

FATIGUE STRENGTH ESTIMATION INCORPORATING RESIDUAL STRESSES: COMPARISON OF DIFFERENT FATIGUE CRITERIA

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ABSTRACT

Attempting to take into account the residual stresses in an estimation of fatigue strength leads to the problem of the calculation criterion. The state of stresses under load in fatigue may be monoaxial, while residual stresses are always multiaxial. If the designer of a part containing residual stresses wants to evaluate its fatigue strength, he must be able to include all stresses in his calculation.

Fatigue tests and measurements of stabilized residual stresses on induction-hardened parts, have shown that the Mises criterion currently used elsewhere does not correctly describe the true behaviour of a material. The criteria of CROSSLAND derived from the Mises criterion, which take into account hydrostatic pressure and the DANG VAN criterion based upon maximal alternate shearing and hydrostatic pressure, give a better correlation with experimental results.

KEYWORDS

Fatigue criterion, residual stresses, multiaxial stresses state.

INTRODUCTION

Although it is now known how to calculate the fatigue strength of part or a structure which is subjected to a uni-axial loading, the same is not true for the more general case when the applied loading is multi-axial and the part is subjected to a complex static stress condition resulting from loadings or residual stresses introduced during manufacture : there is in fact no recognised general method of calculation when the applied cyclic loadings are multi-axial.

However, simplified methods exist which allow an approximate calculation to be carried out. They generally consist of the use of plasticity criteria such as that of Von Mises, and their extrapolation in fatigue. Although these methods, in simple loading, can result in a satisfactory approximation of the fatigue behaviour of the structure or the part, they are quite unable to take into account complex loading, and other criteria must then be used.

In this study, it is therefore intended to examine the various possibilities which are offered to the engineer either for the calculation of the strength of a part which is subject to multi-axial fatigue loadings, or for taking residual stresses into account.

From the results of residual stress measurements and fatigue tests carried out on round bars of XC42 steel, superficially induction-heated and hardened it has been possible to demonstrate the inconvenient features of the MISES criterion, and the usefulness of those criteria which include the effect of hydrostatic pressure and the local shearing.

INFLUENCE OF RESIDUAL STRESSES ON FATIGUE LIFE IN THE CASE OF SURFACE TREATMENT

The macroscopic aspect of residual stresses means that the fundamental equations of elasticity can be applied to them, in particular the theory of superimposition. This means that, if a part is subjected to a residual stress field σ_R and to an operating stress σ_S , the part is in effect subjected to the loading $\sigma_S + \sigma_R$ (fig. 1).

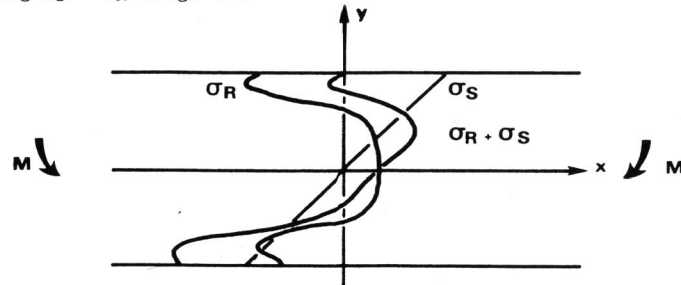


Fig. 1 : superimposition of residual stresses and operating stresses

With regard to the strength of a structure in service, a residual stress will thus behave in the same way as a static or mean stress. The influence of the residual stress on the fatigue strength can then be quantified by using a HAIGH or GOODMAN endurance diagram, in which the mean stress for the fatigue loading σ_m is replaced by the new mean stress $\sigma'_m = \sigma_m + \sigma_R$ (fig. 1 and 2).

If $\sigma_R > 0$ (as in the case of machining stresses due to grinding or turning), the alternating stress which the part can accept will be reduced. On the other hand, if $\sigma_R < 0$ (as in the case of residual stresses produced by shot-peening, hammering, carburizing or induction hardening), the fatigue limit will be improved (fig.2).

