STRESS INTENSITY FACTORS AT THE TIPS OF CRACKS EMANATING FROM HOLES WITH MISFIT FASTENERS

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ABSTRACT

Stress intensity factors at the tips of cracks emanating from circular holes filled with misfit fasteners is presented. Misfit fasteners could be of interference or clearance type. The problem considered is a filled hole with two diametrically opposite symmetric radial cracks in a rectangular plate loaded along the edges in uniaxial tension. The main aspect of interest is the non-linear behaviour arising due to varying contact at pin-hole interface with increasing load. Such non-linearity is handled by an "Inverse" postulation of the problem. Finite Element Method of analysis using isoparametric quadrilateral and sector elements is employed. The stress intensity factor is evaluated from the stress field using a virtual crack closure concept. Numerical results are presented to bring out the effect of the crack length, the hole size and the pin-hole interfacial conditions on the load-contact relations and the stress intensity factors. Concise data presentation was achieved by an appropriate choice of non-dimensional parameters.

KEYWORDS

Cracks around holes, Misfit fasteners, Stress intensity factors, Finite element methods.

INTRODUCTION

Free or filled holes in structures are sources of stress concentration and potential locations of initiation of failure. When the structure is subjected to static or fatigue loading cracks would initiate at these locations and grow to cause final fracture. Based on Linear Elastic Fracture Mechanics the most important parameter influencing this fracture behaviour is the Stress Intensity Factor (SIF) at the tips of these cracks.

Many earlier workers (Bowie, 1956; Owen, 1973; Cartwright, 1972; Broek, 1975) made analytical and experimental investigations of the problems of plates with radial cracks emanating from free circular holes. They studied the effect of geometric and material parameters on the SIF in both infinite and finite plates. Infinite domains were treated with analytical methods, whereas finite domains were analysed using finite element methods. Experimental determination of

SIF in finite strips were made using compliance measurements.

The problem of plates with cracks around holes filled with misfit pins (interference or clearance) presents particular analytical difficulty. When such plates are subjected to increasing load, the pin-hole interface exhibits partial contact/separation and the extent of contact varies non-linearly with the applied load. This leads to moving contact problem and the SIF at crack tip varies non-linearly with applied load. Special techniques are required for analysis of such problems. The only contribution to problems with misfit pins appear to be by Grandt, Jr. (1975). This is limited to cracks from holes filled with interference fasteners and they have not extensively dealt with the non-linear partial contact behaviour at the pin-hole interface. A comprehensive summary of design charts based on all the above contributions is given by Broek (1975) and Karlsson (1978). It can be seen from these papers that the information on the problem of cracks around circular holes with misfit pins is scanty and meagre.

In this paper, a two-dimensional plane stress problem of a finite plate with radial cracks emanating from a circular hole filled with misfit pins is considered. Finite Element Method of analysis using isoparametric quadrilateral and sector elements is employed. The SIF at the tip of the crack is evaluated from the stress field using virtual crack closure method (Rybicki, 1977). A comprehensive and extensive treatment of the problem of a misfit pin in a plate is reported earlier by the authors and co-workers (Eshwar, 1979; Rao, 1978; Mangalgiri, 1980, 1983). The present paper is based on a similar concept. The non-linear behaviour arising due to varying contact at pin-hole interface is handled by an Inverse postulation of the problem. The inverse formulation permits a unified method of analysis for interference and clearance pins. Numerical studies are carried out for various geometric parameters of the problem. Also the limiting cases of completely smooth (zero friction) and rough (infinite friction) interfacial conditions are studied. Results are presented using an appropriate choice of non-dimensional parameters which enabled concise data presentation.

CONFIGURATION, FEM MODELS AND BOUNDARY CONDITIONS

A finite rectangular plate with a circular hole of diameter 2R is filled with a rigid pin of diameter $2R(1+\lambda)$ as shown in Fig. 1a. The plate dimensions are 2H, 2W. The parameter λ, called proportional interference is positive for interference fit, zero for push fit and negative for clearance fit. Two symmetrically placed cracks are assumed as shown in the figure, such that the distance between the crack tips is '2a'. The plate is assumed to be isotropic (E, ν) and subjected to uniform edge tractions on the edges $x = \pm H$.

Because of double symmetry, it is sufficient to analyse a quarter of the domain (Fig. 1b). Consider the hole to be filled with an interference fit. Even at zero load, there are initial compressive stresses all round the hole periphery and the cracks open in mode I. As the plate is loaded, there is a redistribution of stresses at hole periphery and the compressive radial stresses are relieved at points A, A'. Above certain load, the pin and the plate would separate from each other and later the contact/separation region would vary non-linearly with applied load. The problem and thus SIF at crack tip are linear only till the initiation of separation.

