THE UNSTABLE EXTENSION OF INCLINED CRACKS UNDER UNIAXIAL TENSION AND COMPRESSION

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ABSTRACT

The path of unstable extension of the inclined cracks subjected to uniaxial tension and compression is determined theoretically using the criteria of maximum tangential stress (MTS), maximum tangential principal stress (MTPS), maximum tangential strain (MTSN), and strain energy density (SED) from the stress distribution existing prior to the onset of extension. Related experimental results (some of which are new) are presented to assess the accuracy. Out of the four criteria the MTPS criterion provides the best prediction of the experimental observations and the SED criterion is the least satisfactory.

KEYWORDS

The inclined crack problem; prediction of unstable crack path; unstable crack extension; criteria for mixed mode fracture; maximum tangential stress criterion; maximum tangential principal stress criterion; maximum tangential strain criterion; strain energy density criterion.

METHOD OF PREDICTION

The unstable crack path can be predicted using the criteria of maximum tangential stress (MTS), maximum tangential principal stress (MTPS), maximum tangential strain (MTSN), and strain energy density (SED). Most of these criteria have been used previously to predict the direction of initial crack extension, when it is postulated that the crack extends in the direction, radially outwards from the crack tip, which corresponds to the location of MTS (Erdogan and Sih, 1963), MTPS (Maiti and Smith, 1983a, 1983b), MTSN (Wu, 1974), and SED (Sih, 1973), respectively. The direction of initial extension (θ_c) according to the

four criteria is illustrated in Fig.1. The same criteria have been extended to identify the crack paths as the locus of the points of MTS (Maiti, 1980; Maiti and Prasad, 1980), or MTPS (Maiti and Smith, 1983a, 1983b), or MTSN (Maiti and Smith,1983a, 1983b), or minimum SED (Kipp and Sih, 1975). In other words, a smooth curve drawn through the points of either the MTS, or MTPS, or MTSN, or minimum SED, on circles of different radii from the original crack tip gives the crack path. Their ability to predict the experimental observations is not equally good (Maiti and Prasad, 1980; Maiti and Smith, 1983a, 1983b); the accuracy depends on the problem at hand. There are situations where predictions based on the SED criterion are the least satisfactory (Maiti and Prasad, 1980; Maiti and Smith, 1983a, 1983b). The theoretical results based on the MTPS criterion are mostly closest to the observed crack paths.

ANALYSIS

The entire stress field for both a slit and an elliptical crack, which is required for a prediction of the crack path, is obtained here using the exact solution given in Chang and Wu (1980). The tangential stress σ_θ, shear stress $\tau_{r\theta}$, tangential strain ϵ_θ, and SED are calculated at intervals of 0.2° in the θ-direction (positive anticlockwise) at different radii (r/a = 0.001, 0.005, 0.01, 0.05, 0.10, 0.20, 0.30, 0.40, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75 and 2.00) to give the locations of MTS, MTPS, MTSN, and minimum SED. The analysis is two-dimensional, describing a state of plane stress. Poisson's ratio ν is taken as 0.35 for all the computations. The crack extends from the tip of a slit (or line) crack and it is assumed to extend from the point of maximum tangential stress on the boundary of an elliptical crack.

RESULTS

Tension

Slit cracks (Fig.2) subjected to unaxial tension at different orientations (β = 7.5°, 15°, 30°, 45°, 60°, 75° and 90°) are studied. The crack paths predicted by the four criteria are shown in Fig.2. The MTS, MTPS and MTSN criteria differ so little that they are shown by single curves, the solid lines. The predictions by the SED criterion are shown by chain dotted lines. Experimental results on perspex (or PMMA) (Kipp and Sih, 1975), for β = 30° and 45°, are included in Fig.2. The predictions by the first three criteria differ more markedly from those based on the SED criterion for $\beta \leq 15^\circ$ than for $\beta \geq 30^\circ$. For β = 30° and 45° all the four criteria are equally good.

Experiments in Tension

Tensile tests were carried out for inclinations β = 7.5° and 15° with 0.304 m square specimens made from 3 mm thick perspex sheet.

Cracks were first cut with a 0.41 mm jeweller's saw and then extended by at least 1 mm at each end by a sharp razor blade. The final crack length (2a) was 90 mm and 60 mm respectively. Specimens were loaded through friction grips by dead weights. Two fractured specimens are shown in Fig.3. The experimental results obtained from at least eight profiles are shown in Fig. 2. For β = 15° all the criteria appear equally good, but for β = 7.5° the SED criterion is the least satisfactory.

Compression

An elliptical crack with b/a = 0.04 (Fig.4) under uniaxial compression at orientations β = 0°, 15°, 30°, 45°, 60° and 75° is analysed. The crack paths are shown upto r/a = 1.0 in Fig.4 and upto r/a = 0.10 in Fig.5. The experimental observations for β = 30° with a photoelastic material (Brace and Bombolakis, 1963) is included in Fig.5.

