DELAYED-ELASTIC MODEL FOR INITIATION AND ACCUMULATION OF CREEP CAVITATION AT HIGH TEMPERATURES

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ABSTRACT

A quantitative model has been developed for stress dependency of the incubation time required for initiation of intergranular cracks under moderate stress during high-temperature creep, the minimum stress required, and the subsequent accumulation of damage from cracking activity. It is based on a combination of theory and observation: cracks form at a critical delayed-elastic strain corresponding to a critical grain-boundary sliding displacement, and further creep cavitation increases exponentially with delayed-elastic strain in excess of this critical value.

KEYWORDS

High temperature creep; intergranular fracture; cavity nucleation; incubation time; creep damage; damage accumulation; delayed elasticity; polycrystals; grain-boundary sliding; grain size; ice.

INTRODUCTION

Deformation as a result of grain-boundary sliding (shearing) plays a dominant role in creep processes at temperatures above about 0.4 T_m, where T_m is melting point in Kelvin. This grain-boundary sliding could result in stress concentrations at triple points or at irregularities or ledges in the grain boundaries sufficient to nucleate cracks (Zener, 1948; Gifkins, 1956). Sinha (1979, 1982a) related grain-boundary sliding to delayed-elastic effect and showed that crack formation during constant-load creep is initiated at a critical delayed-elastic strain corresponding to a critical grain-boundary sliding displacement. The present paper extends this hypothesis and establishes an interdependence between damage accumulation during creep and the delayed-elastic strain under conditions where the rate-controlling mechanisms are both grain-boundary sliding and dislocation creep (Gandhi and Ashby, 1979; Mukherjee, Bird and Dorn, 1969).

EXPERIMENT

Gold (1967, 1972a) investigated cracking activity in transversely isotropic, columnar-grained S-2 ice subjected to constant compressive load applied perpendicular to the long direction of the grains at 0.96 T_m. The cracks were clearly visible; they were long and narrow, with their long direction in the long direction of the grains, and their plane tended to be parallel to the direction of compressive stress. Formation of first cracks for stresses in the range of $\sigma = 5 \times 10^{-5}$ to 2×10^{-4} E (where E is Young's modulus) was, however, reported to be a reasonably well-defined event in previously undeformed specimens (Fig. 1). The first three cracks were analysed for statistical significance. Stress (σ) dependence of the time of formation of the first crack (t_{fc}) was similar to that of creep failure time on applied load for other materials (Bartenev and Zuyev, 1968; Zhurkov, 1965)

$$t_{fc} = t_o \exp\left[(Q_o - \alpha\sigma)/kT\right] \tag{1}$$

where t_o and α are constants, k is the Boltzmann constant, and Q_o is the apparent activation energy at zero stress. The physical absurdity in this relation is the prediction that cracks would develop even near zero stress ($\sigma = 0^+$). Gold (1967, 1972a) did not observe cracking activity in ice within the experimental time for stresses less than about 0.6 $MN \cdot m^{-2}$ ($\approx 6 \times 10^{-5}$ E).

Gold (1972a, 1972b) also studied the accumulation of damage during creep in S-2 ice. The dependence of crack density on time at 0.96 T_m(−10°C) is shown in Fig. 2, where each curve represents the average of six tests. Crack density is given as the number of cracks per unit area because of the two-dimensional nature of the deformation and crack formation.

DELAYED ELASTICITY AND CRACK INITIATION

The author has already discussed the hypothesis that shear or sliding in the grain-boundary regions gives rise to delayed elastic effect (Sinha, 1979) and developed formulations for stress (σ), time (t), temperature (T), and grain size (d) dependence of the delayed-elastic strain (des), ε_d, under uniaxial loading conditions

$$\varepsilon_d = c_1 \left(\frac{d_1}{d}\right) \left(\frac{\sigma}{E}\right)^s \left[1 - \exp\left\{-(a_T\, t)^b\right\}\right] \tag{2}$$

where E is Young's modulus and c_1 is a constant corresponding to the unit or reference grain size, d_1; b and s are constants and $1/a_T$ is the temperature-dependent relaxation time. The primary assumptions were

$$\varepsilon_d = \varepsilon_{gbs} \tag{3}$$

and

$$\varepsilon_{gbs} = K\bar{x}/d \tag{4}$$

where ε_{gbs} is the strain induced by grain-boundary sliding (gbs), $\bar{x}$ is the average grain-boundary displacement, and K is a constant nearly equal to 1 (Gifkins, 1956). The values of both s and K are given as 1 in Table 1 for the present analysis for ice. In generalizing, however, both constants are retained in this paper.

