MULTIAXIAL FATIGUE LIMIT: A NEW APPROACH

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ABSTRACT

A 3-D fatigue limit criterion based on evaluation of microscopic steady state stresses in presented. This criterion has been checked by tests done by IRSID and Renault. Applications have been done to predict fracture or safety of industrial structures undergoing 3D complex loadings.

KEYWORDS

3D fatigue limit criterion ; initiation of cracks, macroscopic-microscopic ; shakedown state.

INTRODUCTION

Fatigue failure is the most frequent and serious incident for a mechanical structure. It corresponds to a sequence of complex phenomena : strain hardening, local damage and creation of microcracks (initiation), then propagation which leads to final fracture. During the last few years, a great deal of research has been carried out on low cycle fatigue. However, high cycle fatigue remains a fundamental field of investigation for mechanical industrial applications. In particular the much studied problem of the fatigue limit under multiaxial loadings has not yet to date been satisfactorily resolved. There is no existing criterion which satisfies the following requirements.

1. The fatigue limit criterion is independent of the choice of coordinate system.
2. The criterion is characteristic of the loading path : the local stress $\sigma(t)$ or strain $\varepsilon(t)$ which are at the origin of damage should be known by the method at any time, t. In the classical approaches (see for instance ref. [1]), the characterization of the cycles by equivalent stress or strain range are not sufficiently precise and are therefore the source of many ambiguities.

3. It should correlate as well as possible all loading paths for which experimental results exist.

We present here a new method initially proposed by Dang Van [2] which is well verified by test results. Furthermore, this method has already been successfully used in industrial applications involving three dimensional complex loading.

GENERALITY ON THE MODEL

Fatigue crack initiation generally occurs in regions of stress concentration. It is a microscopically localized phenomenon which can be observed in some plastically deformed grains. At this scale the material is neither isotropic not homogeneous and the local responses (σ, ε^P, ...) can be quite different from the macroscopic corresponding quantities (Σ, E^P, ...) considered by engineering-type calculations. The quantities we mesure are already "filtered" by the macrovolume element corresponding for instance to the dimension of the strain gauge. In our approach, local microscopic variables in apparent steady state are used to postulate the initiation criterion ; a method to compute these variables is given from the usual engineers'parameters.

The different calculations to be done are summarized in figure 1. The first step is solved by using for example the finite element method. The second step which corresponds to "macroscopic to microscopic passage" is solved by an approximate modelling. For this purpose, it is necessary to distinguish two scales (figure 1) :

- A "macroscopic" scale characterized by an elementary representative volume V(x) surrounding the point x. It is the usual engineer's scale and the corresponding parameters Σ_{ij} ... are assumed to be homogeneous in V(x).
- A "microscopic" scale of the order of grain size which corresponds to a subdivision of V(x). The microscopic parameters σ, ε_p, ... are not homogeneous from one grain to an other and they are different from Σ and E_p. In particular, even if mean value of σ equals Σ, the local stress can fluctuate. More precisely,

$$\sigma(y,t) = A(x,y)\ \Sigma(x,t) + \rho(y,t) \qquad y \in V(x)$$

In the above equation, $\rho(y,t)$ is the residual stress field and A(x,y) is the elastic localization tensor [3]. For the sake of simplicity we put A(x,y) equal to identity.

Near the fatigue limit the applied stresses are low and it is reasonable to suppose that σ tends toward a pseudo shake down state (strictly shake down corresponds to no damage). Then Melan's theorem states that their must exists a residual stress state $\rho^*(y)$ independent of time t such that

$$\sigma(y,t) = \Sigma(x,t) + \rho^*(y) \qquad \forall\ y \in V(x)$$

We propose an initiation criterion in terms of the microscopic steady state stresses :

Failure occurs if $f(\sigma(y,t)) \geqslant 0$ for least at one instant t of the cycle.

