UNILATERAL CONTACT BETWEEN THE SIDES OF AN ELASTIC CRACK

M. Birades, Ph. Destuynder**, P. Mialon* and J. P. Lefebvre***

ABSTRACT

The purpose of this paper is to analyse the effect of unilateral contact between the lips of a crack in elasticity theory. Usually one considers that the effect of this phenomenon imposes the safety of a structure. It is shown in this paper that it not always true and that unilateral contact can be at the origin of a buttress which changes the stress distribution.

KEYWORDS

Fracture, contact, energy release rate, pipes.

INTRODUCTION

In linear elastic fracture mechanics, it is well known that the rate of energy release in a crack propagation is a characteristic of the crack propagation. A large number of contributions have been devoted to the determination of this quantity which is connected to Rice's integral or stress intensity factors. But the contact between the lips has been very seldom discussed. In such a case, the rate of energy release as a governing quantity in the evolution of the crack has to be resumed. This is the aim of the first section of the paper. Then the numerical implementation of methods taking the unilateral contact into account is presented in the second section. Numerical results are given in the third section.

* Electricité de France - 1, avenue du Général de Gaulle
92141 - CLAMART (France).

** Ecole Centrale des Arts et Manufactures - Grande Voie des Vignes
92290 - CHATENAY-MALABRY (France) and EDF.

I - THE ENERGY RELEASE RATE

Let us consider a cracked body which is supposed to behave as an elastic medium. The displacement field is u and the stress field c. Let us assume - for sake of simplicity - that the solid is two dimensional (figure 1). There exists a crack in the solid, the length of which is ℓ. If the potential energy of the solid is considered as a function of the crack length, we set :

(1) $$F(\ell) = \frac{1}{2} \int_{\Omega(\ell)} \mathrm{Tr}\ (c\ \gamma\ (u)) - \int_{\Gamma_o} \overline{g}\ u$$

where g is the set of external surface forces, $\gamma(u)$ is the strain defined by:

(2) $$\gamma(u) = \frac{1}{2}\ \left(\frac{\partial u}{\partial M} + \overline{\frac{\partial u}{\partial M}}\right)$$

and the stress c is connected to the strain by the classical relationship :

(3) $$c = R\ \gamma(u)$$

where R is the stiffness operator of the medium. In the formulation (1) of the potential energy, the displacement field and the stress field are solutions of an elastic problem which can be formulated as follows. Let $\underset{\sim}{V}$ be the space of all the kinematically admissible displacement fields on Ω. Then the stress field c is such that :

(4) $$\forall\ v \in \underset{\sim}{V}\ ,\ \int_{\Omega} \mathrm{Tr}\ \left(c\ \frac{\partial v}{\partial M}\right) - \int_{\Gamma(\ell)} \overline{n}\ \ c\ |v| = \int_{\Gamma_o} \overline{g}\ v$$

where $|v|$ is the discontinuity in the displacement field on the crack lips $\Gamma(\ell)$.

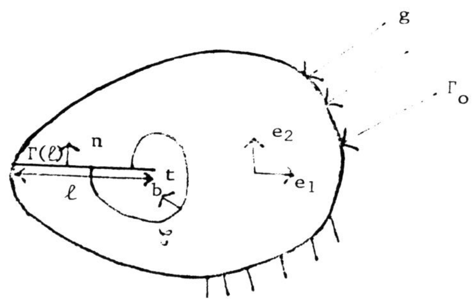

Figure 1

If there is no friction between the lips of $\Gamma(\ell)$ the tangential stress $\overline{n}\ c\ t$ on $\Gamma(\ell)$ is zero. Furthermore the normal discontinuity in the displacement field is such that :

$$|u|_n \ > \ 0 \quad \text{on } \Gamma(\ell)$$

(the notation $|u|_n$ denotes the normal discontinuity). The normal component of the stress is negative or zero ; ie :

$$\overline{n}\ c\ n\ < \ 0 \quad \text{on } \Gamma(\ell)$$

The energy release rate is then by definition (0) :

(5) $$G = - \frac{\partial F}{\partial \ell}\ (\ell)$$

The computation of G is not straightforward even if the result is quite obvious from a physical point of view. The difficulty arises from the unilateral condition. But the contact force does no work in a displacement field. Hence one can guess that the energy balance equation leads to the same expression of G as in classical elasticity. The result would be slightly different if friction were to be taken into account. From a mathematical point of view, it is proved in (1) that :

(6) $$G = - \frac{1}{2} \int_{\zeta} \mathrm{Tr}\ \left(c\ \frac{\partial u}{\partial M}\right)\ \overline{b}\ e_1 + \int_{\zeta} \overline{b}\ c\ e_1$$

where ζ is a path surrounding the crack (see figure 1) and e_1 is the unit vector parallel to the crack, b being the normal unit vector to the curve ζ. This expression is the classical Rice's integral (2). But c and u are now solutions of the unilateral problem. As usual, this expression is only true in plane elasticity. For thermal problems or axisymmetrical structures, one has to add a surface integral on the area enclosed by the curve ζ. From a computational point of view, many papers have shown that the expression (6) of G leads to accurate values of the energy release rate as far as it is possible to choose a curce ζ far enough from the crack tip. But that is not always possible, for instance when there are several cracks very near to one another or in the vicinity of a material discontinuity (bimaterial crossing (4)). In such a case, the computation of G with Rice's integral appears to be very difficult and inaccurate. In other respects, it would be very convenient in a finite element code to be able to compute G with the help of quantities defined near the crack tip, because the description of the nodes on the curve is easier to obtain automatically. This is particularly important when we are wishing to study crack propagation through a structure with an automatic remeshing. For all these reasons, it appears interesting to have a more accurate expression to G (for numerical purpose).

