FATIGUE GROWTH OF PART-THROUGH CRACKS: A NEW APPROACH

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ABSTRACT

The most common flaws in structural components are part-through surface and internal cracks of more or less elliptical or semi-elliptical shape. For some years the practical importance of such cracks has been recognized, especially in the context of fatigue life prediction. The new procedure presented hereafter facilitates the use of stress intensity K (and in particular its double amplitude ΔK) as a tool to estimate the fatigue life of cracked plates under tensile load. This is made possible by determining a set of parameters so as to obtain equal ΔK values and analogous crack propagation rates in a plate and in a standard (e.g. CT) specimen. Comparisons of cycle numbers obtained with equivalent crack growth in both types of specimens indicate that, in using the proposed procedure, fatigue life predictions taken from CT specimens are normally on the safe side.

KEYWORDS

Three-dimensional crack problems, fatigue, specimen to structure correlation, vessels.

TREATMENT OF 3-DIMENSIONAL PROBLEMS

The evaluation of the stress intensity factor K along the crack front of a part-through crack is a complex 3-dimensional problem. The main topic of the present paper is the fatigue analysis of such cracks by means of readily available fracture mechanics concepts. Figure 1 shows a cross section in the plane of the crack and illustrates the local application of the crack propagation formula

$$\frac{dl(\varphi)}{dN} = f\left[\Delta K(\varphi)\right] \quad (1)$$

under the assumption that ΔK plays a dominant role in describing crack growth and predicting fatigue life.

Several calculation methods for stress intensity factors are proposed in the literature. Especially those of Raju and Newman [1] and Irwin [2] are well-known and extensively applied. The proposed procedure uses the results thereof. To calculate the growth of a crack, an iterative scheme with equidistant steps in the direction of the crack depth is used. The following operations are necessary to get from state (i-1) to state (i).

a) Calculate the crack width

$$c_i = c_{i-1} + (a_i - a_{i-1})\left(\frac{a_{i-1}}{c_{i-1}}\right)^{m/2} \quad (2)$$

from the known crack depth a_i and the results of the previous step, c_{i-1} and a_{i-1}, using a material-dependent exponent m/2 (2<m<5, e.g. m = 3). Formula (2) is deduced from well-known equations established by Irwin [2] and Paris [3].

b) Calculate ΔK_i from $\Delta\sigma$, c_i and a_i using the Finite Element solution of Raju and Newman [1] or Irwin's equation [2].

OPTIMIZATION METHOD

A sufficient number of value pairs (ΔK_i, a_i) being determined, the parameters of the equation

$$\Delta K = \frac{\Delta F}{t\sqrt{W}}\, f\left(\frac{a}{W}\right) \quad (3)$$

(as currently used for CT specimens) can be adjusted to fit this set of information as well as possible. ΔF, t, a and W are the respective notations for load double amplitude, thickness, crack length, and width of the CT specimen. The expression

$$f\left(\frac{a}{W}\right) = \frac{\left(2+\frac{a}{W}\right)\left(0{,}886+4{,}64\left(\frac{a}{W}\right)-13{,}32\left(\frac{a}{W}\right)^2+14{,}72\left(\frac{a}{W}\right)^3-5{,}6\left(\frac{a}{W}\right)^4\right)}{\left(1-\left(\frac{a}{W}\right)\right)^{2/3}} \quad (4)$$

is taken from ASTM E 399-81 (with $0{,}2 \le \frac{a}{W} \le 0{,}8$).

By combining the two purely multiplicative factors in (3) and transforming the co-ordinate origin

$$\Delta K = \frac{\eta}{\sqrt{W}}\, f\left(\frac{a_o + a'}{W}\right) \quad (5)$$

is obtained, where $\eta = \frac{\Delta F}{t}$, $a_o + a' = a$, $a' = \sum_{i=1}^{n} (a_i - a_{i-1})$.

Thus W, a_o, and η can be used to reproduce the key values ΔK and a of a plate in a CT specimen. For optimization of the process the steepest descent method and a modified Newton-Raphson minimization scheme were used. The influence of each independent variable is described as follows.

(i) W influences both the vertical and the horizontal scales of the function $\frac{\eta}{\sqrt{W}} f\left(\frac{a_o+a'}{W}\right)$.

(ii) η influences the vertical scale alone.

(iii) a_o allows to find the section of the $\Delta K(a)$ curve of the CT specimen where the slope is nearest to that of the plate.

As regarding initial estimates for W, a_o and η, the following suggestions are useful.

(i) Equal thickness for CT specimen and plate is recommended. Then $W \ge \frac{5}{3}$ t will be valid.

(ii) $a_o \ge 0{,}2$ W.

