CYCLIC GROWTH AND COALESCENCE OF MULTIPLE FATIGUE CRACKS

A. F. Grandt, Jr., R. Perez and D. E. Tritsch

School of Aeronautics and Astronautics, Purdue University West Lafayette, IN 47907, USA

ABSTRACT

This paper describes an analysis scheme for predicting the fatigue life of components which contain multiple cracks. Fracture mechanics techniques are used to predict the initial growth and coalescence of two or more cracks located along the bore of a hole. The results of experiments with multiple cracked specimens are described and are compared with crack growth predictions obtained by the analysis method.

KEYWORDS

Fatigue cracks, crack coalescence, fatigue life predictions

INTRODUCTION

Fatigue cracks frequently initiate simultaneously near stress concentrations formed by notches. These cracks may grow independently for a period, and then, depending on their initial spacing, may join into a single dominant crack which causes eventual failure. The objective of this paper is to describe an experimental and numerical program directed at predicting the growth and coalescence of multiple fatigue cracks located at holes in plates.

The particular geometries of interest are shown in Fig. 1. Here 2r is the diameter of an open hole in a plate of thickness 2t. Initial surface and corner flaws, which are described by portions of ellipses with semi-axis dimensions a and c, are located along the bore of the hole. In the present work, the two separate cracks are assumed to be coplanar, and a remote tensile stress is applied perpendicular to the crack plane. Various combinations of surface and corner cracks are considered. The next section presents a life prediction scheme for calculating the time required for the initial flaws to grow together, coalesce into a single crack, and extend to final failure. The following section then describes experiments with multiple cracked specimens conducted to evaluate the numerical analysis.

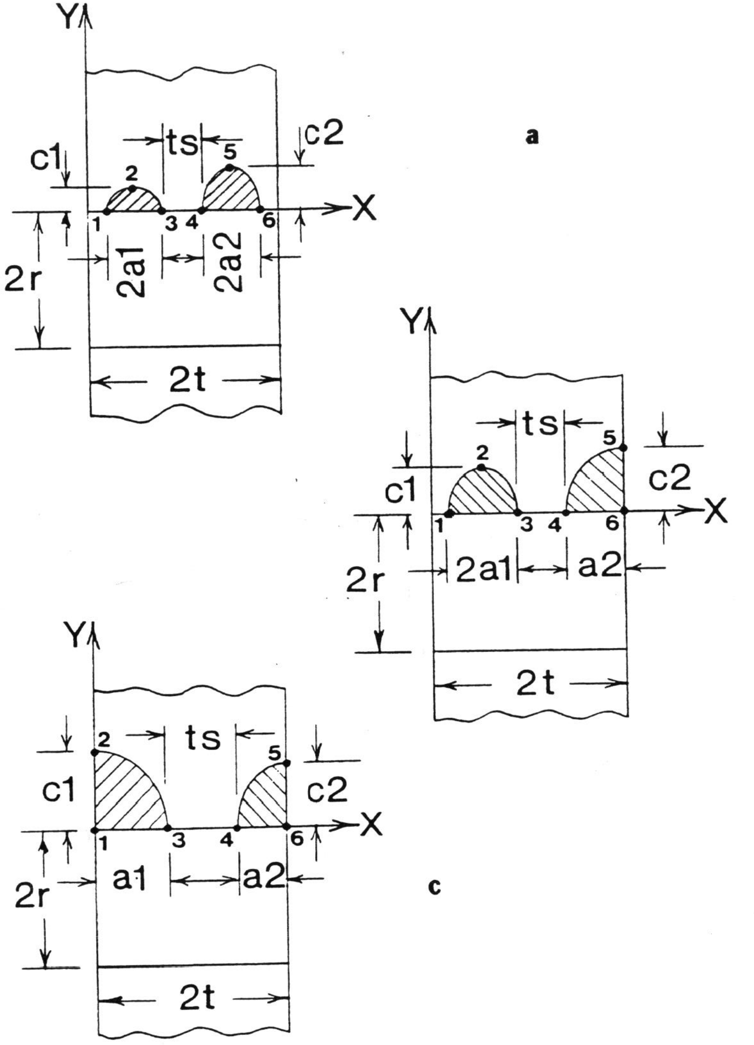

Fig. 1. Two cracks at the bore of a hole defining crack lengths and crack tip locations.

NUMERICAL ANALYSIS

The goal here is to describe an approach for predicting the initial growth and eventual coalescence of separate cracks located along the bore of a hole. The geometries of interest are summarized in Fig. 1. The semi-axis dimension along the hole bore is given by a, while the crack depth is specified by c. Subscripts 1 and 2 refer to cracks 1 and 2 respectively. Crack tip locations are defined by points 1 thru 6 as shown in Fig. 1.