Under compressive loading, the tangential stress σ_θ becomes gradually compressive at a finite distance from the point where extension starts. Under tensile loading, σ_θ is positive and almost constant as the distance from the point of initiation increases. Because of this difference, tensile loading is inherently less stable than compressive loading so the extent of unstable extension is always less under compressive states of stress. For this reason, the experimental and theoretical results are compared in Fig.5 upto r/a = 0.10 only. The predictions by the four criteria are very different except for β=0°. The trend of the predictions by the criteria of MTS, MTPS and MTSN is the same and it is different from that by the criterion of SED. The experimental result for β = 30° is closest to the prediction by the criterion of MTPS.

DISCUSSION

The prediction for crack paths based on the criteria of MTSN and SED are dependent on the magnitude of Poisson's ratio. (For Poisson's ratio ν = 0 the predictions based on the criteria of MTS and MTSN are the same because the two criteria are then the same). Notwithstanding this fact two points are quite clear. Firstly, in the case of tension the difference between the criteria of MTS, MTPS and MTSN on the one hand and SED on the other increases as the inclination of the crack with the direction of loading is reduced. For inclinations $\beta \geq 15^\circ$ all four criteria give almost equally good matching to experiment, but for β = 7.5° the SED criterion is less satisfactory. Secondly, in the case of compression the predictions of the four criteria are different; except for an orientation aligned with the direction of loading, the difference between any two criteria is much larger than in the case of tension.

In the case of compression, the SED criterion indicates a trend of extension which is different from that of the other three. The available (though limited) experimental data favour the MTPS criterion. It is relevant to note here that this criterion also gives the best description for bar shearing (Maiti and

Prasad, 1980) and crack-face-point-loaded geometries (Maiti and Smith, 1983b).

The present study together with those reported earlier (Kipp and Sih, 1975; Maiti, 1980; Maiti and Prasad, 1980; Maiti and Smith, 1983a, 1983b) indicate that the stress field existing prior to the start of extension of a crack has a dominant influence on the path of extension. It may be possible to improve the accuracy of the results using a step-by-step analysis, thereby allowing changes in the prior stress field to influence the prediction. Such approaches present considerable mathematical difficulties for a rigorous treatment, and demand considerable computer time. For this reason, the simple approximate method given here has attractions.

CONCLUSIONS

1. The predictions for crack paths by the criteria of MTS, MTPS and MTSN are almost the same in the case of tension, but they differ considerably in the case of compression.

2. The trend of predictions for the crack paths by the SED criterion is the same as the other three criteria in the case of tension, but it is different in the case of compression.

3. In the case of tension, the ability to predict the experimental observations on the crack path is equally good for all the criteria except the SED criterion whose performance deteriorates as the inclination of the crack is reduced below 15^{0}.

4. In the case of compression, the MTPS criterion provides the best prediction of the experimental observations.

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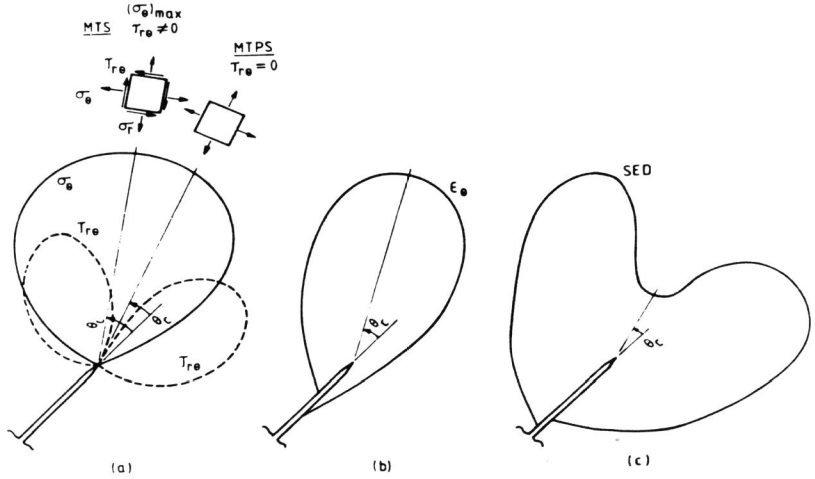

Fig.1. Direction of initial crack extension θ_c according to the criteria of (a) MTS and MTPS, (b) MTSN and (c) SED.