This can be obtained from (6) by an integration by parts (Stokes'formula). As a matter of fact, it has been proved in (1) and (3) that :

$$G = - \frac{1}{2} \int_{\Omega(\ell)} \mathrm{Tr}\ \left(c\ \frac{\partial u}{\partial M}\right)\ \mathrm{div}\ \theta + \int_{\Omega(\ell)} \mathrm{Tr}\ \left(c\ \frac{\partial u}{\partial M}\ \frac{\partial \theta}{\partial M}\right)$$

where θ is any vector field satisfying the following properties :

(i) on a neighbourhood of the crack tip, say V_0, θ is a constant vector, the components of which are : (in the axes of figure 1).

$$\theta = \binom{1}{0}$$

(ii) outside of another neighbourhood V_1 of the crack tip, containing V_0, θ is identically equal to zero.

The computation of G from (8) is as a matter of fact, limited to the computation of an integral over a surface surrounding the crack (excepted for thermal or axisymmetrical problems). It is well known in numerical analysis that such an integral is able to be computed with higher accuracy, than an integral over a curve. Hence, this enables us to compute G closer to the crack tip than with Rice's integral. The reader will find comparisons of the two methods in the references (3) and (4). Let us now consider an application of this method to a particular problem arising in a pipe of a nuclear reactor. It is an axisymmetrical situation.

II - THE PROBLEM TO BE SOLVED

Let us consider a piece of pipe having the shape drawn in figure 2. The reinforcements inside the pipe are used as flanges to support a circular valve in order to block the pipe when one wants to isolate the reactor from the rest of the circuit. A simplified model of this structure is given in figure 3. The vicinity of the two cracks (four crack tips) involves a mesh refinement as in figure 4.

a) Boundary conditions (figure 3)

At the end A, we assume that the longitudinal displacements are locked and a reaction - due to the rest of the pipe - is introduced at the other end B. An internal pressure is applied on the pipe. When the valve is open, this pressure is P = 30 bars. When the valve is closed, the internal pressure is still 30 bars on the side of the reactor and 1 bar on the other side. The closed valve presses over the reinforcements. The pressure is approximately 230 bars (in order to equilibrate the valve).

b) Computations

The energy release rate G has been computed for each crack tip when the valve is closed. In order to compare the effect of the unilateral contact two models have been studied :

- with unilateral contact W.U.C.

- without unilateral contact W.N.U.C.

III - THE RESULTS

The numerical results obtained are summarised in table 1.

Crack tip	1	2	3	4
G.WUC (no unit)	1.09	0.35	$1{,}7\,10^{-2}$	54
G.WNUC	0.99	0.53	$1{,}7\,10^{-2}$	51

Table I

It appears from these results that unilateral contact produces a buttress which transfers a part of the force, acting on the crack tip 2, onto crack tip 1. The difference in the present computation is about 10% on G. If use is made of a Paris law to characterise fatigue propagation, one has to consider an expression such as.

$$\frac{d\ell}{dN} = c\,(\sqrt{G})^n$$

where n would be between 4 and 6 (and N is the number of cycles, ie the number of closings of the valve). The error, if we neglect the unilateral contact, would be between 20% and 30% (furthermore in the wrong sense for crack tip 1). Obviously, the conclusions are the opposite for crack tip 2.

The conclusion of this paper would still apply if thermal problems were considered. Then the heat transfer would certainly be different depending on whether the two lips of a crack were in contact or not.

REFERENCES

(0) H.D. BUI : Mécanique de la rupture fragile - Masson editor (1978).

(1) Ph. DESTUYNDER : Calcul des forces d'avancement d'une fissure en tenant compte du contact unilatéral entre les lèvres de la fissure. C-R Acad. Sc. Paris t. 296, Série II, p. 745-749.

(2) J. RICE : Fracture, vol III, Liebowitz, ed. Academic Press 1968 p. 191.

(3) Ph. DESTUYNDER, M. DJAOUA, S. LESCURE : Quelques remarques sur la mécanique de la rupture élastique - J.M.T.A. Vol. 2 N° 1 (1983) p. 113 à 135.

(4) Ph. DESTUYNDER, J.P. LEFEBVRE, S. LESCURE, P. MIALON : Nouvelles méthodes en mécanique de la rupture numérique. Conférence invitée au 3ème colloque international sur les Méthodes Numériques de l'Ingénieur Paris (1983), edited by Masson, Paris.