When subjected to an applied stress, cracks 1 and 2 will grow. Let the fatigue crack growth rates for crack tips 1, 2, ..., 6 be d1/dN, d2/dN, ..., d6/dN. Note that, in general, these crack growth rates will have different values. The crack growth increments for a specified number of cycles ΔN of loading are, however, related. Assume, for example, that the depth c2 of crack 2 extends a small amount $\Delta 5$ during the interval ΔN. Now, ΔN is given by Eq. 1.

$$\Delta N = \frac{\Delta 5}{\frac{d5}{dN}} \qquad (1)$$

The growth Δi of crack tip i is given by

$$\Delta i = \frac{di}{dN} \cdot \Delta N = \frac{di}{dN} \cdot \frac{\Delta 5}{\frac{d5}{dN}} \qquad (2)$$

where i takes the values 1 to 6 as appropriate.

Now, if the cyclic stress intensity factors ΔKi are known at crack tips 1, 2, ..., 6, the conventional fracture mechanics approach may be used to compute the corresponding fatigue crack growth rates (at the various crack positions) from Eq. 3:

$$\frac{di}{dN} = F(\Delta Ki) \qquad (3)$$

Here di/dN is the crack growth rate at location i and F (ΔKi) is the appropriate fatigue crack growth model for the material of interest.

Stress intensity factors for the multi-cracked holes were obtained in the following manner. First, the Newman and Raju (1981) solutions for single surface or corner cracks located at holes were used to compute K at crack tip locations 1 thru 6. Newman and Raju (1981) have fit empirical equations to earlier finite element results obtained for the single crack geometries. Their empirical solutions are readily coded for computer use, and provide an effective means for interpolating between the original finite element results obtained for discrete crack shapes.

The effect of the adjacent flaw was accounted for by an "interaction factor" obtained previously by Heath and Grandt (1984). In that work, the three-dimensional finite element- alternating method was used to compute stress intensity factors for symmetric corner cracks located on opposite sides of the plate thickness (the symmetric version of Fig. 1c). The double crack K was then divided by the corresponding result for a single flaw to obtain the dimensionless interaction factor γ given in Fig. 2. Here the

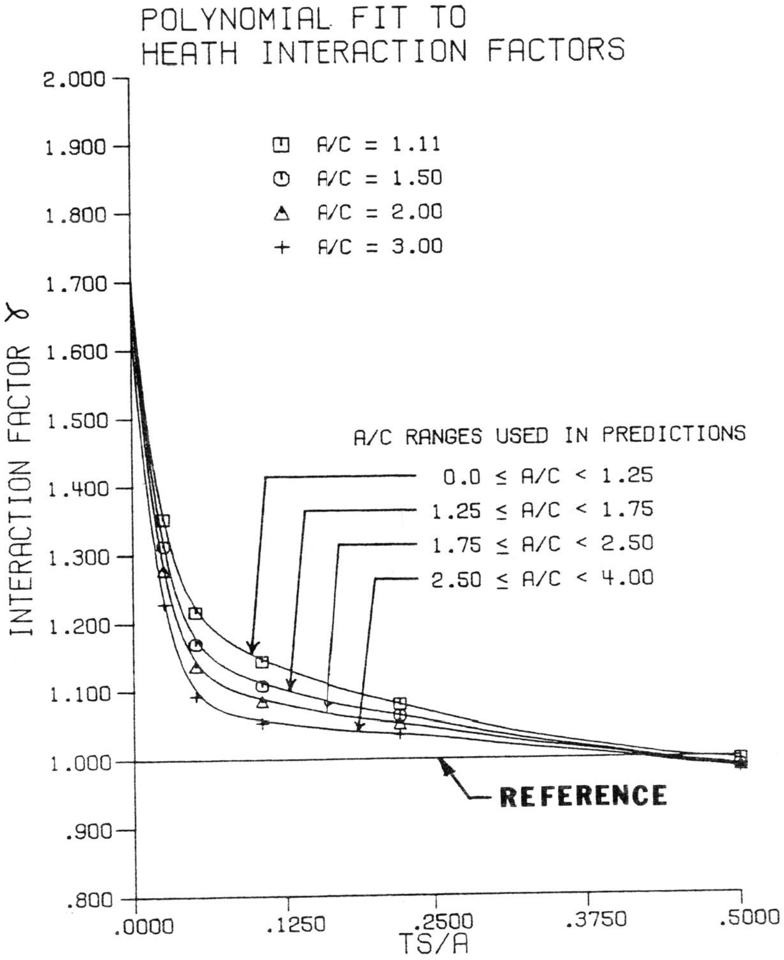

Fig. 2. Summary of the effect of crack spacing ts/a and crack shape a/c on stress intensity interaction factor γ at hole bore crack location.

increase in K at the point where the corner crack intersects the hole bore (locations 3 and 4 in Fig. 1c) is given as a function of dimensionless crack spacing ts/a and crack shape a/c. Note in Fig. 2 that as the cracks approach coalescence (ts/a < 0.3), K is increased significantly along the hole bore. In addition, flaw shape has a relatively minor effect on the increase in K. Since, Heath and Grandt showed that the stress intensity factor at free surface locations 2 and 5 is relatively unaffected by the approaching crack, no interaction was considered at those points, and the single crack Newman-Raju (1981) solutions were used to compute K there.