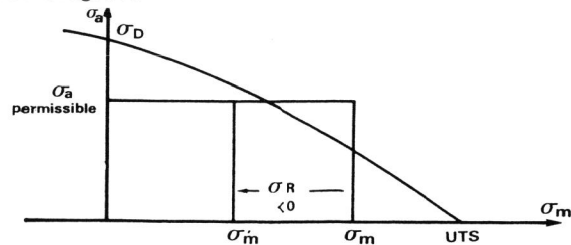


Fig. 2 : Use of the HAIGH diagram with residual stresses present

When taking the residual stresses into account in a calculated prediction of fatigue life, the following considerations must be born in mind :

- When the residual stresses are subjected to cyclic loading they are greatly reduced. Depending on the applied loading, the type of treatment which produced the residual stresses, and the material concerned, this reduction may be total or partial. It is therefore important to know, for the part being considered, the nature of the reduction, in order to take into account the stabilised residual stress in the calculation. In order to estimate the reduction in the residual stresses, it is necessary to make measurements after fatigue testing, or to consult some existing published results.

- In practice, an uni-axial residual stress cannot exist. Any surface treatment which generates residual stresses in a mechanical part will produce a bi-axial residual stress field at the surface and a tri-axial field in the sub-layers. If it is then wished to superimpose this complex residual stress field on to an uni-axial alternating loading, the application of a fatigue criterion is necessary. The use of only the residual stress which lies in the same direction as the fatigue stress can only be an approximation.

- The residual stresses are not the sole factor to be taken into account. A surface treatment, of whatever kind, will also result in a change of :

- . the roughness
- . the superficial metallurgical structure
- . the true loading, by the creation of residual stresses.

These three factors must thus be taken into account in fatigue calculations.

RESULTS OF FATIGUE TESTS AND MEASUREMENTS OF RESIDUAL STRESSES ON CYLINDERS INDUCTION-HEATED AND HARDENED

The fatigue tests involved repeated plane bending applied to cylinders, 36 mm in diameter made from XC42 steel, induction-heated and hardened. To eliminate the influence of the surface finish, the test specimens were machined by fine turning, to give a surface roughness before treatment of 5 to 7 microns Rt (total height from highest peak to lowest valley). For each treatment examined, measurements of the residual stresses were made on test specimens subjected to $5 \cdot 10^6$ fatigue cycles, with an applied loading which corresponded to the fatigue limit for each treatment. Since the fatigue failures were initiated at the surface of the test specimens, the measurements were made at the surface. It is, in fact, the surface stress condition which, in this case, controls the fatigue strength. The measurements were made by means of X-rays. For each test specimen examined, the longitudinal residual stress (σ_{R1}) and the tangential residual stress (σ_{Rt}) were measured. (see table I).

In order to carry out a calculation of the fatigue strength in which the residual stresses can be taken into account, it is necessary firstly to know the fatigue strength of the steel forming the hardened case, in the absence of residual stresses. For this reason, repeated tensile fatigue tests were carried out on test specimens in XC42 steel, uniformly hardened to 52 HRC, and approximating to the hardness of the treated case, obtained by induction hardening (54-56 HRC). The ultimate strength of these uniformly-treated test specimens was then determined by means of a tensile test. From these results, it was possible to draw the approximate endurance diagram (HAIGH type), which is probably close to the fatigue behaviour of a case obtained by induction hardening, with no residual stresses present.

Table I : below are summarised the various results obtained for the four treatments

Ref. of treatment	Type and depth of treatment for 45 HRC	surface hardness HRC	Fatigue limit at 5.10^6 cycles in MPa		Stabilised residual stress at the fatigue limit in MPa	
			σ_m	σ_a	σ_{R1}	σ_{Rt}
A	2,7 mm (induction)	55-56	596	584	- 128 - 243	- 468 - 571
B	4,2 mm (induction)	55-56	623	610	- 273 - 341	- 583 - 676
C	4,7 mm (induction)	54-59	670	660	- 655	- 603
D	3,5 mm Water-quenched after heating without stress relief annealing	60-61	780	750	- 863 - 777	- 1132 - 1156

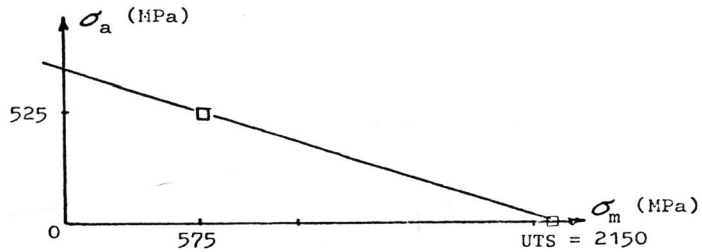


Fig. 3 : HAIGH endurance diagram for XC 42 steel with a hardness of 52 HRC

CRITERIA DERIVED FROM THE MISES PLASTICITY CRITERION

As for incipient fatigue cracks, plastic deformation originates with shearing on crystallographic planes. It would thus appear correct to look for criteria which would attempt to take into account the maximum shear stress produced by the fatigue loading(s). The MISES criterion, which only uses this octahedral stress, takes a mean value of the shear stress on differently-orientated grains and therefore gives fairly good results in the case of pure alternating radial loading.