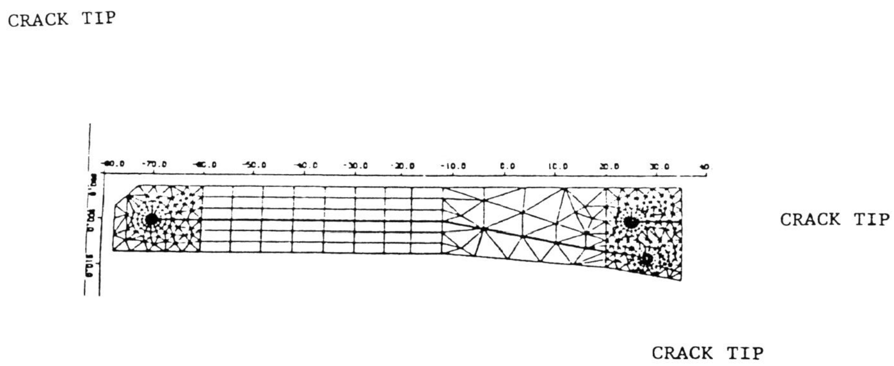

Figure 4
Mesh refinement near the crack tips

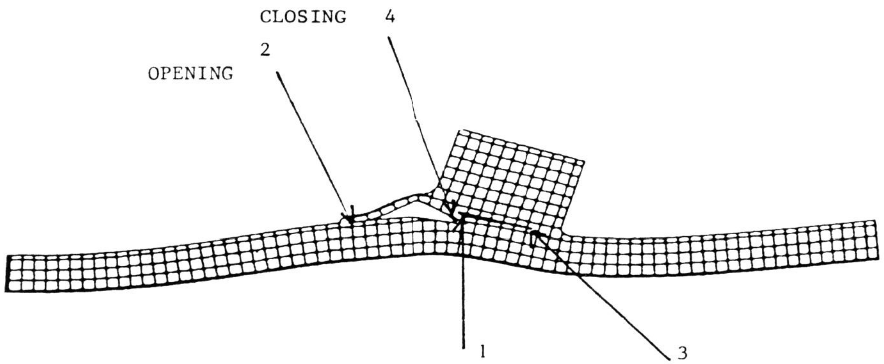

Figure 5
Deformed configuration when the valve is closed

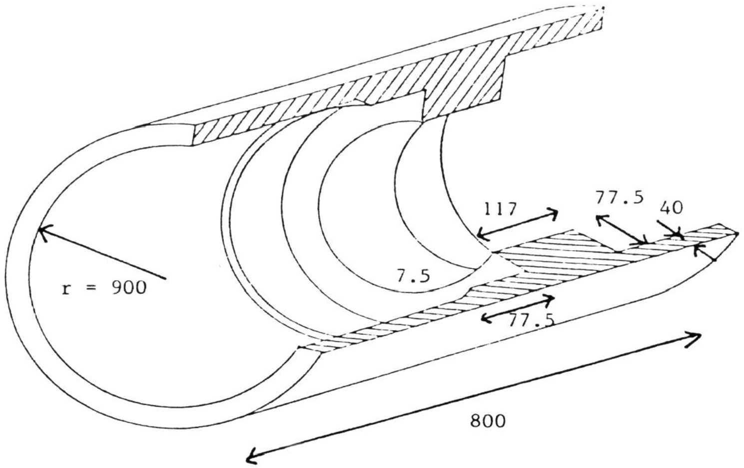

Figure 2
A section of the pipe

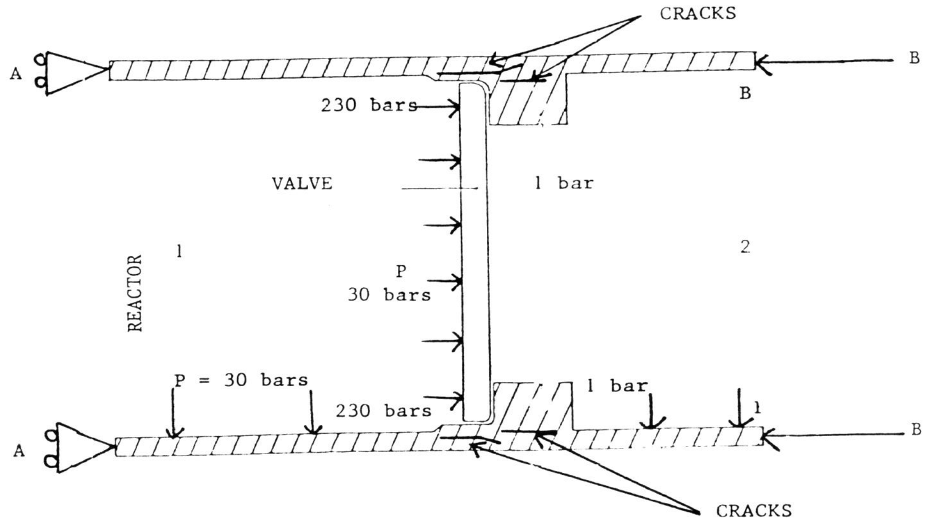

Figure 3
The axisymmetrical model