SOME PROBLEMS ON MEETING, DEVIATION AND BRANCHING OF SLIP-LINES

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ABSTRACT

The paper embraces some problems on meeting, deviation and branching of slip-lines in the conditions of plane strain.

The first section deals with the classical mechanism of cracking in metals according to which the cleavage crack emerges at the meeting of two dislocations or slip-lines. The theory of this phenomenon is considered with the assumption that the length of the emerging opening mode crack is small in comparison with the length of the meeting slip-lines. The boundary problem of the plane theory of elasticity thus obtained is solved by means of Wiener-Hopf method. The formula for the stress intensity factor at the end of the crack has been obtained. The dependence of total opening of slip-lines at the point of their meeting on operating stresses has been found.

The second section treats with the solving of the question about the possible local deviation or branching of slip-lines. With this aim the authors consider the singular problem of the theory of elasticity for semi-infinite straight slip-lines with a branch emerging from its apex and running at a certain angle with its extension. The precise closed solution of the problem is obtained by Wiener-Hopf method. The analysis of this solution shows that in homogeneous and isotropic bodies corner points and branching points on the contour of the slip--line are impossible.

KEYWORDS

Slip-line, dislocation, crack, theory, meeting, deviation, branching, fracture mechanics.

INTRODUCTION

There are many works devoted to the branched (or kinked) crack problem in fracture mechanics. We believe the most correct approach to this problem is that based on the viewpoint of singu lar perturbation and of vanishing small scale formulated first by one of the authors (see Cherepanov, 1979). This approach is used below for solution of two similar problems concerning slip--lines.

EMERGENCE OF A CRACK AT THE MEETING OF TWO SLIP-LINES

Let two slip-lines meet at a point O (Fig.1). We shall assume that the conditions of plane deformation are valid. Then, due to the concentration of stresses at the point O, there appears a possibility for emergence of a cleavage crack at this point and in its vicinity. This mechanism of crack formation in metals (Kipnis and Cherepanov, 1983) is similar to that of Stroh (1954) and Cottrell (1958), see, in detail, Honeycombe (1968).

Let us consider only the initial development of the crack. We may assume that its length is small in comparison with the characteristic length of slip-lines and dimensions of the body itself. Confining ourselves to a symmetrical case, we come to the singular boundary problem of the plane theory of elasticity whose boundary conditions have the shape (Fig.2)

$$\theta=\alpha,\ [\sigma_\theta]=[\tau_{r\theta}]=0,\quad [u_\theta]=0,\ \tau_{r\theta}=\tau_s$$

$$\theta=0,\ \tau_{r\theta}=0;\quad \theta=\pi,\ \tau_{r\theta}=0,\ u_\theta=0$$

$$\theta=0,\ r<\ell,\ \sigma_\theta=0;\quad \theta=0,\ r>\ell,\ u_\theta=0$$

Here $\sigma_\theta, \tau_{r\theta}, \sigma_r$ are stresses, u_θ, u_r - displacements, $[N]$ - jump of value N, τ_s - shear yield point. Due to symmetry only half-space $0<\theta<\pi$ is considered. It is assumed that $\pi/2<\alpha<\pi$.

At infinity the given asymptotics which represents the solution of the symmetrical problem of the theory of elasticity concerning the plane which contains two slip-lines meeting at the point O without a cleavage crack is realized (Fig.2). This solution is determined by the formulae:

$$\sigma_\theta=q+\tau\cos 2\theta,\quad \tau_{r\theta}=\tau\sin 2\theta,\quad \sigma_r=q-\tau\cos 2\theta$$
$$u_\theta=\frac{1+\nu}{E}\tau r\sin 2\theta,\quad u_r=\frac{1+\nu}{E}[(1-2\nu)q-\tau\cos 2\theta]r \qquad (1)$$
$$\tau=\tau_s/\sin 2\alpha$$

where q is an arbitrary real constant characterizing the intensity of uniform two lateral extension, E - Young's modulus, ν - Poisson's ratio. The constant q is considered as given in the formulation of this problem.

As we see, the solution (1) in stresses is continuous like in strains. For stresses it has the form characteristic of the general solution of the plane problem of the theory of plasticity with the condition of plasticity $(\sigma_r-\sigma_\theta)^2+4\tau_{r\theta}^2=4k^2$ where k is the plasticity constant equal to $-\tau_s/\sin 2\alpha$ for solution (1) Value k equals τ_s only at $\alpha=3\pi/4$, at other α will be $k>\tau_s$. Value τ_s characterizes a well-developed plastic deformation along the slip-line, and value k characterizes the initial "delayed" plastic deformation of a body element of a larger size (Cherepanov, 1979). Macroplasticity is realized as a result of forming and developing of many slip-lines which divide elastic zones (Cherepanov, 1979). In physical terms it means that on the scale large in comparison with the emerging crack (but small in comparison with the characteristic size of the body and slip-lines) the beginning of plastic deformation is the case, corresponding to the angle point M of the diagram σ-ε (Fig.3).