(iii) $\eta = \frac{\Delta\sigma (W-a_o)^2}{2(2W+a_o)}$, where $\Delta\sigma$ is the double amplitude of stress in uncracked region of the plate. This expression is derived from an elementary consideration of the stress distribution in the CT specimen.

Starting from the values of W, a_o and η thus obtained, the optimization algorithms are repeatedly applied until either a chosen local sum of the squares of the deviations $\sum_{0}^{n} \left[(\Delta K_{Plate} - \Delta K_{CT})/\Delta K_{Plate}\right]^2$ is obtained or a predetermined number of iteration steps is accomplished.

COMPUTATIONAL AND EXPERIMENTAL RESULTS

To illustrate the procedure described hereabove, an example is shown in Fig. 2. The material used was a fine-grain structural steel BH 43W (St.E.43) [4]. The ΔK values of the growing part-through crack in the plate were calculated using the Finite Element solution of Raju and Newman [1]. The goal is obviously a one-to-one correlation between the fatigue growth rates of the cracks in the plate and in the CT specimen. As far as the crack shape (Fig. 1) was concerned, good agreement was obtained on the basis of the local Paris law [3]. However the comparison of the cycle numbers for plates and optimally corresponding CT specimens shows a certain discrepancy (Fig. 3a). This may be explained by various reasons, ranging from locally varying stresses and material properties to incomplete crack propagation algorithms and insufficient control of experimental conditions (e.g. unwanted bending effects).

Further investigations with the proposed method might contribute to a quantification of the mentioned influence of a locally varying crack resistance along the crack front. Fig. 3a shows, however, that, by using CT specimen results for the prediction of fatigue growth in surface cracks, results on the safe side are most probable.

With reference to Fig. 3a the following remark is of a certain importance. By starting the count of load cycles N at the moment when both cracks have a depth (a for the plate, $a-a_0$ for the CT specimen) of 4 mm a substantial reduction of scatter and discrepancies is obtained. This manipulation, though somewhat arbitrary at first sight, makes clear that the congruence between the two types of experiments is remarkably good if their initial phases are neglected (Fig. 3b).

Fig. 4 shows a comparison of the observed crack growth with the assumption of a constant a/c ratio as postulated in the ASME Boiler and Pressure Vessel Code, Section XI. Obviously this assumption may sometimes be very far from correct (depending on the crack depth-to-length ratio) and consequently the number of cycles between the observation of a crack and its critical state may be dramatically overestimated.

CONCLUSIONS

The significance of ΔK values in a part-through crack is closely related to that in a CT specimen. The experimental results of fatigue tests with plates and conveniently adjusted CT specimens are a promising contribution to the use of standard techniques when investigating the fatigue behaviour of structural components.

ACKNOWLEDGEMENTS

The authors extend their thanks to the Director and staff of the Swiss Federal Institute for Reactor Research (EIR), Würenlingen, where a substantial part of this work was carried out.

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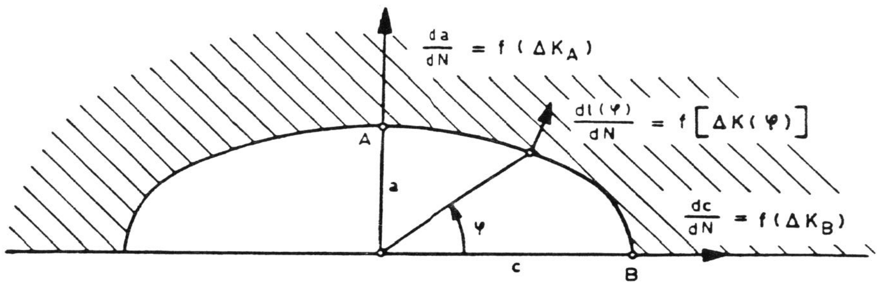

Fig. 1 Assumption for the fatigue growth of surface cracks in isotropic material (the resistance to crack propagation is considered as independent from direction)

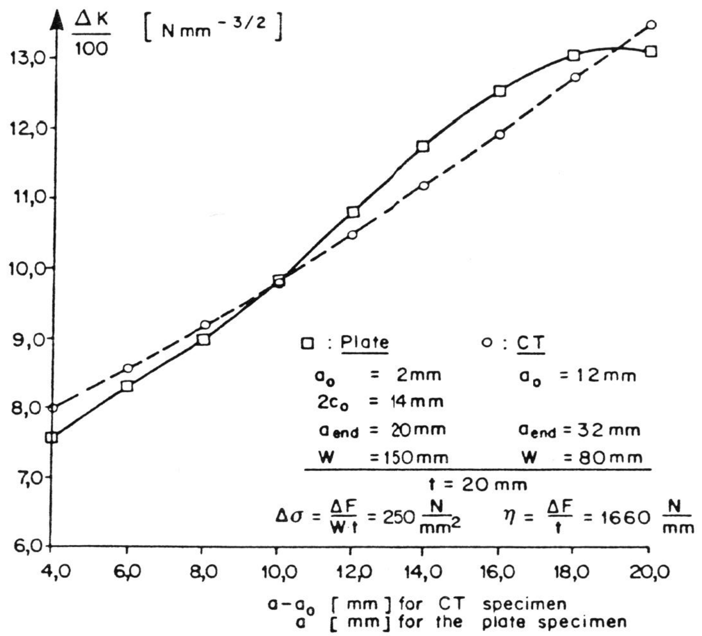

Fig. 2 Calculated ΔK values of a growing surface crack (at its deepest point) and of an optimally adjusted CT specimen

