

# ELASTIC-PLASTIC ANALYSES FOR SHORT FATIGUE CRACK AT NOTCH

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## ABSTRACT

Elastic-plastic analyses for the crack growth rates of short fatigue crack at notch are presented. The work shows detailed features of monotonic plastic zone, reversed plastic zone, residual stress, etc. It is found that the interactions of short crack and notch have significant effect on the local stress-strain curve and cyclic plastic strain range. The decreasing/increasing trends in growth rates of short crack at sharp notch have been explained. In addition, using the equivalent plastic strain range as a critical parameter, the theoretical prediction for the size effect on threshold stress of short crack shows good agreement with test data.

## KEYWORDS

Short fatigue crack; Elastic-plastic analysis; Notch root.

## INTRODUCTION

When a microcrack is born at notch root within a single grain, the crack length is less than the grain size and the crack growth may be controlled by the metallurgical effects (Lankford,

1982). After that a short crack will grow within a local plastic strain fields of the notch. Here 'short' means that the crack length is rather small compared with the notch plastic zone size. In this situation, the growth rates of short cracks are much faster than those expected on the basis LEFM (Broek, 1972). This paper presents an elastic-plastic analysis for the growth of short cracks at notches. The work shows in detail features of monotonic plastic zone, reversed plastic zone, residual stress, etc. The significance of short crack-notch interactions is explained.

Using cyclic plastic strain criterion and crack opening displacement criterion, several theoretical predictions are made and compared with experimental results.

#### EFFECT OF CRACK LENGTH ON THRESHOLD STRESS RANGE

Kitagawa and Takahashi (1976) indicated that the threshold stress  $\Delta\sigma_{th}$  approaches the fatigue limit  $\sigma_f$  of the material as the crack length trends to zero. For short cracks, it is reasonable to use the equivalent plastic strain range  $\Delta\bar{\epsilon}_p$  as a parameter to characterize the crack tip stress-strain fields instead of the stress intensity factor range. The mechanism of repeated plastic blunting and sharpening of the crack tip is based on slip along planes oriented at  $45^\circ$  to the crack line. We choose the equivalent plastic strain range  $\Delta\bar{\epsilon}_p$  along this direction at a distance  $r_0$  ahead of the crack tip as a critical parameter. The  $r_0$  should be of the same order as the grain size. In order to get the equivalent plastic strain  $\Delta\bar{\epsilon}_p$ , the finite element method has been employed (Wang and Miller, 1982). The power law dependence

$$\sigma_{ea} = \sigma_{cy} (\bar{\epsilon}_{pa} / \bar{\epsilon}_{oa})^{n'} \quad (1)$$

has been used in the calculation. Here  $\sigma_{cy}$  is the cyclic yield strength and  $n'$  is the cyclic strain-hardening exponent. For the G 40.11 steel the material properties are  $2\sigma_{cy} = 532\text{MPa}$  and  $n' = 0.25$  (Haddad and Co-workers, 1980). For symmetrical cyclic loading, we assume that when the equivalent plastic

strain range  $\Delta\bar{\epsilon}_p$  as defined above reaches a threshold value  $\Delta\bar{\epsilon}_{pth}$ , the crack propagation-nonpropagation transition will occur.

The calculation results have shown in Fig.1, where the solid line predicts the theoretical curve given by this paper. Agreement of the theoretical prediction and the test data is quite good.

#### PLASTIC ANALYSES OF CRACK PROPAGATION FOR SHORT CRACKS AT NOTCHES

##### Interaction of Notch and Short Crack

At stress levels causing local plasticity, a plastic zone develops at the notch root. Small crack may be imbedded in such a plastic zone. During tensile loading, the crack tip stress fields is dominated by the crack. The act of the notch seems as a stress raiser. Conversely during push loading, the crack will close and the crack tip fields is controlled by the notch which leads to yielding in compression and resulting in rather large plastic strains. The effect of the notch seems to be not only a stress raiser but also a cyclic plastic strain range raiser. The plastic zones for a long crack emanating from a circular hole are shown in Fig. 2. The monotonic plastic zone is concentrated on the crack tip region and there is no any plastic zone near the notch root due to the traction free condition on the crack face.

The reversed plastic zone is separated into two parts. One is near the crack tip with a size which is much smaller than the monotonic plastic zone. Another is near the notch root and is very similar with the reversed plastic zone at the circular hole without the crack. In Fig.3, one can find the interaction between the notch and a short crack.

##### Residual Stress

As pointed by Klesnil and Lukas (1980), the residual stress ahead of the crack tip after cyclic loading is always com-

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pressive due to the effect of the crack closure. It is the interaction of a notch and a short crack that cause a sign change of residual stress. During tension loading-unloading, the crack plays a main role. As a result, one gets compressive residual stress near the crack tip after unloading to zero. During push loading, the local notch plasticity is more important. Therefore one may get a tensile residual stress near the crack tip after unloading to zero.

Fig.4 shows the residual stress  $\sigma_y^r$  as a function of distance from the crack tip for a short crack emanating from a circular hole.

#### THEORETICAL PREDICTION OF SHORT CRACK PROPAGATION RATES

The most important property of a material subjected to cyclic loading is the hysteresis loop, which can be characterized by the plastic strain range.

In this section, we choose the equivalent plastic strain range  $\Delta \bar{\epsilon}_p$  at a fixed distance  $r_0$  ahead of the crack tip as a critical parameter. The crack opening displacement range is also used as a critical parameter.

Thus we can suppose that the rate of fatigue crack propagation is governed by the plastic strain range  $\Delta \bar{\epsilon}_p$  or by the crack opening displacement range  $\Delta \delta$ .