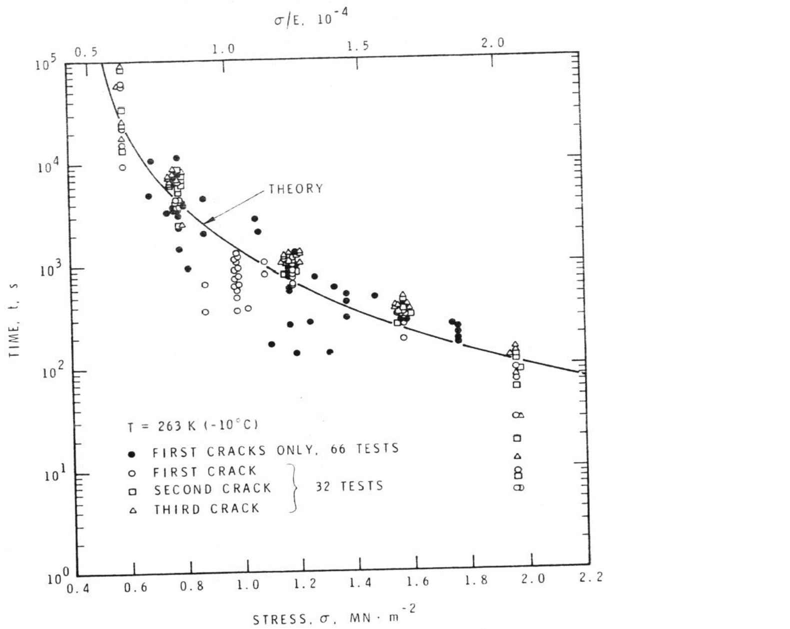

Fig. 1. Stress dependence of average times to formation of first three cracks in S-2 ice under compressive stress (Gold 1967, 1972a). The solid line is based on present theory.

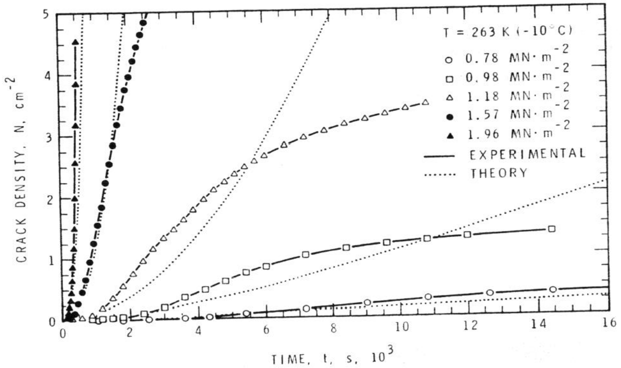

Fig. 2. Time dependence of crack density for compressive stress (Gold 1972a). Broken lines are based on present theory.

Cracks can develop at triple points or irregularities in the grain boundaries during creep if the stress concentrations produced by grain-boundary shearing are not relaxed by the processes of internal accommodation (Sinha, 1984). A critical grain-boundary displacement, $\bar{x}^c$, might be required before cracks are initiated. Equations (3) and (4) give the critical gbs, ε^c_{gbs}, and the critical des, ε^c_d as

$$\varepsilon^c_d = \varepsilon^c_{gbs} = K\bar{x}^c/d \qquad (5)$$

Delayed-elastic strains calculated for all observations (σ, t_{fc} pairs) in Fig. 1 are presented in Fig. 3. Calculations were made on the basis of equation (2) for a grain size of 4.5 mm and the values of the material constants in Table 1. The first cracks seem to form, irrespective of stress level, for $\varepsilon^c_d = 1.04 \times 10^{-4}$ (with a scatter of 10%). According to equation (5) this gives $\bar{x}^c = 0.47$ μm for K = 1 and d = 4.5 mm.

It should be mentioned (and can be shown by substituting ε_d in equation (2) by the right side of equation (5)) that the calculated value of $\bar{x}^c$ was not affected by the somewhat arbitrary choice of grain size. This choice was made because of the extensive grain diameter determinations carried out during strength tests (Sinha, 1981, 1982b) on ice produced, essentially, by the method used by Gold (1972a).

