STUDY OF ELASTIC FATIGUE CRACK GROWTH IN STAGE II USING THE SIMILARITY CRITERION OF LOCAL FRACTURE

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ABSTRACT

In this communication the I-similarity of fatigue fracture in stage II and cyclic fracture toughness criterion K_{IS} is proposed. The parameter K_{IS} corresponds to the threshold $\Delta K = K_{IS}$, at which the self-similar elastic-plastic transition at $I = I_*$ is effected under maximum plastic strain constraint factor.

KEYWORDS

Cyclic fracture toughness; strain energy density; crack tip; self-similar crack growth; similarity criterion; plastic strain constraint factor.

INTRODUCTION

The advent of the linear mechanics has made it possible to apply the similarity theory for solving the engineering problems of materials' fatigue. Thereby, one of the important problems is the comparison of the cyclic fracture toughness under conditions of the local fracture similarity. The purpose of this experiment is to compare cyclic fracture toughness of steel with that of titanium and aluminium alloys at the moment of the local plastic instability under cyclic loading.

THE CONCEPT OF THE CYCLIC FRACTURE TOUGHNESS

The idea of cyclic fracture toughness has been introduced by Yokobori and Aizawa (1970) due to analysis of the critical length of smooth specimens, tested for fatigue. To develop these studies it was proposed to estimate the cyclic fracture toughness parameter K^S_{I*} using the fatigue limit (σ_w) of smooth specimens and the length (l_s) of a stable crack (Ivanova, Maslov, Botvina, 1972):

$$K^{s}_{I*} = \sigma_w \sqrt{\pi\, l_s} \qquad (1)$$

On the other hand, the cyclic fracture toughness is usually estimated with the help of the threshold value K_{fc}, corresponding to the final fatigue fracture (Fig. I)

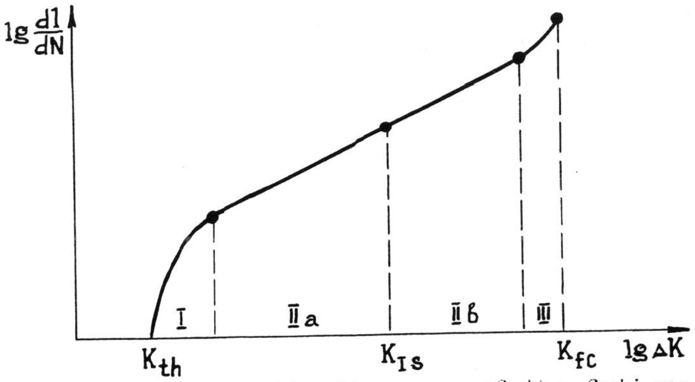

Fig. I. The kinetic diagramms of the fatigue crack growth.

In all the cases the conditions of similarity of local fracture for the accepted threshold stress-intensity factor should be provided in order to obtain the comparable cyclic fracture toughness for different materials. In order to analyse the fracture conditions for the local metal volumes near the tip of the fatigue crack developed, let us determine the critical dimensions of the self-similar precracking zones. In accordance with the methods of linear fracture mechanics the size of such zone is determined by maximum distance from the crack tip $r = r_c$ within which the strain-stress fields may be described with the help of a singular term $K / \sqrt{2\pi r}$. The maximum size of the self-similar precracking zone (r^{max}_c) at the static loading may be calculated with the help of equation (Ivanova, 1979):

$$r_c^{max} = \left[\frac{(K^R_{I*})_{max}}{\sigma_{11}}\right]^2 \cdot \frac{1}{2\pi} = \left[\frac{(K^R_{I*})_{max}}{\sigma_y}\right]^2 \cdot \frac{1}{2\pi} \qquad (2)$$

Here $(K^R_{I*})_{max}$ is the value of K, controlling the distribution of strains and stresses near the crack tip at the moment of the plastic instability under static loading.
The maximum size (r^{max}_c) of the self-similar zone is important parameter, controlling the maximum distance from crack tip r^{max}_{oc} at which a critical strain energy density $(\Delta W/\Delta V)_c$ is achieved. The conception of the critical strain energy density has been developed by G.Sih (1974). One of his basic concepts is that materials in the immediate vicinity of the crack tip, marked as the core region (Fig. 2) will behave differently from that of the bulk.

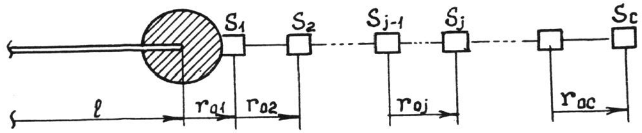

Fig. 2. Slow crack growth prior to rapid crack extension.