Let us now consider a clearance fit pin. When the plate is not loaded, there is no contact around the hole boundary. As the plate is loaded, contact would initiate first at points B, B' at a particular level of load. Till then the problem is linear and corresponds to the case of cracks emanating from a free circular hole. Above the load for initiation of contact, there is a non-linear growth of

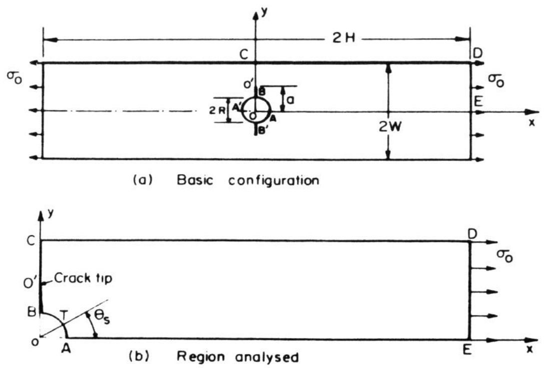

Fig. 1. Plate with filled hole and diametric cracks.

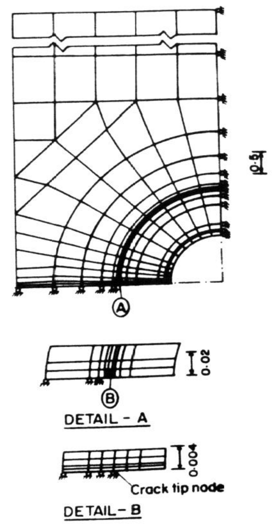

Fig. 2. Typical FEM mesh

contact region and the SIF at crack tip varies non-linearly with applied load.

A typical finite element model used for the analysis is shown in Fig. 2. Since the hole boundary is circular, a number of layers of isoparametric sector elements are used in an annular region containing the hole. In the space remaining between this region and the rectangular outer boundary, Isoparametric quadrilaterals and CST elements are used. High stress gradients occur near the periphery of the hole and near the crack tip and the finite element mesh is made finer in these regions. The smallest sector element used near the crack tip has a dimension ($\Delta r, \Delta\theta$ = 0.002, 0.05°). In all the problems studied, the degrees of freedom used in the finite element mesh varied between 600-850.

The nodes on the hole boundary are numbered consequtively from 1 to M. Node 1 to node T represents the separation region and contact extends between node T to node M. Thus, T is the transition node. The radial boundary conditions on hole periphery to be imposed can be written as

$$F_{ri} = 0 \, , \, U_i \geq a\lambda \, , \, 1 \leq i \leq T$$
$$U_i = a\lambda \, , \, F_{ri} \leq 0 \, , \, T \leq i \leq M \qquad (1)$$

The tangential boundary conditions depend on the interfacial frictional conditions (smooth or rough). For example in an advancing separation problem such as that in an interference fit,

$$\text{Smooth } (\mu = 0){:} \quad F_{\theta i} = 0 \, , \, 1 \leq i \leq M$$

$$\text{Rough} \ (\mu = \infty) : F_{\theta i} = 0 \ , \ 1 \leq i \leq T$$
$$V_i = 0 \ , \ T \leq i \leq M \qquad (2)$$

where F_r, F_θ are radial and tangential nodal forces and U, V are radial and tangential nodal displacements. The remaining boundaries of the problem are the external boundary, axes of symmetry and crack faces. The applied force boundary conditions are imposed on external boundary, zero shear and zero normal displacements are imposed on axes of symmetry and stress free boundary conditions are imposed on crack faces.

METHOD OF SOLUTION : INVERSE FORMULATION

A direct method of analysis of partial contact could be as follows: For a given applied load what is the extent of separation (θ_s) and contact ($\theta_c = \pi/2 - \theta_s$). Such an approach requires an iterative method of solution. Since the contact/separation regions progress monotonically with applied load the problem could be effectively posed in an inverse way as follows: Given an arc of separation (θ_s) and an arc of contact ($\theta_c = \pi/2 - \theta_s$) what is the load required to maintain the configuration ? When the problem is posed this way the solution is direct and avoids the complicated iterative procedure.

Considering the configuration and the boundary conditions of the problem, the method of analysis using inverse formulation is direct except for the satisfaction of inequality constraints in Eq.(1). It is seen that they require the satisfaction of two radial conditions, viz., $U = a\lambda$ and $F_r = 0$ at the transition point T. A procedure for dealing with such problems is given by Mangalgiri (1983) and is not repeated here. This procedure minimises computational effort compared to direct iterative procedure. The strain energy relese rate and the corresponding SIF were evaluated from the stress field based on the virtual crack extension method proposed by Rybicki (1977).