KIOCECIOGLU criterion [1]

KIOCECIOGLU [1] has proposed the extension of the MISES criterion to fatigue for multi-axial loadings alternating about a mean value.

In the case of fatigue cracks initiated at the surface, the stress condition is bi-axial, and σ_{eqa} and σ_{eqm} are then calculated from the relationships :

$$\sigma_{eqm} = \pm (\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m} \cdot \sigma_{2m})^{1/2} \quad (1)$$

$$\sigma_{eqa} = \pm (\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a} \cdot \sigma_{2a})^{1/2} \quad (2)$$

The alternating stress obtained on a 5.10^6 cycles fatigue test (endurance limit) was plotted on the HAIGH diagram for each equivalent mean stress (fig.4). A very considerable scatter in the test results was noted, particularly in the case of those treatments which generate the highest residual compressive stresses.

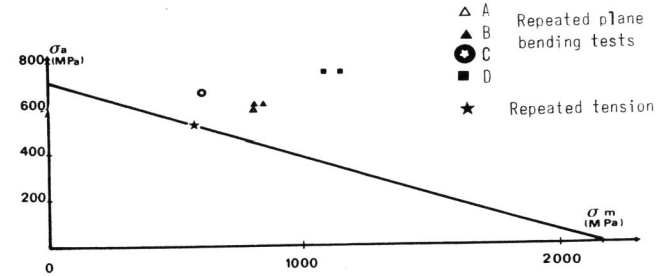


Fig. 4 : Comparison of the experimental results obtained by the MISES criterion with the reference endurance diagram.

The MISES criterion cannot therefore be used where there are high residual stresses or high mean stresses. On the other hand, it seems to be suitable for purely alternating combined loadings.

SINES criterion (2)

Starting from the finding that a mean torsional stress has only a minor influence on the fatigue strength, SINES (2) has proposed a criterion which takes into account the effect of the different combinations of mean and alternating stresses. The criterion is a relationship between the alternating octahedral stress and the nominal mean octohedral stress :

$$\tau_{octa} + \alpha \sigma_{octm} = \beta \quad (3)$$

The constants α and β are determined by fatigue tests for two values of the mean stress.

If the equivalent alternating stress in the MISES criterion sense and the mean hydrostatic pressure (p_m) are used instead of the normal mean octahedral stress ($\sigma_{oct} = p_m = \frac{\sigma_{1m} + \sigma_{2m} + \sigma_{3m}}{3}$) the following equation may be written

$$\sigma_{eqa} \text{ (MISES)} + \alpha' \cdot p_m = \beta' \quad (4)$$

CROSSLAND criterion (3)

CROSSLAND has proposed a criterion similar to that of SINES, but in which the maximum rather than the mean hydrostatic pressure is used. The criterion is written :

$$\tau_{octa} + \alpha \cdot \sigma_{oct \max} = \beta \quad (5)$$

If the equivalent MISES stress and the hydrostatic pressure p_{max} are used, the relationship (5) becomes :

$$\sigma_{eqa} + \alpha' \cdot p_{max} = \beta' \quad (6)$$

If the relationships (4) (6) are used to take the experimental results examined into account, figure 5, below, is obtained.

This representation provides a good correlation between the fatigue strength and the residual stresses. The CROSSLAND criterion, as for that of SINES, clearly shows the improvement in the fatigue strength when the residual compressive stresses are increased. With these experimental results, an approximately linear limiting curve is obtained for the two criteria, well justifying a relationship of the type (4) or (6). The repeated tensile test without residual stresses lies, on the limiting curve for the criterion. The difference between the two criteria reside in the value of σ_a corresponding to the pure alternating torsion ($p_m = p_{max} = 0$)