The amount of energy dw/dv stored in one of these elements becomes exceedingly large as r is made smaller and smaller reaching a limit on the boundary of the core region $r = r_o$. The critical value of the energy ΔW accumulated in the volume ΔV is denoted as $(\Delta W / \Delta V)_c$. The last increment of slow crack growth terminates at r_{oc} which corresponds to the ones of instability, i.e. when

$$S_c = r_{oc}\ (\Delta W/\Delta V)_c \qquad (3)$$

Here S_c is the strain-energy density factor; S_c is directly related to the critical stress intensity factor K_{IC} by the relation as follows (Sih, 1974)

$$S_c = \frac{1}{2\pi E}\cdot(1+\nu)\cdot(1-2\nu)\cdot K^2_{Ic} \qquad (4)$$

where ν is the Poisson coefficient, E is the elasticity modulus From eq. (4) and (3) follows that C.O.D. is the function of the critical strain energy density.
Basing on the concept about the constancy of a critical strain energy density it is possible for fatigue crack growth condition to write

$$S_1/r_{01} = S_2/r_{02} = S_3/r_{03} = \dots = S_s/r_{0s} = (\Delta W/\Delta V)_c = const \qquad (6)$$

thereby, taking into account (4) S_s this may be expressed as:

$$S_s = \frac{1}{2\pi E}\cdot(1+\nu)\cdot(1-2\nu)\cdot K^2_{Is} \qquad (7)$$

With its physical meaning r_{os} characterizes the maximum value of the core region which is realized during selfsimilar elastic-plastic transition at $\Delta K = K_{Is}$ (see Fig. I).
The crack growth rate per cycle (dl/dN) on stage II (Fig. I) is primarily controlled by the alternating stress intensity (ΔK) factor through an expression of the form (Paris, 1964):

$$dl/dN = C(\Delta K)^n \qquad (8)$$

Where C and n are scaling constants, If one takes into account the elastic-plastic transition it is possible to divide stage II into two stages: stage II-a characterising quasielastic selfsimilar crack growth at $\Delta K < K_{Is}$ and stage II-b characterised elastic-plastic crack growth at $\Delta K > K_{Is}$ (Gurevich, 1981). Recent studies

(Jarema, 1981; Toth and Nagy, 1982) have shown that there is correlation between C and n parameters as follows:

$$C = B/A^{n} \tag{9}$$

This made it possible to represent the relation between dl/dN-ΔK in the form:

$$dl/dN = B(\Delta K/A)^{n} \tag{10}$$

On the other hand under self-similar condition of the crack growth the parameter n are shown to be invariable to load condition. It is possible in this case to calculate K_{IS} value by use next equation (Ivanova, 1982)

$$K_{IS} = K_{IS}^{max}\left[\frac{n_{max} - n}{n_{max} - 2}\right] \tag{11}$$

Where n_{max} is maximum value of the parameter n up to mode I may be realised, K_{IS}^{max} is maximum value of the parameter K_{IS} at n = 2. It allows to calculate maximum value of the r_{os} for alloys on the given base by use following equation:

$$r_{os}^{max} = \frac{S_s^{max}}{(\Delta W/\Delta V)_c^{max}} = \frac{(1+\nu)\cdot(1-2\nu)}{(\Delta W/\Delta V)_c^{max}}\cdot(K_{IS}^{max})^2 \tag{12}$$

Dividing eq (2) on eq (12) we obtain:

$$\frac{(K_{I*}^{R})^{max}}{K_{IS}^{max}} = \sigma_y\sqrt{\frac{r_{oc}^{max}\cdot(1+\nu)\cdot(1-2\nu)}{r_{os}^{max}\cdot E\cdot(\Delta W/\Delta V)_c^{max}}} \tag{13}$$

According to this equation $f = (K_{I*}^{R})^{max}/K_{IS}^{max}$ characterises the static core region to the cyclic core region square root ratio. This ratio is constant for alloys on the given base as $(K_{I*}^{R})max$ = const and K_{IS}^{max} = const. It allows to use $(K_{I*}^{R})max$ and K_{IS}^{max} parameters as a parameters controlling cyclic and static core region size at elastic-plastic transition.

ANALYSIS OF DIMENSIONAL CONSTANTS, CONTROLLING THE SELF-SIMILAR GROWTH OF FATIGUE CRACK (STAGE II)

According to the theory of L.I.Sedov (1981) for realizing the self-similarity it is sufficient that the system of dimensional determining parameters, given by additional conditions, and, in particular, by the limiting and initial ones should contain not more that two constants with independent dimensions, differing from length and time.

Let us show, that during the self-similar growth of the crack such dimensional constants are A and B in relation (10), having the dimensions $[A] = P\,L^{-3/2}$ and $[B] = LT^{-1}$ are the constants controlling the self-similar growth of the crack in stage II. Since in stage II the crack moves in a single direction, its velocity depends only upon a single self-similar variable:

$$\lambda = x/bt^{\delta} \tag{14}$$

where x is the coordinate of the crack's tip, b is the constant equal to B and t is time, δ is coefficient ($\delta \neq 0$). Taking into account, that the fatigue crack moves discontinuosly it is convinient to consider the original as moving together with the crack; then λ may be represented as follows:

$$\lambda = \Delta l/(\Delta N\cdot b) \tag{15}$$

where Δl is the descrete increment of the crack in the direction of the x axis at the moment when a given number of cycles is accumulated, ΔN is the duration of propagation of the crack by Δl.

