracture surface energy is obtained

$$R = \frac{F^2 e^2}{32 E I b} \left(2 - kc \tan \frac{kc}{2}\right) \sec^2 \frac{kc}{2} \qquad (3)$$

It is evident that R depends not only on w/d but also on the crack length c/d. It should be mentioned that the directional stability of the compression crack follows from the minimum energy considerations (Kendall 1978). It is worth re-emphasizing that the above expression for R is valid only for $w/d \le ½$ and $1 < c/d < 3/2$ when axial splitting occurs without either yielding under the platen or buckling of the struts. Outside of these ranges there is substantial yielding under the platen and/or buckling of the specimens so that R does not reflect fracture surface energy alone. The values of R for various c/d and for two values of w/d are shown in Table 1 for Perspex. Also shown is the corresponding critical buckling force F_{cr} ($F_{cr} = 2\pi^2 E I/c^2$). It is evident from the values of F_{cr} that no appreciable buckling of the struts occurs in the range of values of c/d and w/d appropriate to region I. It should be pointed out that for $c/d \ge 1$ narrower specimens of Perspex were tested (d = 18 mm as against d = 40 mm for other values of c/d). The values of R are not reported if the mode of failure was buckling, i.e. $F > F_{cr}$.

If plane-strain conditions are assumed to prevail in the specimen, the fracture surface energy R may be re-written in terms of the stress intensity factor, K_I,

$$ER = K_I^2 (1 - \nu^2), \qquad (4)$$

whence it follows that

$$\frac{K_I^2}{(F/bd^2)d} = \frac{3}{16}\frac{(1 - w/d)^2}{(1 - \nu^2)} \left(2 - kc \tan \frac{kc}{2}\right) \sec^2 \frac{kc}{2} \qquad (5)$$

TABLE 1 Fracture Surface Energy, R, and Stress Intensity Factor, K_I, for PMMA (Perspex).

$\frac{w}{d}$	$\frac{c}{d}$	F kN	F_{cr} kN	R J/m^2	$\frac{K_I^2}{(\frac{F}{bd})^2 d}$	K_I MPa m^{½}
	¼	79.5	13152	9920	0.2511	4.98
	½	77.0	3288	9331	0.2514	4.43
	5/8	75.5	2104	8911	0.2497	4.72
	3/4	74.0	1461	8500	0.2481	3.61
¼	1	32.5	185	3648	0.2489	3.02
	3/2	28.8	82	2767	0.2410	2.63
	2	23.7	46	1764	0.2266	2.10
	5/2	21.2	30	1211	0.1942	1.74
	3	19.2	20.5	686	0.1336	1.31
	¼	126.0	13152	11067	0.1116	5.26
	½	106.0	3288	7815	0.1111	4.42
	5/8	96.0	2104	6368	0.1105	3.99
	3/4	86.0	1461	5069	0.1098	3.56
½	1	48.2	185	3528	0.1093	2.97
	3/2	40.8	82	2343	0.1015	2.42
	2	36.0	46	1444	0.0801	1.90
	5/2	30.0	30	586	0.0412	1.21
	3	30.5	20.5	-	-	-

The values of K_I are shown in the table. It is seen that K_I decreases with increasing c/d and attains as asymptotic value at c/d > 1. The variation of K_I with c/d is easily explained by the fact that for small c/d there is substantial plastic deformation prior to axial splitting, whereas the present model completely ignores any plastic deformation. The asymptotic values attained by K_I at c/d > 1 are indeed the correct ones; this was confirmed by three-point bend tests performed on both materials. These values are also in good agreement with those reported elsewhere in literature (Marshall et al 1973).

In conclusion it may be pointed out that fracture surface energy rather than compressive strength gives a better description of the fracture behaviour of polymers under compression. The compression splitting test provides a reliable method for determining the fracture surface energy of polymers, provided the compressing platen is sufficiently narrow ($w/d \le ½$) and the pre-crack fairly long ($c/d \ge 3/2$). In this range, the linear elastic fracture mechanics seems to predict reasonably accurate values of R (or K_I). Outside of this range the deformation is highly elasto-plastic for which an elastic-plastic fracture mechanics analysis is appropriate (Kells and Mills 1982).

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