A LOCAL APPROACH TO CRACK GROWTH IN CREEPING MATERIALS BY A SHEAR DCB MODEL

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ABSTRACT

A local analysis of fracture in creeping materials, based upon a critical strain, is proposed. We modellize the process zone by a viscous strip and the elastic continuum by a double cantilever shear beam.

Closed form solutions for creep crack growth under particular loading conditions are given. Numerical solutions for studying cyclic loading and crack arrest are also proposed.

KEYWORDS

Creep crack growth ; local analysis ; double cantilever beam ; arbitrary loading ; crack arrest ; cyclic loading.

INTRODUCTION

Crack growth in creeping materials have been investigated in fracture mechanics, following different points of views : near-tip singularity analysis (Hui and Riedel, 1981), path independent C^* integral (Riedel and Rice, 1980 ; Ellison and Harper, 1978).

These approaches are similar to that of the linear fracture mechanics in the sense that the criterion of rupture is expressed in terms of a crack parameter.

An alternative approach to fracture mechanics consists of describing the continuum and the process zone by the same stress-strain law taking account of damage.

A simple model of damage, with a sudden drop of stress $\sigma_{ij} = 0$ when a critical strain is reached $|\varepsilon| \geqslant \varepsilon_R$, has been used by Bui et Ehrlacher (1981) who studied crack propagation in elastic and elastic-plastic materials.

A similar approach to creep crack growth was proposed by Bui, Dang Van and de Langre (1984) who modellized the process zone by a viscous strip and the continuum by a classical double cantilever beam in bending.

It is found that the crack length a(t) is completly determined by the loading conditions and that closed forms solution to a non-linear fourth order partial differential equation where obtained for various loading conditions.

In this paper, we investigate another class of solutions for a double cantilever shear beam. If v(x,t) is the displacement of the upper beam, only the shear strain $\gamma = \frac{1}{2}\,\partial v/\partial x$ is considered. This shear model, which can be used for studying fracture of composites with fibers parallel to Oy was first introduced by Freund (1977) in dynamic fracture.

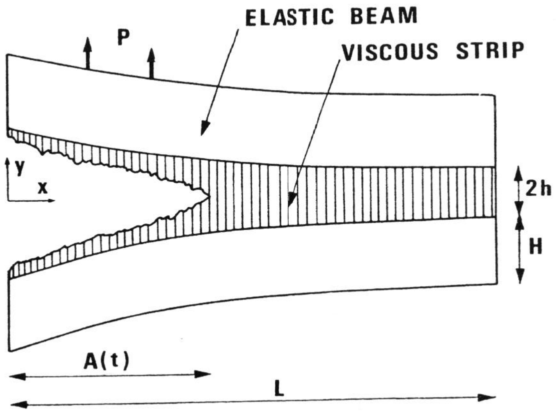

Fig. 1. DCSB model with viscous strip.

DOUBLE CANTILEVER SHEAR BEAM (DCSB) MODEL

Let us consider the double cantilever shear beam with a linear viscous strip (Fig. 1), the behaviour of which is governed by the following equations :

$$\begin{cases} \sigma = \frac{1}{B}\frac{\partial \varepsilon}{\partial t}(x,t) & \text{if} \quad \varepsilon < \varepsilon_R \\ \sigma = 0 & \text{if} \quad \varepsilon \geqslant \varepsilon_R \end{cases} \tag{1}$$

The damaged part of the strip, where $\sigma \equiv 0$, can be considered as a crack of length A(t). The tensile strain ε of the strip is expressed in terms of the displacement v of the upper beam as

$$\varepsilon = v/h \tag{2}$$

The equation of the shear beam is written as follows :

$$\mu H \frac{\partial^2 v}{\partial x^2} + P(x,t) - \frac{1}{B}\,Y(x - A(t))\frac{\partial \varepsilon}{\partial t} = 0 \tag{3}$$

where μ is the shear modulus, Y(u) is the Heaviside step function, P is the applied load per unit thickness. From (2) and (3), the equation of the strain is

$$\mu H h \frac{\partial^2 \varepsilon}{\partial x^2} + P - \frac{1}{B}\;Y(x - A(t))\frac{\partial \varepsilon}{\partial t} = 0 \tag{4}$$

We shall introduce dimensionless variables ($\xi = x/h$, $\ell = L/h$, $a = A/h$, $p = Ph/\mu H$, $\tau = B\mu H\,t/h$) so that (4) can be written simply as :

$$\frac{\partial^2 \varepsilon}{\partial \xi^2} + p - Y\,[\xi - a(\tau)]\;\frac{\partial \varepsilon}{\partial \tau} = 0 \tag{5}$$

with the criterion :

$$\varepsilon(a(\tau),\ \tau) = \varepsilon_R \tag{6}$$

and the boundary conditions of the beam.

For example, the boundary conditions are either

$$\text{(Free ends)} \qquad \frac{\partial \varepsilon}{\partial \xi}(0,\tau) = \frac{\partial \varepsilon}{\partial \xi}(\ell,\tau) = 0 \tag{6'}$$

or

$$\text{(Prescribed displacement v)} \qquad (0,\tau) = \Delta_1 \;;\; \varepsilon(\ell,\tau) = \Delta_2 \tag{6''}$$

The set of equations (5)(6), with appropriate boundary conditions implicitly determines the crack length a(τ), hence the crack growth rate $\dot{a}(\tau)$ is completly determined by the loading conditions.