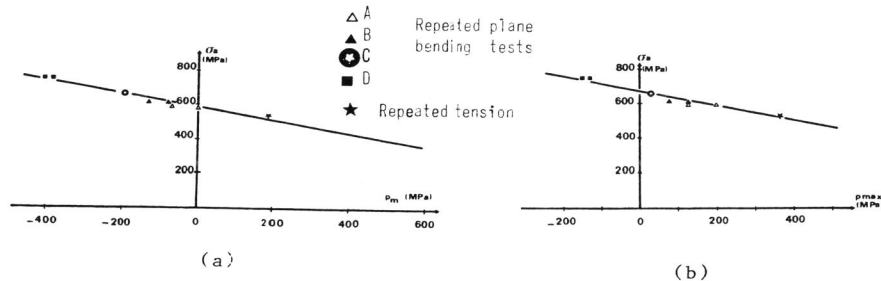


Fig. 5 : Application of the experimental results with the SINES criterion (a) the CROSSLAND criterion (b).

FATIGUE CRITERIA MAKING USE OF THE MAXIMUM SHEAR STRESS

Criteria of FINDLEY - NAKANISHI - YOKOBORI - MATAKE - STANFIELD (4)

These various authors have separately proposed a criterion of the same type, making use, for the plane of maximum shearing, of :

- the alternating shear stress τ_a acting on this plane
- the normal shear stress σ_n acting on this plane

The criterion may then be written :

$$\tau_a + B \cdot \sigma_n < A \tag{7}$$

The result obtained by means of this criterion on surface-hardened test specimens are plotted in figure 6(a).

The distribution of the experimental results reveals no linear correlation between the fatigue strength and the maximum normal stress because for pure alternating torsion ($p_{max} = 0$) the value of τ_a will be too high relating to the real value.

DANG VAN criterion (5)

The limiting curve for this criterion can be represented by a linear relationship of the form :

$$\tau_a + \alpha p_{max} = \beta \tag{8}$$

The constants α and β are determined from reference fatigue tests. If, for example, the fatigue limits in pure alternating tension (σ_D) and in pure alternating torsion (τ_D) are known, the relationship (8) may be written :

$$\tau_a + 3 \left(\frac{\tau_D}{\sigma_D} - \frac{1}{2} \right) p_{max} = \tau_D \tag{9}$$

The results obtained, using this criterion, on surface-hardened test specimens are plotted on figure 6(b). In the case of simple fatigue tests, as for plane bending for example, the DANG VAN criterion gives the same results as the CROSSLAND criterion.

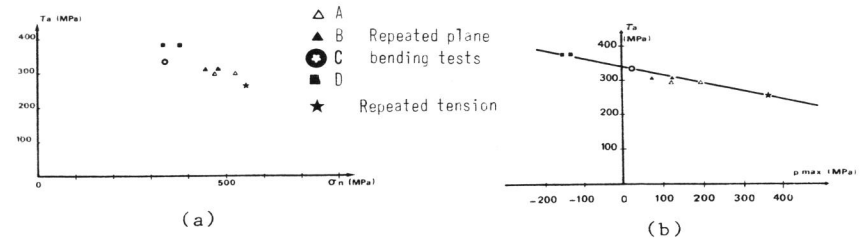


Fig. 6 : Application of the experimental results using the FINDLEY criterion (a), the DANG VAN criterion (b)

CONCLUSION

Several criteria which may be used for the prediction of fatigue behaviour in complex stress conditions have been examined. From the results of fatigue tests and the measurements of residual stresses carried out on test specimens in XC42 steel superficially induction-hardened, it has been possible to draw the following conclusions :

- The MISES criterion cannot be used with loading which include mean or residual stresses. This criterion is acceptable only in the case of purely alternating loadings.
- The criteria of CROSSLAND and the DANG VAN criteria give a good estimation of fatigue strength when incorporating residual stresses. These criteria are similar and must be utilised in nonproportionnal loading after some modification.
- The comparison of the FINDLEY and DANG VAN criteria shows that the fatigue behaviour of the parts studied is dependent on the maximum hydrostatic pressure, rather than the stress normal to the plane of maximum shear.
- The results presented and the criteria examined do not include the effect of the stress gradient (fatigue stress gradient or residual stress gradient) In attempting to predict the fatigue strength of parts with residual stresses and, for example, stress concentrations, other criteria should be used (6).

ACKNOWLEDGEMENTS

The authors are grateful to P. Barbarin, F. Convert, and B. Miede for residual stress measurements using X-ray techniques.

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