Then the rate of the crack's growth may be expressed in terms of the self-similar variable as

$$\Delta l/\Delta N = B\cdot\lambda \tag{16}$$

Here

$$\lambda = (\Delta K/A)^{n} \tag{17}$$

For the microscopic growth rate of the crack parameter n in the eq (8) is equal 2; then λ may be represented in the form as follows:

$$\lambda = (\Delta K/A)^{2} \tag{18}$$

Taking into account this equation, the microscopic rate may be represented in the form as follows:

$$\Delta l/\Delta N = B(\Delta K/A)^{2} \tag{19}$$

Then, for a threshold value $\Delta K = K_{IS}$ we shall obtain:

$$(\Delta l/\Delta N)_s = v_s = B(K_{IS}/A)^2 = B\cdot\lambda_s \tag{20}$$

and

$$(K_{IS})_1/(v_s^{1/2})_1 = (K_{IS})_2/(v_s^{1/2})_2 = \cdots = A/B^{1/2} = const \tag{21}$$

Here $(v_s)_1$; $(v_s)_2$ etc. are the threshold rates, corresponding to $(K_{IS})_1$; $(K_{IS})_2$ etc. for the alloys 1,2,3 etc. Relation (21) implies, that for these alloys the dependence between K_{IS} and $v_s^{1/2}$ is a linear one with the slope tangent equal to $A/B^{1/2}$.

The regularities established made it possible to include into the system of determining parameters, controlling the boundary of a self-similar elastic-plastic crack growth in stage II for the limiting case, related to the plastic instability the parameters and dimensional constants are as follows:

$$A;\ B;\ \Delta l;\ \Delta N;\ \Delta K;\ \sigma_y;\ (K_{I*}^{R})_{max};\ K_{IS}^{max};\ \Delta$$

Here Δ including main elastic and thermodinamical constants, determining the ability of the alloy material to accumulate the elastic distortion energy. For alloys on the given base Δ is constant (Ivanova, 1979). Then only four parameters ΔK, Δl, ΔN and σ_y remain with three independent dimensions from which only one dimensionless combination may formed as follows:

$$I = \frac{\Delta K\cdot\Delta N^{-1/2}}{(\Delta l/\Delta N)^{1/2}\,\sigma_y}\cdot\frac{(K_{I*}^{R})_{max}}{K_{IS}^{max}}\cdot\Delta \tag{22}$$

When ΔK is equal K_{IS} then the striation spacing is $S_* = \Delta l$ at $\Delta N = 1$ and $I = I_*$:

$$I_* = \frac{K_{IS}}{S_*^{1/2}}\cdot\frac{\Delta}{\sigma_y}\cdot f \tag{23}$$

Since for alloys on the given base $K_{IS}/S_*^{1/2} = A/B^{1/2}$ = const; Δ = const; f = const (see Table I) then the product $I_* \cdot \sigma_y$ = = const. Table I gives the values of $I_* \cdot \sigma_y$ while Fig. 3 shows I_* versus σ_y .

TABLE I The Values of the Parameters, Controlling of the Elastic-Plastic Transition in stage II

Alloy basis	A MPa·m$^{1/2}$	B m·cycle^{-1}	Δ	f	$I_* \cdot \sigma_y$ MPa
Fe	27.2	I IO^{-7}	0.II	1	9467
Ti	I7.7	2.3 IO^{-}	0.I2	$\Delta^{-1/4}$	7605
Al	5.I	5.I IO	0.22	$\Delta^{-1/2}$	I0633

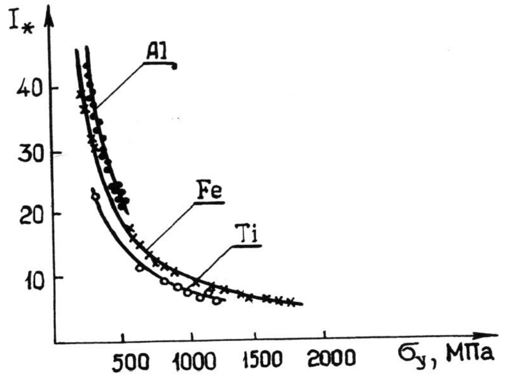

Fig. 3. I_* criteria versus σ_y for steels, aluminium and titanium alloys.

One can see, that despite the difference between the crystalline structure of materials, their chemical composition, strength level and other factors the uniform dependence $I_* = f(\sigma_y)$ is observed at $2 \leq n \leq 4$ where n = 4 characterizes the upper limit up to f = const.

CONCLUSION

The analysis carried out has revealed the regularity, according to which during the self-similar elastic-plastic transition the

product of the critical number I_* and the yield point of this material is the constant value.

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