Substituting ε^c_d for ε_d and t_{fc} for t in equation (2) and rearranging gives

$$t_{fc} = a_T^{-1} \left[-\ln\left\{1 - \frac{\varepsilon^c_d}{c_1}\left(\frac{d}{d_1}\right)\left(\frac{E}{\sigma}\right)^s\right\}\right]^{1/b} \qquad (6)$$

Thus, on substitution of ε^c_d from equation (5),

$$t_{fc} = a_T^{-1} \left[-\ln\left\{1 - \frac{K\bar{x}^c}{c_1 d_1}\left(\frac{E}{\sigma}\right)^s\right\}\right]^{1/b} \qquad (7)$$

Thus t_{fc} is independent of grain size. Calculations based on Table 1 and equation (6), with $\varepsilon^c_d = 1.04 \times 10^{-4}$ and d = 4.5 mm (or equation (7), with $\bar{x}^c = 0.47$ μm) are compared with the experimental results in Fig. 1. The rapid increase in t_{fc} with decrease in σ, particularly at the lower end of the stress, is now represented more realistically than by the Zhurkov type equation (1). Substitution of $t_{fc} = \infty$ in equations (6) and (7) gives the minimum stress, σ_{min}, for cracking as

$$\sigma_{min} = E\left[\frac{\varepsilon^c_d\, d}{c_1 d_1}\right]^{1/s} \quad \text{or} \quad \sigma_{min} = E\left[\frac{K\bar{x}^c}{c_1 d_1}\right]^{1/s} \qquad (8)$$

This is independent of grain size and gives $\sigma_{min} = 0.5$ MN·m^{-2} for $\bar{x}^c = 0.47$ μm and other constants in Table 1. It agrees well with Gold's (1967, 1972a) observation of minimum stress of 0.6 MN·m^{-2} for cracking.

CREEP DAMAGE ACCUMULATION

It is possible that more and more cracks will develop for $\sigma > \sigma_{min}$ and $t > t_{fc}$ if the number of sites of stress concentration become critical owing to increased strain (increase in grain-boundary sliding). This possibility can be examined, using equation (2), by computing ε_d for all the

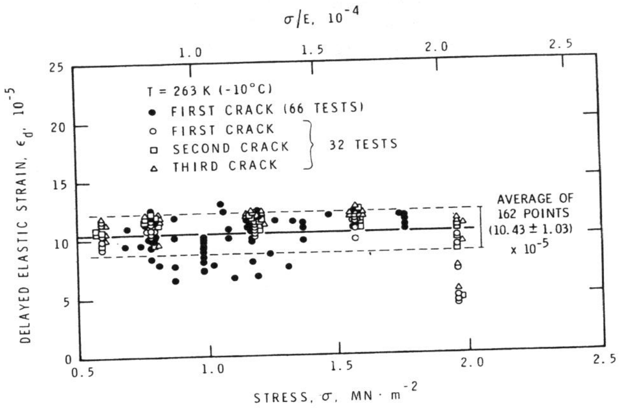

Fig. 3. Computed delayed-elastic strain versus stress for all experimental points in Figure 1.

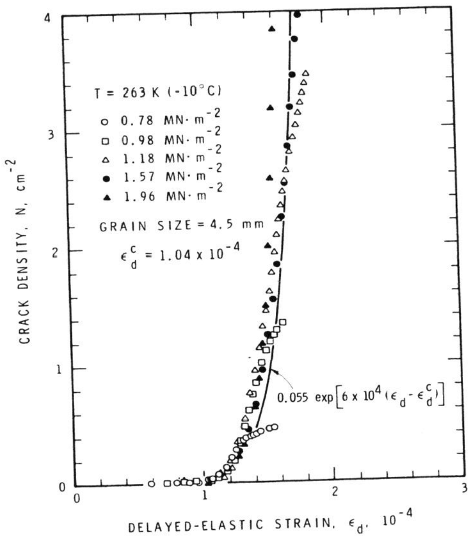

Fig. 4. Dependence of experimentally observed crack density on computed delayed-elastic strain for experiments in Figure 2.

experimental results in Fig. 2 and comparing the dependence of crack density with computed values for each stress level. Results shown in Fig. 4, calculated for d = 4.5 mm and information in Table 1, indicate a strong dependence between cracking activity and delayed-elastic strain (or grain-boundary displacement) irrespective of stress level. There is deviation from this dependence after longer periods, depending on the stress level. This is to be expected because the potential sites for crack nucleation would decrease with time and, moreover, the accumulated damage should have a profound influence on the processes of further cavitation dependency. The damage accumulation during creep may be expressed by

$$N = N_c \exp\left[\xi\left(\varepsilon_d - \varepsilon_d^c\right)\right] \quad (9a)$$

or

$$N = N_c \exp\left[\psi\left(\bar{x} - \bar{x}^c\right)\right] \quad (9b)$$

where ξ is a constant and $\psi = \xi\,K/d$, and N_c is the critical crack density corresponding to the critical value of ε_d^c or $\bar{x}^c$.