NUMERICAL RESULTS

Numerical investigations are conducted for a rectangular strip of H/W = 4, containing filled holes of sizes R/W = 0.25 and 0.5, with different crack lengths. Results are presented for both smooth and rough interfacial conditions. In all the cases the transition point is progressively changed to get the load-contact relations and the correspondig SIF values. These results are presented using non-dimensional parameters. $E\lambda/\sigma_o$ and $K_I/(\sigma_o + \sigma_i)\sqrt{\pi a}$ where $\sigma_i = [E\lambda/(1-\nu)]$, is the hoop stress around the circular hole due to initial compressive radial stresses in interference fit and $\sigma_i = 0$ for clearance fit. The results are presented in Table 1 and Figs. 3-10.

The first step is to compare the present results with existing solutions and also to study the convergence of present solution with incresing refinement of the mesh, in particular with the size of the smallest element used at crack tip. Table 1 shows the results for the case of radial cracks emanating from free circular hole. The solution is exhibiting an oscillatory convergence and is within 1 % of the results from Karlsson (1978).

Fig. 3 shows the load-contact relations for the case of R/W = 0.25 for various crack lengths. The results above x-axis (+ve $E\lambda/\sigma_o$) correspond to the case of

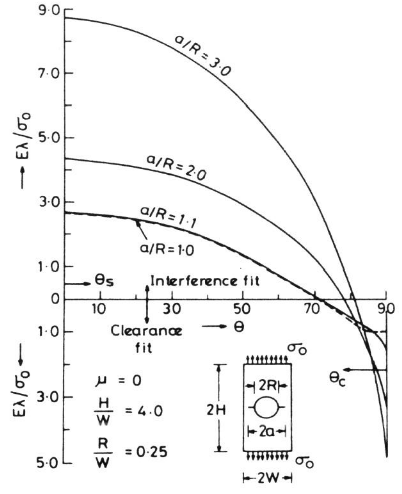

Fig. 3. Effect of crack length on load-contact behaviour.

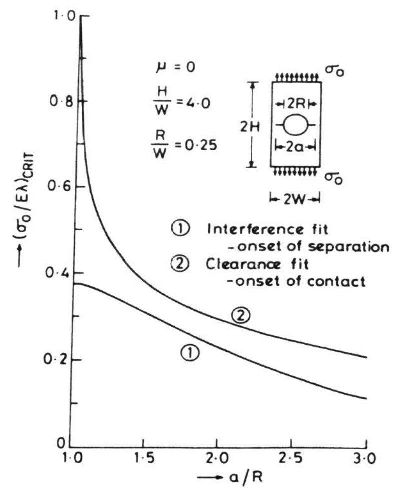

Fig. 4. Variation of onset of contact or separation load with crack length.

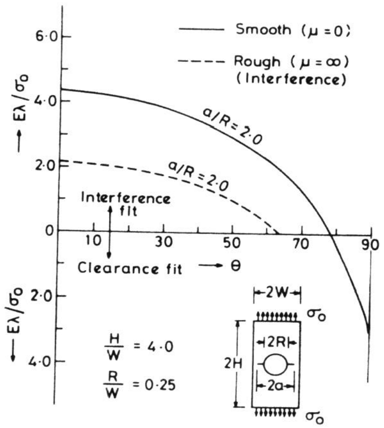

Fig. 5. Comparison of load-contact behaviour for smooth and rough interfaces.

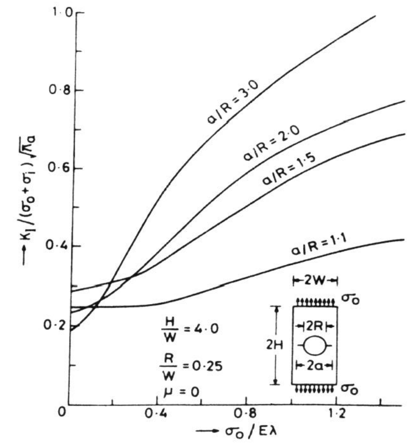

Fig. 6. Nonlinear SIF variation with applied stress: Interference fit.

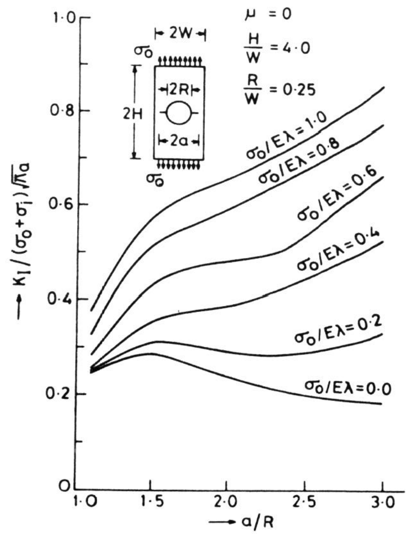

Fig. 7. SIF variation with crack length: Interference fit.