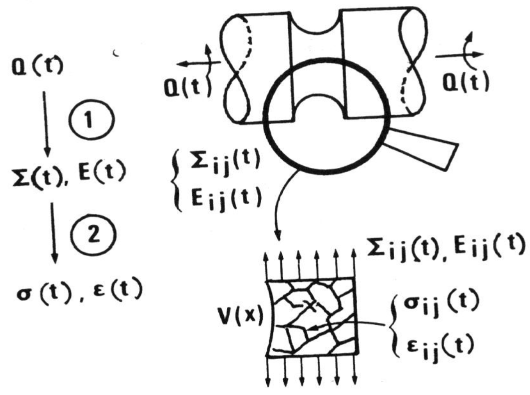

Fig. 1.

MACROSCOPIC-MICROSCOPIC PASSAGE

Evaluation of shake down state is a difficult problem which has not yet received general solution. The simplified method proposed by Zarka et al [4] could be very helpful in the future.

Generalizing Orowan's model [5], Dang Van [2] has given a method to estimate local stress acting on misoriented microelements and in particular the local shear stress $\tau(t)$. Some physical assumptions are needed.

- microelements undergo the macroscopic deformation E_{ij}. This is reasonable since the plastic strain E^P is negligeable at the fatigue limit.
- microscopic strains show isotropic hardening.

It can therefore be demonstrated that under cyclic deformation a residual stress state $\rho(x,t)$ appears which tends toward a stationary state $\rho^*(x)$.

These residual stresses cause shake down and contribute to symmetrization of the local shear stress τ as shown in fig. 2.

When only one gliding system (defined by $\vec{n}$ normal to the gliding plane, and $\vec{m}$ the glide direction) is active, the results are :

$$\sigma_{ij}(t) = \Sigma_{ij}(t) - 2\ \alpha_{ij}\ T_0$$

where $\alpha_{ij} = \frac{1}{2}\ (n_i m_j + m_i n_j)$

and $T_0 = \frac{1}{2}\ \alpha_{ij}\ [\ (\Sigma_{ij})_{max} - (\Sigma_{ij})_{min}\ ]$

T_0 is the mean shear stress acting on the gliding plane.

For any time t, the shear stress is :

$$\tau(t) = \alpha_{ij}\ \sigma_{ij}(t) = \alpha_{ij}\ \Sigma_{ij}(t) - T_0$$

Here T_0 is the shake down residual shear stress.

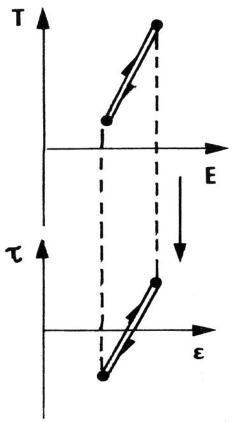

Fig. 2.

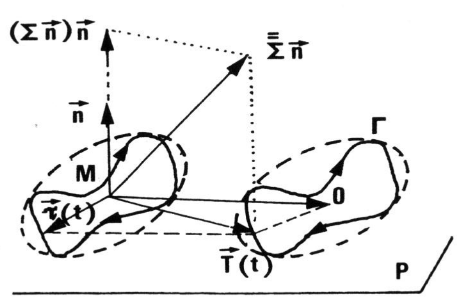

Fig. 3.

Thanks to the remark on the symmetrization of the shearing cycles, the method can be extended to any arbitrary loading. Let $P(M,\vec{n})$ be the physical plane associated with a point M in the critical zone which undergoes a tangential stress $\vec{T}(t)$ (fig. 3). For a sequence of cyclic loading, the extremity of vector $\vec{T}(t)$ follows a closed curve Γ in the P plane, the diameter of which is the amplitude of the shear stress. To find the residual stresses $\rho^*(x)$ leading to symmetry of the cycle, the following method may be used.

1. The smallest circle surrounding Γ is found.
2. Let o be the center of the circle. The local shear stress $\vec{\tau}(t)$ is deduced from $\vec{T}(t)$ by the vector $\vec{OM}$.

The physical meaning of this method is the following : the local plastic yield condition can be written $|\vec{\tau}(t)| \leqslant R$, where R is the yield limit in the stabilized state so that $\vec{\tau}(t)$ has to be inside the circle of radius R.

The remaining problem is the choice of the plane ; usually this choice depends on the chosen criterion. For the particular criterion that we propose, the right plane is most often the one in which the amplitude of the macroscopic shear stress is maximum. For a plane problem, this amplitude could easily be derived through the method of Mohr's circles.