By means of Mellin's transform the initial problem is reduced to the following functional Wiener-Hopf equation:

$$\frac{\sigma}{p+1}+\Phi^+(p)=-\operatorname{tg} p\pi\, G(p)\Phi^-(p) \qquad (2)$$
$$(-\varepsilon_1<\operatorname{Re}p<\varepsilon_2,\quad 0<\varepsilon_{1,2}<1)$$

$$\Phi^+(p)=\int_1^\infty[\sigma_\theta(\rho\ell,0)-q-\tau]\rho^p d\rho$$

$$\Phi^-(p)=\frac{E}{2(1-\nu^2)}\int_0^1\left.\frac{\partial u_\theta}{\partial r}\right|_{\substack{r=\rho\ell\\ \theta=0}}\rho^p d\rho$$

$$G(p)=\frac{2\delta_2(\sin^2 p\alpha-p^2\sin^2\alpha)+\Delta_1\Delta_2}{\operatorname{tg}p\pi(\delta_1\Delta_2+\Delta_1\delta_2)},\quad \sigma=-q-\tau$$

$$\Delta_1=\sin 2p\alpha+p\sin 2\alpha,\quad \Delta_2=\sin 2p(\pi-\alpha)-p\sin 2\alpha$$

$$\delta_1=\cos 2p\alpha-\cos 2\alpha,\quad \delta_2=\cos 2p(\pi-\alpha)-\cos 2\alpha$$

The solution of the equation (2) is as follows (Kipnis and Cherepanov, 1982):

$$\Phi^+(p)=-\frac{\sigma p\, G^+(p)}{(p+1)K^+(p)}\left[\frac{K^+(p)}{pG^+(p)}+\frac{K^+(-1)}{G^+(-1)}\right]\ (\operatorname{Re}p<0) \qquad (3)$$

$$\Phi^-(p)=\frac{\sigma K^+(-1)\,K^-(p)\,G^-(p)}{(p+1)\,G^+(-1)}\qquad (\operatorname{Re}p>0)$$

$$\exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\ln G(t)}{t-p}dt\right]=\begin{cases}G^+(p), & \operatorname{Re}p<0\\ G^-(p), & \operatorname{Re}p>0\end{cases} \qquad (4)$$

$$K^\pm(p)=\frac{\Gamma(1\mp p)}{\Gamma(1/2\mp p)}\quad (\Gamma(z)\text{- Euler's gamma function}).$$

Making use of solutions (3), (4), known asymptotics of the elastic field in the vicinity of the crack end and the theorem of Abel's type we get the formula for the coefficient K_I (stress intensity factor)

$$K_I=\gamma\left(\frac{\tau_s}{\sin 2\alpha}+q\right)\sqrt{\ell} \qquad (5)$$
$$\gamma=\gamma(\alpha)=2\sqrt{2}/[\sqrt{\pi}\,G^+(-1)]$$

The value of the function $\gamma(\alpha)$ at some values α are given in Table.

The dependence of the crack opening V at point O on stresses τ_s and q and the length of the crack ℓ is expressed by the formula

$$V=\frac{1-\nu^2}{E}\,\frac{\gamma(\alpha)}{\lambda(\alpha)}\left(\frac{\tau_s}{\sin 2\alpha}+q\right)\ell \qquad (6)$$

$$\lambda(\alpha)=\frac{[8\sin^2\alpha\,(\alpha^2-\sin^2\alpha)+2(2\alpha+\sin 2\alpha)(2\pi-2\alpha-\sin 2\alpha)]^{1/2}}{8\sqrt{\pi}\,\sin\alpha}$$

According to (5), (6) we have

$$K_I=\lambda E V/[(1-\nu^2)\sqrt{\ell}] \qquad (7)$$

Formulae (6), (7) solve the problem of the quasi-brittle crack formation in metals according to the Cottrell mechanism.

In the simplest case of a quasi-brittle crack of Griffith-Irwin $K_I=K_{Ic}$ (K_{Ic} - fracture toughness) and from (5) it follows that

$$q+\tau_s/\sin 2\alpha=K_{Ic}/(\gamma\sqrt{\ell}) \qquad (8)$$

Dependence (8) is presented qualitatingly in Fig.4, where $q_F=-\tau_s/\sin 2\alpha$. It is obvious that the development of the initial crack of any length will be unstable since value q is reduced with the growth of ℓ. Consequently, the final size of the crack developing at the meeting of two slip-lines is determined by the structural characteristics of the material (the crack is stopped by some barrier, peculiar to the given structure and absent in the problem formulation considered). Besides, it follows from (8) that the problem has a solution only under the condition $q>q_F$. Hence, the external field of extension should be sufficiently intensive.