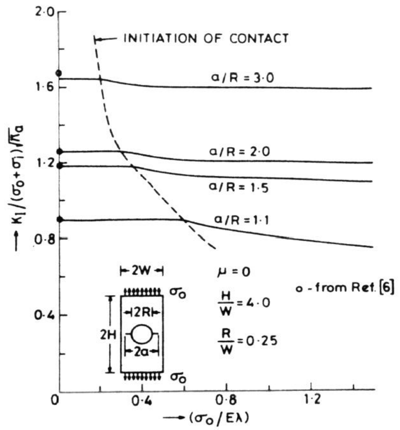

Fig. 8. Nonlinear SIF variation with applied stress: Clearance fit.

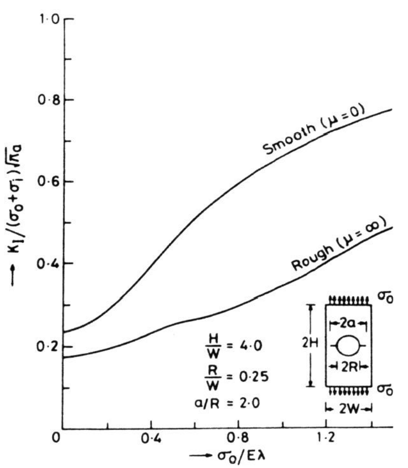

Fig. 9. Nonlinear SIF variation with applied stress: Smooth and rough interfaces.

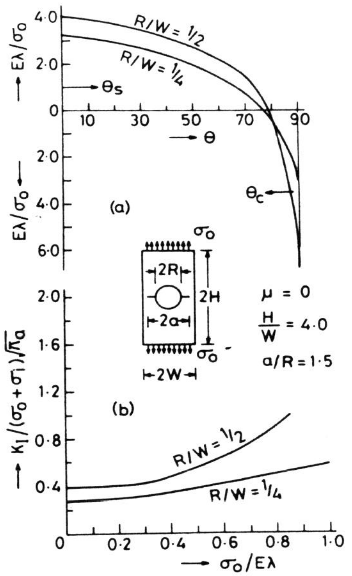

Fig. 10. Effect of ratio of hole radius to plate width.

TABLE 1 Convergence of SIF with Mesh Refinement: Free Circular Hole with Radial Cracks

Size of smallest element at crack tip	$K_I/\sigma_o\sqrt{\pi a}$
0.004	1.257
0.002	1.245
0.001	1.255
From Karlsson (1978)	1.25

interference (+ve λ) and those below x-axis (-ve $E\lambda / \sigma_o$) correspond to the case of clearance fit (-ve λ). The results for the case of no crack (a/R = 1.0) are also shown in dotted line to compare the limiting case as crack length tends to zero. Since the presence of crack relieves the initial compressive stresses for the case of iterference fits, it is seen that the separation initiates at a lower load and progresses faster with increasing crack length (Fig. 4).

Fig. 5 presents the difference between the load-contact relations for smooth and rough interfaces for a given crack length. For the case of no crack, Eshwar (1979) showed that the effect of roughness was to delay separation. Now it is seen that the same is also true in the presence of a crack. The variation of SIF with applied load for interference fits is seen in Fig. 6. The SIF varies linearly till the onset of separation and in the partial contact region the variation is non-linear. It is clearly seen that the partial contact significantly increases the SIF and it is more pronounced at higher crack lengths. The results of Fig. 6 are cross-plotted to provide the variation of SIF with crack length as shown in Fig. 7. Even at zero applied stress K_I is significant due to the initial stress caused by the interference. As the crack length tends to zero it is expected that K_I tends to zero very sharply.

The SIF variation for the case of clearance fit for different crack lengths is seen in Fig. 8. Till the plate and the pin are in contact with each other (onset of contact) $K_I/\sigma_o\sqrt{\pi a}$ remains constant and later varies non-linearly with applied stress. Comparitive values from the figures given by Karlsson (1978) for the case of free circular hole are shown on the y-axis ($\sigma_o/E\lambda = 0$). The two results compare very well.

Comparing SIF variation with applied stress for smooth and rough interfaces in Fig. 9, it is seen that presence of interfacial friction significantly lowers the SIF. The effect of hole diameter on the load-contact relations and the SIF variation is shown in Fig. 10. The influence of the diameter of the hole is prominent and as expected, increase in hole size leads to larger values of SIF at any given load.

CONCLUSIONS

An accurate method of estimating SIF for the case of radial cracks emanating from holes filled with misfit pins in two-dimensional plates under plane stress is presented. The problem is non-linear and is effectively solved by using an inverse formulation. It is shown that partial contact behaviour at pin-hole interference has significant effect on the SIF. The effect of geometric parameters and the limiting interfacial frictional conditions (smooth and rough) are studied. The results are of considerable interest to the problem of fatigue crack growth in structures with fastener joints. The present method of analysis is being extended for practical problems involving cracks around pin-loaded lugs and eyebars.

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