Thus, Equations 1 through 3 were solved in an iterative manner which allows the cracks to grow naturally. When the crack tips 3 and 4 touch, a single corner, surface, or through-the thickness flaw was assumed as appropriate,and the corresponding single crack K solution was used to continue the analysis. Other possible combinations, such as penetration of surface crack locations 1 or 6 through a free surface to form corner flaws were also considered in the algorithm. Additional details are given by Tritsch (1983). Sample calculations obtained by this multi-degree-of-freedom analysis are compared with experimental results in the following section.

EXPERIMENTAL PROCEDURE

This section describes a set of experiments conducted with multi-cracked holes to provide initial evaluation of the predictive scheme. The double corner crack geometry given in Fig. 1c was considered here.

Test specimens were machined from a single sheet of polymethylmethacrylate (PMMA), a transparent polymer. The specimens were 25.4 cm (10.0 in.) long, 8.9 cm (3.5 in.) wide, 1.9 cm (0.75 in.) thick, and contained a central 0.95 cm (0.375 in.) diameter hole. The specimens were annealed to relieve possible residual stresses, and the ends were polished to transparency. Small notches were cut with a razor blade along the hole bore (on opposite sides of the plate thickness) to provide crack initiation sites. The specimens were then precracked in cyclic bending until fatigue cracks had completely surrounded the pre-notches.

Following precracking, grips were bonded to the ends of the specimens and a constant amplitude tensile load was applied at 2 Hz with a closed loop electrohydraulic fatigue machine. Strain gages were mounted to the front and back sides of the specimens to verify that uniform tension was applied without bending. A cutout in one end of the grips allowed a mirror to be placed at an angle over the transparent specimen end and provided optical access to the crack plane. Crack growth was recorded with a 35 mm camera, and the film strips measured to give crack dimensions as a function of applied load cycles. Additional experimental details are given by Perez (1983).

Fatigue crack growth curves for two specimens are given in Figs. 3 and 4. Here the experimental measurements are given by the data points defined in the legends on Figs. 3 and 4. Measurements are given for the four crack dimensions a1 and c1 (crack 1) and a2 and c2 (crack 2) prior to coalescence. Following link up, the surface dimensions c1 and c2 of the through-the-thickness flaw are also given.

The solid and dashed lines in Figs. 3 and 4 are predictions obtained by the numerical scheme discussed in the last section. Baseline fatigue crack growth data (da/dN versus ΔK) were obtained with additional through-cracked

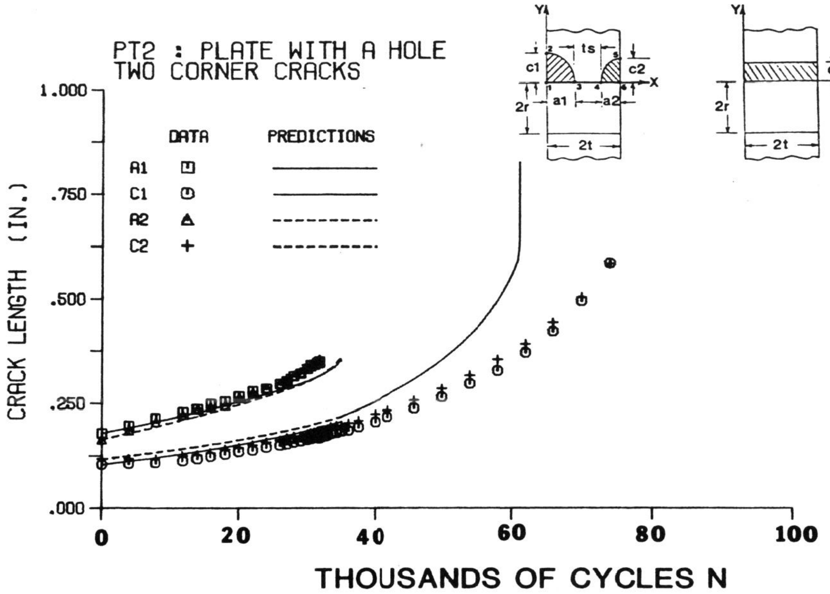

Fig. 3. Comparison of experimental and numerical results for growth of two corner cracks at open hole.