FATIGUE INITIATION CRITERION AND APPLICATIONS

The use of local parameters to write the fatigue criterion allows a good description of the main features of the cyclic loading. As the initiation happens in the gliding strips, the shear stress τ(t) acting on this plane is an important parameter. Hydrostatic tension $p = \sigma_{kk}/3$ is another important parameter in opening of the cracks. Thus we propose a linear relationship between τ and p . For example (cf. ref [2])

$$|\tau| + ap - b = 0$$

where a and b are positive constants. For such a criterion the endurance domain is limited by the two symmetric straight lines D and D' in τ , p space.

For a plane problem with one gliding system if the curve (τ(t), p(t)) intersect D or D' fracture could occur. For a general 3D problem, it is more convenient to plot $(|\vec{\tau}(t)|, p(t))$. Such a curve is represented in dotted line on figure 4. If this curve intersect D, then fracture will occur.

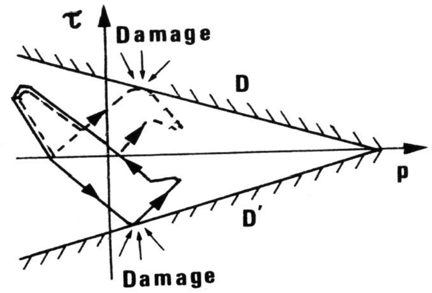

Fig. 4.

In view of automatic computation, it is possible instead of $\vec{\tau}(t)$ to use a derived octaedral shear stress $J^*(t)$ defined as follows.

(1) Choose a fixed material coordinate system.

(2) Compute

$$\Delta J_2 = \underset{t_1}{\text{Max}} \left[\underset{t_2}{\text{Max}} \left[J_2(\Sigma_{ij}(t_2) - \Sigma_{ij}(t_1)) \right] \right]$$

where $\Sigma_{ij}(t_\ell)$ = stress component at time t_ℓ.

(3) $J_2^*(t)$ is defined by :

$$\tau = J_2^*(t) = J_2 \left[\Sigma_{ij}(t) - \Sigma_{ij}(t_1) \right] - \frac{\Delta J_2}{2}$$

- Here, $\frac{\Delta J_2}{2}$ is a measure of the shake down equivalent residual stress.

The criterion has been checked on three different steels at IRSID : an E.36 grade, a 35 CD 4 grade and a rail steel UIC 70. Four different cyclic loadings were used :

- plane and rotative bending ;
- uniaxial or biaxial tension with different mean stresses ;
- torsion with different mean values ;
- in phase bending and torsion.

Test results are represented in the (p,τ) diagram on figures 5, 6 and 7. For E36 and 35 CD 4 steel, the limited life curves (10^5 and 10^6 cycles) are plotted as well as the fatigue limit (10^7 cycles). It could be seen that the different tests results are in good agreement with the proposed criterion. Similar agreement was found for other metals tested in Renault's Laboratory.

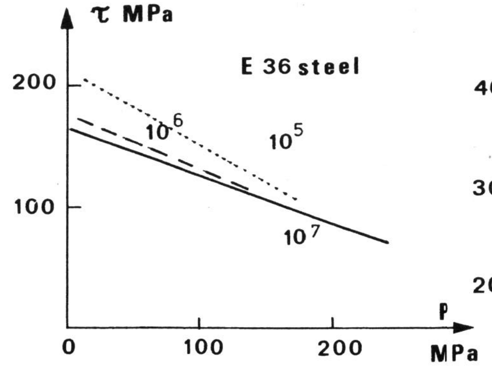

Fig. 5.

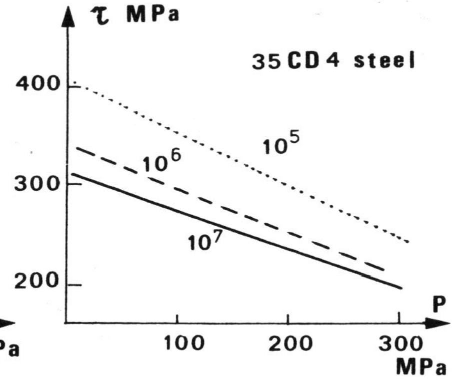

Fig. 6.