#### Calibration Curve

A typical center cracked panel with a long crack has been chosen as a calibration specimen. From a series of simulated calculations for the stress-controlled cyclic loads, one can get a set of relationships between the plastic strain range  $\Delta \bar{\epsilon}_p$  (as well as the crack opening displacement range  $\Delta \delta$ ) and the stress intensity factor range  $\Delta K_I$ .

#### Prediction of Fatigue Crack Growth Rates at Notches

In order to study the crack growth rates for short cracks at

notches, a careful experimental work has been carried out by Haddad and co-workers (1980). Fig.5 shows a typical result. The  $\Delta \bar{\epsilon}_p$  and  $\Delta \delta$  values were calculated for test specimens. Using the calculated values and the calibration curves, one can get theoretical predictions for short crack propagation rates at notches.

In Fig.5, the theoretical predictions obtained from the criteria of the equivalent plastic strain range  $\Delta \bar{\epsilon}_p$  are compared with experimental results. For cracks initiating from the a 9.50mm diameter circular notch, the  $\Delta \bar{\epsilon}_p$  criteria based on different material points (with different distances ahead of the crack tip) gave nearly same predictions, which are in good agreement with the test data. The predictions for the crack initiating from the elliptical notch show a growth rate that initially decreases, reaches a minimum and then increases. This decreasing/increasing trends in growth rate are coincided with the test results. But there are some discrepancies between the predictions and the test data when the crack length is less than 1mm.

The  $\Delta \delta$  criteria anticipated at different points on the crack faces gave eventually identical results which are much lower than the test data. For the cracks emanating from the elliptical notch, the prediction exhibits continuously increasing crack growth rates which is in contrary with test observation (see Fig.7).

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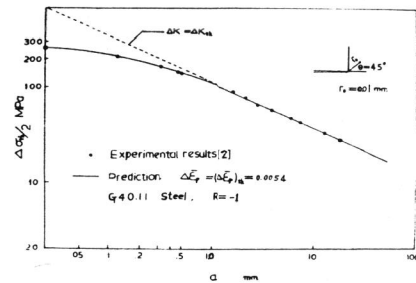


Fig. 1. Effect of crack length on threshold stress range

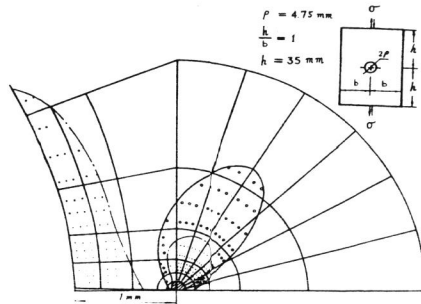


Fig. 2 Plastic zone for a crack emanating from a circular notch in G40.11 steel.  $\Delta\sigma = 269\text{MPa}$ ,  $R = -1$ . The zone with solid line border is monotonic plastic zone. The zone with dashed line border is the reversed plastic zone.

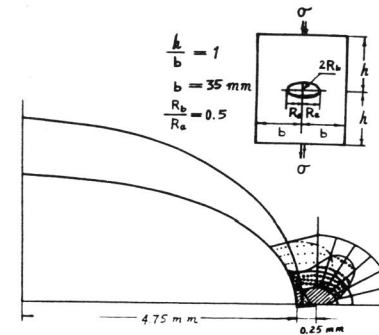


Fig. 3. Plastic zone for a short crack emanating from an elliptical notch in G40.11 steel.  $\Delta\sigma = 269\text{MPa}$ ,  $R = -1$ .

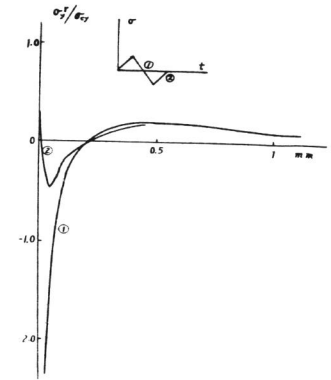


Fig. 4. Residual stress  $\sigma_r$  distribution near crack tip. The specimen dimensions and load condition are as same as in Fig. 2.

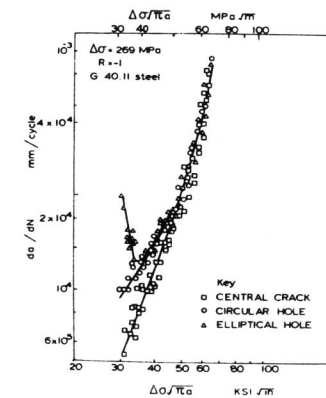


Fig. 5. Experimental results for G40.11 steel after Haddad and coworkers (1980).

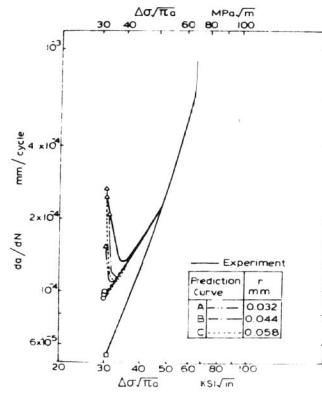


Fig.6. Comparison of theoretical prediction with the experimental results for  $\Delta\bar{\epsilon}_p$  criteria.

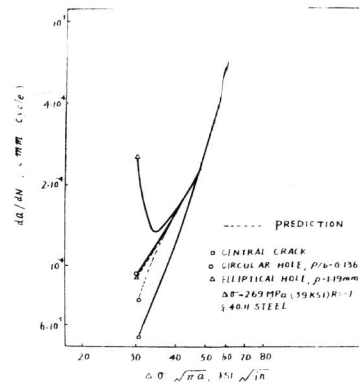


Fig.7. Comparison of theoretical prediction with the experimental results for  $\Delta\delta$  criteria.