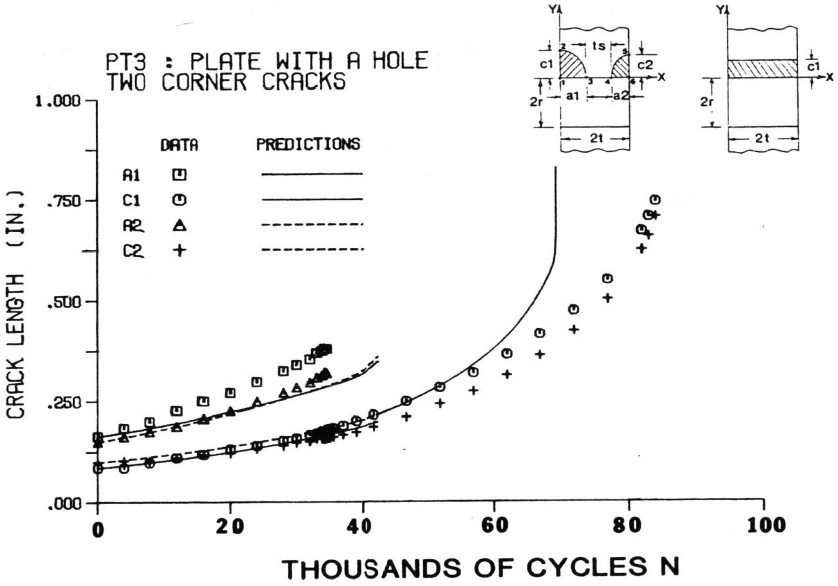

Fig. 4. Comparison of experimental and numerical results for growth of two corner cracks at open hole.

specimens obtained from the same sheet of PMMA and loaded at the same cyclic frequency. A simple power law was used to fit these baseline data and provided the crack growth model used for Eq. 3. Care was taken not to extrapolate the fatigue crack growth law for the baseline data beyond its limits of validity.

Note that the predictions of Figs. 3 and 4 generally agree quite well with the experimental data. In both cases, coalescence occurred slightly before the prediction, but actual fracture occurred shortly following the analysis. The latter result is more likely due to conservatism in the analysis scheme resulting from the fact that transition to a uniform through-the-thickness flaw was assumed to occur immediately following coalescence (joining of points 3 and 4). In actuality, a short period of time was required for the joined cracks to grow into a uniform front. More detailed measurements of the transition crack growth are given by Perez (1983).

CONCLUDING REMARKS

The objective of this paper has been to describe a procedure for predicting the lives of components which contain two or more adjacent cracks. A multi-degree of freedom model has been described for computing the initial growth, coalescence, and final extension of corner and surface cracks located along the bore of a hole in an open plate. Experiments described here with corner cracked holes in transparent polymer specimens gave excellent agreement with the predictive model. Similar predictions have been made by the authors for experiments in more conventional structural metals, and also show good agreement between experiment and analysis. (It is planned to describe those results in a separate paper.)

Although the present model is limited to two initial cracks, the procedure is readily extended to additional flaws. The stress intensity factor calculations for interacting cracks, based on the single crack solutions given by Newman and Raju (1981) and modified by the crack interaction factor obtained by Heath and Grandt for symmetric corner cracks, worked well for the experiments which have been examined to date. It may be necessary to develop alternate interaction factors for crack combinations which differ greatly in size, as the Heath and Grandt result assumes symmetric flaws. In addition, cracks which initiate in different planes may need to be analyzed by an alternate procedure.

ACKNOWLEDGMENTS

Portions of this research were sponsored by the US Air Force Office of Scientific Research, Air Force Systems Command, USAF under grant Number AFOSR-82-0041. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon. Major D.A. Glasgow was the technical monitor. The authors gratefully acknowledge the technical assistance provided by G. Beeker with the experiments.

REFERENCES

Heath, B.J., and Grandt, A.F., "Stress-Intensity Factors for Coalescing and Single Corner Flaws Along a Hole Bore in a Plate," Engineering Fracture Mechanics, Volume 19, No. 4, 1984, pp. 665-673.

Newman, J.C. and Raju, I.S., "Stress-Intensity Factor Equations for Cracks in Three-Dimensional Finite Bodies," NASA Technical Memorandum 83200, NASA Langley Research Center, Hampton, Virginia 23665, August 1981.

Perez, R., "Initiation, Growth, and Coalescence of Fatigue Cracks," M.S. Thesis, School of Aeronautics and Astronautics, Purdue University, W. Lafayette, IN 47907, August 1983.

Tritsch, D.E., "Prediction of Fatigue Crack Lives and Shapes," M.S. Thesis, School of Aeronautics and Astronautics, Purdue University, W. Lafayette, IN 47907, August 1983.