Regression analysis of the results in Fig. 4 for equation (9a) and previously obtained $\varepsilon_d^c = 1.04 \times 10^{-4}$ ($\bar{x}^c = 0.47$ μm) and d = 4.5 mm gave $N_c = 0.055$ and $\xi = 6.0 \times 10^4$ ($\psi = 1.33 \times 10^7\ m^{-1}$), with a correlation coefficient of 0.97, giving

$$N = 0.055 \exp\left\{6 \times 10^4\left(\varepsilon_d - 1.04 \times 10^{-4}\right)\right\} \quad (10a)$$

or

$$N = 0.055 \exp\left\{1.33 \times 10^7\left(\bar{x} - 0.47 \times 10^{-6}\right)\right\} \quad (10b)$$

This is shown in Fig. 4 by the solid line.

The dependence of cracking activity on stress and time can be obtained by eliminating ε_d in equation (9a), using equation (2)

$$N = N_c \exp\left[\xi\left(\frac{c_1 d_1}{d}\left(\frac{\sigma}{E}\right)^s\left\{1 - \exp\left[-\left(a_T\,t\right)^b\right]\right\} - \varepsilon_d^c\right)\right] \quad (11a)$$

or

$$N = N_c \exp\left[\psi\left(\frac{c_1 d_1}{K}\left(\frac{\sigma}{E}\right)^s\left\{1 - \exp\left[-\left(a_T\,t\right)^b\right]\right\} - \bar{x}^c\right)\right] \quad (11b)$$

Equation (11a) with the above values of N_c, ξ, ε_d^c and the values of other constants from Table 1 is compared in Fig. 2 with the experimental results.

TABLE 1 Creep Parameters for Ice Obtained from Earlier Creep Experiments (Sinha, 1979).

E	$= 9.5\ GN\cdot m^{-2}$; $Q = 67$ kJ/mol (16 kcal/mol);
c_1	$= 9$; $d_1 = 1$ mm; $s = 1$; $K = 1$; $n = 3$;
b	$= 0.34$; a_T (T = 263 K) $= 2.5 \times 10^{-4}\ s^{-1}$;
$\dot{\varepsilon}_{v1}$	$= 1.76 \times 10^{-7}\ s^{-1}$; $\sigma_1 = 1\ MN\cdot m^{-2}$, T = 263 K

DISCUSSION

Grain-boundary shearing in polycrystals is a complex process depending on external conditions of stress and temperature and internal conditions such as crystalline structure of the matrix, type of defect, texture and fabric of the material, grain size and its distribution, impurities in the material and inclusions at the grain boundaries. The analysis, however hypothetical, resulted in a meaningful way of handling experimental observations, particularly the stress and time dependence of the onset of crack formation and subsequent creep damage. Although the present analysis is based mainly on speculation, it is not without direct experimental evidence. The dependence of cavity formation on amount of grain-boundary sliding was reported first by Intrater and Machlin (1959) in copper bicrystals. Similar observations have been reported by Fleck, Taplin and Beevers (1975) in a copper alloy. That creep fracture takes place when the amount of grain-boundary sliding, as predicted here, reaches a given value

has also been observed directly from experiments on copper bicrystals (Watanabe, 1983). Bicrystal results cannot, however, be directly applied to polycrystalline materials.

A further significant test of the model developed in this paper is to use it to predict the strain dependence of cracking activity. Creep strain, ε, was described (Sinha, 1979) as composed of three components

$$\varepsilon = \varepsilon_e + \varepsilon_d + \varepsilon_v \quad (12)$$

where ε_e is pure elastic strain (= σ/E), ε_d is the des described by equation (2), and ε_v is viscous or permanent deformation [= $\dot{\varepsilon}_{v1}\ t\ (\sigma/\sigma^1)^n$, where $\dot{\varepsilon}_{v1}$ is the viscous strain rate for unit or reference stress, $\sigma^1 = 1\ MN\cdot m^{-2}$, and n is a constant].