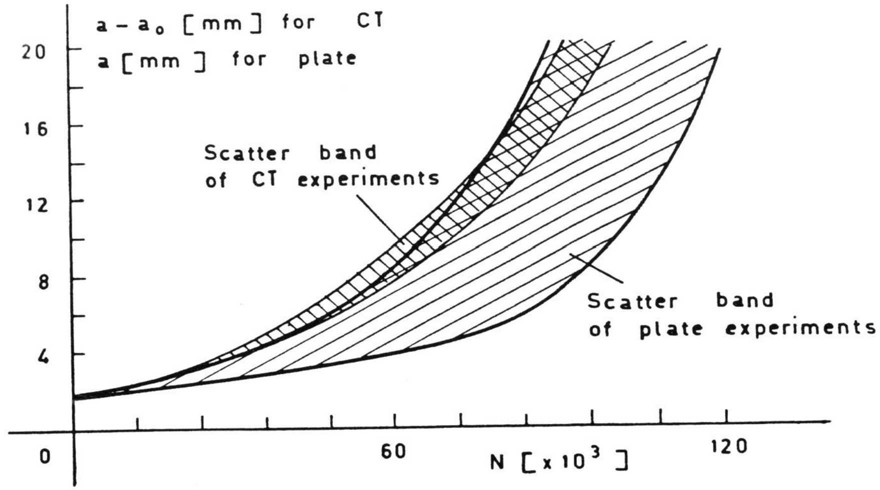

Fig. 3a Crack propagation in tensile plates and optimally adjusted CT specimens

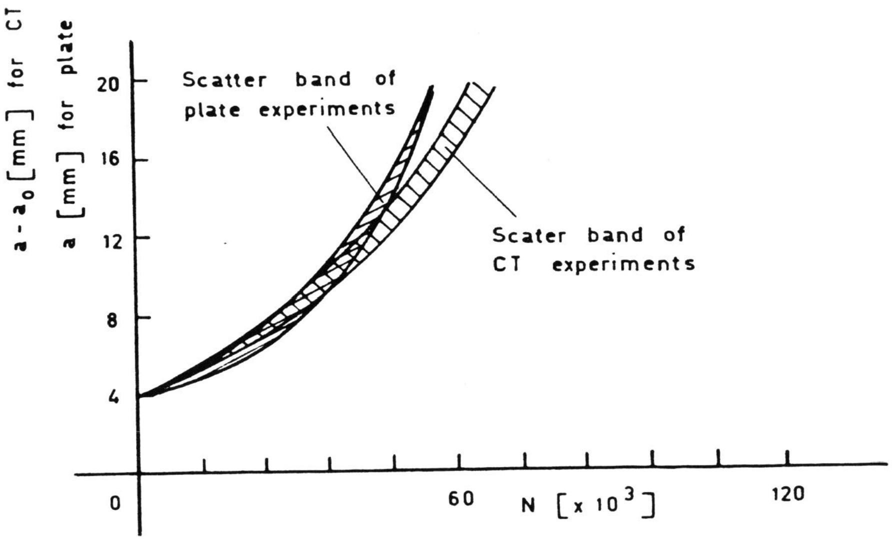

Fig. 3b Diagram as represented in Fig. 3a adjusted so as to have the origin transformed N=0 when crack depth equals 4 mm, in both specimens (see text)

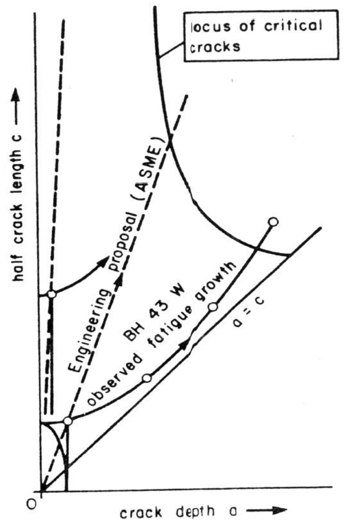

Fig. 4 Fatigue growth of surface cracks towards critical size. Comparison of experimental results with assumptions made in ASME code.