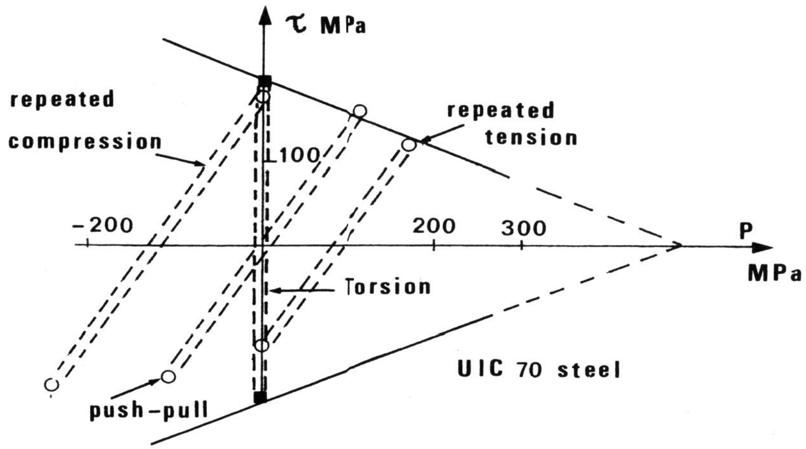

Fig. 7.

The criterion has also been used for real structures under complex loadings. The bending and torsional stresses measured on the hot spot of a crank-shaft of an engine are shown in figure 8. Predictions of the model for this loading was checked by the tests on the real structure. The limiting straight lines represents fatigue limit obtained by Laboratory tests on smooth specimens. When the criterion is reached (> 5500 rev/min) fatigue failure occurs as predicted, figure 9.

This method has also successfully been used to study the rolling fatigue problem [6].

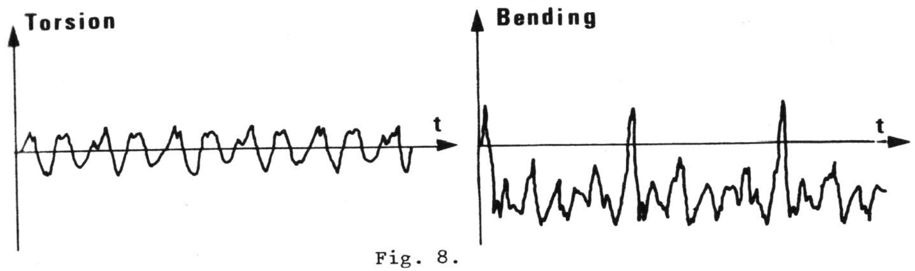

Fig. 8.

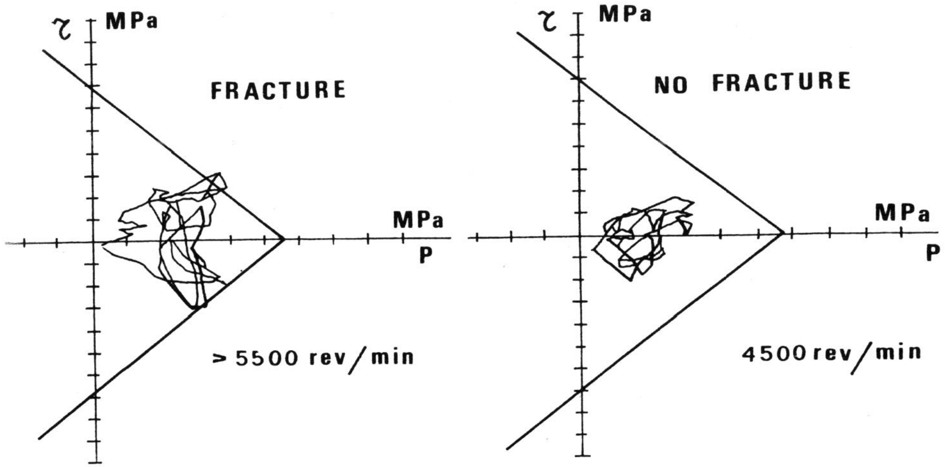

Fig. 9.

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