ANALYTICAL SOLUTION

For a semi-infinite beam $\ell = \infty$, the solution to equation (5)(6) can be given in closed form, for the second loading conditions, i.e. with the boundary conditions

$$\begin{cases} \varepsilon(0,\tau) = \Delta \\ \varepsilon(\infty,\tau) = 0 \end{cases} \qquad \tau > 0$$

the solution of (5)(6) is given by

$$\varepsilon(\xi,t) = \begin{cases} \Delta - (\Delta - \varepsilon_R) \qquad \xi\tau^{-1/2}/k & \text{if } \xi \leqslant a(\tau) \\ \varepsilon_R - \sqrt{\pi}((\Delta - \varepsilon_R)/k)\; e^{k^2/4}\;[\operatorname{erf}(\xi\tau^{-1/2}/2) - \operatorname{erf}(k/2)] & \text{if } \xi \geqslant a(\tau) \end{cases} \tag{7}$$

where k is the root of the equation

$$\Delta/\varepsilon_R = 1 + k\,\pi^{-1/2}\,e^{-k^2/4}\,(1 - \operatorname{erf}(k/2))^{-1} \tag{8}$$

This problem corresponds to a relaxation test when a crack displacement is prescribed at the left end of the DCSB.

The crack length is found to be

$$a(\tau) = k\,\tau^{1/2} \qquad (9)$$

Hence the crack growth rate decreases with time as $\tau^{-1/2}$. This solution is very useful for checking a numerical solution using the finite difference method. Comparison between the analytical solution of a semi-infinite beam and the numerical solution of a finite beam, of length ℓ, is shown in Fig.2.

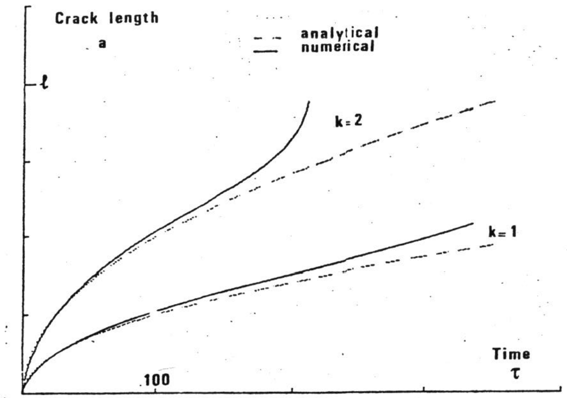

Fig. 2. Numerical and analytical crack length in a relaxation test.

The deviation is due to the right free end of the beam.

Now we made use of the numerical method for solving (5)(6) with arbitrary loading conditions $p(\xi,\tau)$.

CYCLIC LOADING

We consider the cyclic loading

$$p(\xi,\tau) = p\,\delta(\xi) \quad \sin^2(\omega\tau) \qquad (10)$$

corresponding to an oscillating point force at $\xi = 0$ where $\delta(\xi)$ is the Dirac delta function.

The numerical solution of (5)(6), with (10) indicates that the crack grows monotonically with time (Fig. 3.).

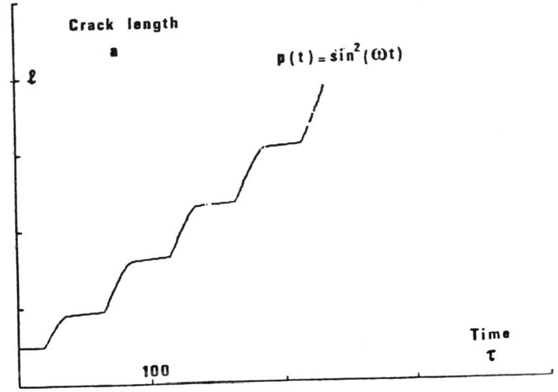

Fig. 3. Crack growth under cyclic loading.

CRACK ARREST

Let the DCSB model be loaded by a constant uniform load p, and at time $\tau = \tau_1$, we reduce the load progressively as follows :

$$p(\tau) = p\,\exp(-\,\alpha(\tau - \tau_1)/\tau_1) \quad , \quad \tau > \tau_1 \qquad (11)$$

Two rates of unloading $\alpha = .5$ (slow) and $\alpha = 1$ (fast) are considered. The results are shown in Fig. 4.

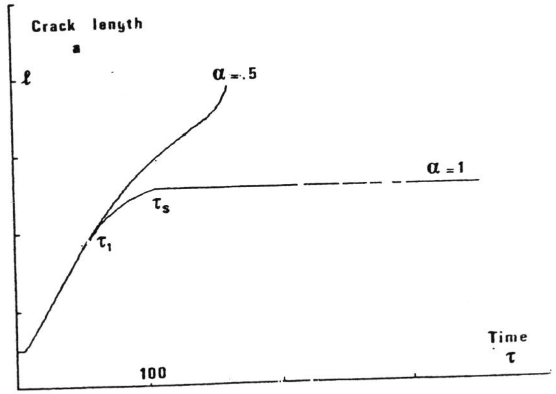

Fig. 4. Crack arrest due to unloading.

For a slow unloading rate (i), the crack is growing up. The effect of unloading is to reduce the crack growth rate during some periode of time. Afterwards the crack accelerates as is shown in Fig. 4.

For a faster unloading rate (ii), the crack decelerates and stops at time τ_s.

CONCLUSIONS

In this paper, we have presented a very simple model of creep crack growth which is based upon a local strain criterion for the viscous strip, and a double cantilever shear beam for describing the continuum.

Simple equations are derived and closed form solutions are presented. This model gives us an useful means for understanding crack growth in viscous materials. The crack growth rate is not generally constant. It can be determined, from the boundary and initial conditions, by solving the coupled equations, where the crack length $a(\tau)$ is an unknown parameter.

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