Thus

$$\varepsilon = \frac{\sigma}{E} + c_1\left(\frac{d_1}{d}\right)\left(\frac{\sigma}{E}\right)^s\left[1 - \exp\left\{-\left(a_T\,t\right)^b\right\}\right] + \dot{\varepsilon}_{v1}\,t\left(\frac{\sigma}{\sigma^1}\right)^n \quad (13)$$

For a given stress, temperature, and grain size, equation (13) gives total strain as a function of time. As ε depends on grain size and stress, it can readily be shown that strain at first cracks will depend on these quantities. Equation (11) gives, for the same imposed conditions, the dependence of crack density on time. Equations (11) and (13) can therefore be used to examine the dependence of cracking activity on strain. A set of calculated results is shown in Fig. 5 that compares well with the experimental observations of Gold (1972b).

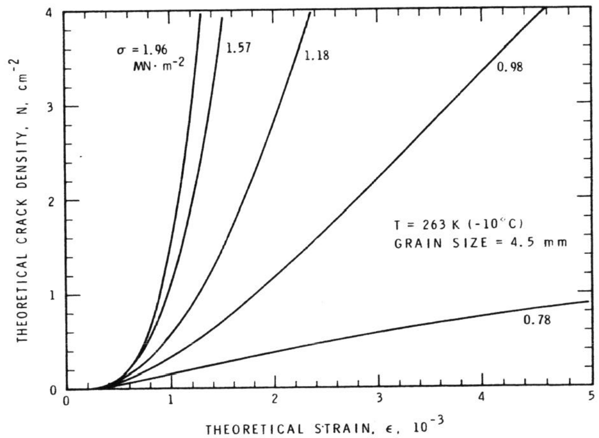

Fig. 5. Theoretical crack density versus strain at −10°C for grain size of 4.5 mm.

CONCLUSION

A simple model has been developed that agrees well with observations of crack formation in polycrystalline ice during creep at elevated temperatures. It describes the stress, time, and strain dependence of cracking activity at a constant temperature. It indicates that cracks are initiated on attaining a critical grain-boundary sliding displacement that is not dependent on grain size or stress; that the corresponding critical delayed elastic strain depends on grain size but not on stress; and that the corresponding total strain depends on both grain size and stress. The model predicts the stress dependence of onset of cracking activity better than the usual Zhurkov type relation. It also predicts that cracks do not develop below a minimum stress, irrespective of grain size. As the analysis is very general, the approach should find application to high temperature engineering problems involving metals and other materials.

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REFERENCES

Bartenev, G.M. and Y.S. Zuyev (1968). Strength and Failure of Viscoelastic Materials, Pergamon Press, 164.
Fleck, R.G., D.M.R. Taplin, and C.J. Beevers (1975). Acta Met., 23, 415.
Gandhi, C. and M.F. Ashby (1979). Acta Met., 27, 1565.
Gifkins, R.C. (1956). Acta Met., 4, 98.
Gold, L.W. (1967). Time to formation of first cracks in ice. In H. Oura (Ed.), Physics of Snow and Ice, Institute of Low Temperature Science, Hokkaido, Vol. 1, Part 1, 359-370.
Gold, L.W. (1972a). The failure process in columnar-grained ice. National Research Council of Canada, Division of Building Research, NRCC 12637.
Gold, L.W. (1972b). Phil. Mag., 26, 311.
Intrater, J. and E.S. Machlin (1959). Acta Met., 7, 140.
Mukherjee, A.K., J.E. Bird, and J.E. Dorn (1969). Trans., Am. Soc. Metals, 62, 155.
Sinha, N.K. (1979). Phil. Mag., 40, 825.
Sinha, N.K. (1981). Exper. Mech., 21, 209.
Sinha, N.K. (1982a). Delayed elastic strain criterion for first cracks in ice. IUTAM Symposium on Deformation and Failure of Granular Materials, Delft, A.A. Balkema Publisher, 323-330.
Sinha, N.K. (1982b). J. Mats. Sci., 17, 785-802.
Sinha, N.K. (1984). Intercrystalline cracking, grain-boundary sliding, and delayed elasticity at high temperatures. J. Mats. Sci.
Watanabe, T. (1983). Met. Trans. A., 14A, 531-545.
Zener, C. (1948). Fracturing of Metals, Amer. Soc. Metals, Cleveland, Ohio, U.S.A., 3, 3-31.
Zhurkov, S.N. (1965). Int. J. Fracture Mech., 1, 311.