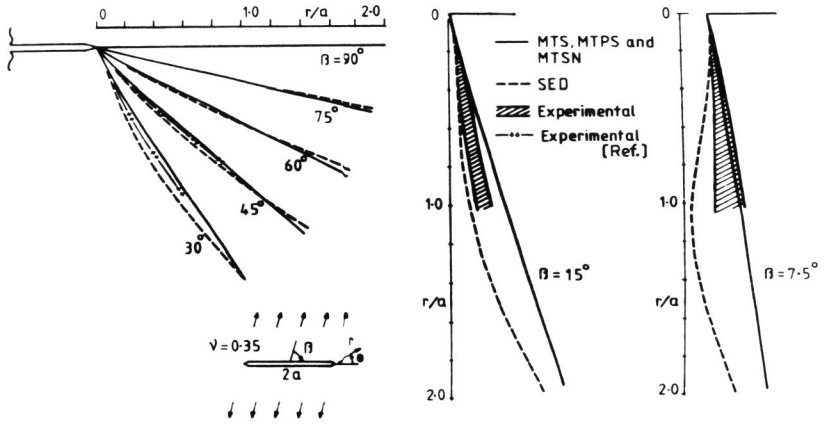

Fig.2. Comparison of predicted and experimental (Kipp and Sih, 1975; Present work) crack paths for a slit crack subjected to uniaxial tension at different orientations.

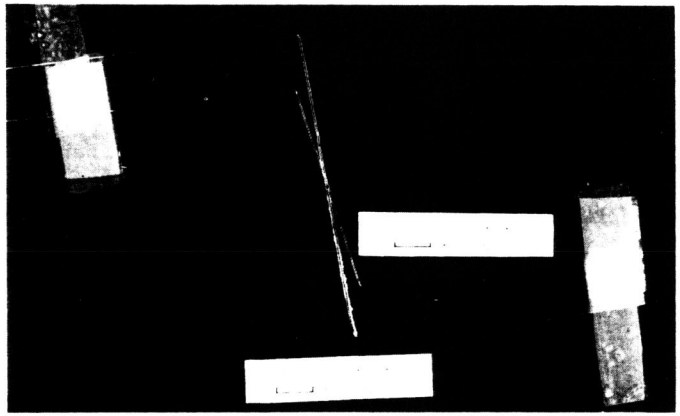

Fig.3. Two typical fractured perspex specimens for tension applied at 7.5° and 15° to the crack.

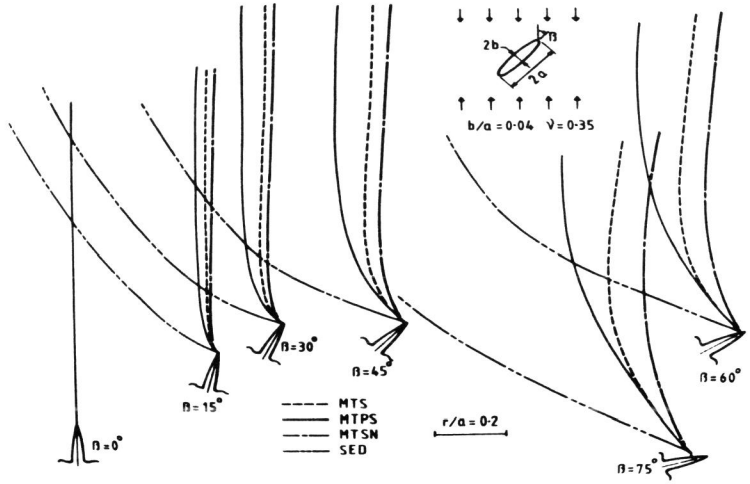

Fig.4. Comparison of predicted crack paths (upto r/a = 1.0) for an elliptical crack (b/a = 0.04) subjected to uniaxial compression at different orientations.

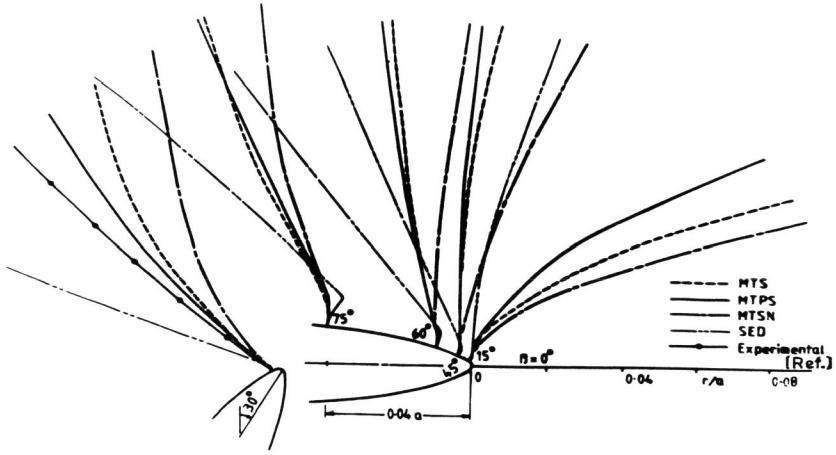

Fig.5. Comparison of predicted and experimental (Brace and Bombolakis, 1963) crack paths (upto r/a = 0.10) for an elliptical crack (b/a = 0.04) subjected to uniaxial compression at different orientations.