When the value of K_{Ic} depends on ℓ for small ℓ vanishing with $\ell\to 0$ faster than $\sqrt{\ell}$, then the crack development will be stable at the first stage with the growth of q. In this case most exactly corresponding to the original Cottrell mechanism there may exist stable brittle cracks.

In the case of $\alpha\approx 110°$ while the value of K_{Ic} being independent on ℓ our model of cracking is most similar to that of Stroh (1954), although with a double slip-band.

If we assume that the slip-lines began to develop from the free straight linear border of the body (Fig. 1a), then, as it is shown (Cherepanov, 1979), angle α will be equal to $3\pi/4$

In this case $K_I=0{,}6252EV/[(1-\nu^2)\sqrt{\ell}]$. Hence we obtain the following expression for the length ℓ of a brittle crack of Griffith-Irwin:

$$\ell=0{,}3909E^2V^2/[K_{Ic}^2(1-\nu^2)^2]$$

THE PROBLEM ON DEVIATION AND BRANCHING OF THE SLIP-LINE

Let us consider the balance of a homogeneous isotropic elastic body being under the conditions of plane deformation and containing the straight linear slip-line. Let us assume that from the apex of the slip-line at some angle with its extension there runs a straight branch whose length is considerably smaller than the length of the slip-line and the dimensions of the body. It is necessary to estimate the angle of inclination of the branch to the extension of the slip-line and to determine its length.

Making use of "the microscope principle" (Cherepanov, 1979), we come to the singular problem of the theory of elasticity with the following boundary conditions (Fig.5)

$$\sigma_\theta(r,\pi)=\sigma_\theta(r,-\pi),\ \tau_{r\theta}(r,\pi)=\tau_{r\theta}(r,-\pi),\ u_\theta(r,\pi)=u_\theta(r,-\pi)$$

$$\theta=\pi,\ \tau_{r\theta}=\tau_s;\ \theta=\alpha,\ [\sigma_\theta]=[\tau_{r\theta}]=0,\ [u_\theta]=0$$

$$\theta=\alpha,\ r<\ell,\ \tau_{r\theta}=\tau_s;\ \theta=\alpha,\ r>\ell,\ [u_r]=0$$

At infinity the solution of this problem behaves like the solution of the problem of the theory of elasticity concerning the plane which contains a semi-infinite slip-line.

This solution has the shape:

$$\sigma_\theta=-\tau_s\sin 2\theta+C_1\cos 2\theta+C_2-\frac{3K_{II}}{4\sqrt{2\pi r}}\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right)$$

$$\tau_{r\theta}=\tau_s\cos 2\theta+C_1\sin 2\theta+\frac{K_{II}}{4\sqrt{2\pi r}}\left(\cos\frac{\theta}{2}+3\cos\frac{3\theta}{2}\right)$$

$$\sigma_r=\tau_s\sin 2\theta-C_1\cos 2\theta+C_2-\frac{K_{II}}{4\sqrt{2\pi r}}\left(5\sin\frac{\theta}{2}-3\sin\frac{3\theta}{2}\right)$$

Here K_{II}, C_1, C_2 are the arbitrary real constants which are considered as given in the problem formulation. They characterize the intensity of the external field and are determined from the solution of the external problem.

Let us assume that at the head of the slip-line (in the vicinity of point $\theta=\alpha$, $r=\ell$) there is some concentration of stresses characterized by the elastic asymptotics for cracks with plastic filler (Cherepanov, 1979). This asymptotics is quite determined only by the stress intensity factor K_{II}. The critical value K_{IIc} of the coefficient K_{II} (slip toughness) determines the resistance of the material to the emergence of new surfaces in it. Value K_{IIc} can be considered as the given constant of the material.

The problem thus formulated is reduced to the functional equation of Wiener-Hopf resulting in the formula for the coefficient K_{II}. Equating the latter to K_{IIc}, we come to the equation for determining the length ℓ of the branch

$$\sqrt{\ell}=\mathcal{D}\frac{K_{II}}{\tau_s}, \quad \mathcal{D}=\mathcal{D}(\alpha)=\frac{G^+(-1)[g(\alpha)-\Lambda]}{\sqrt{2\pi}\,\sin\alpha(\sin\alpha-\Lambda_1\cos\alpha)}$$

$$g(\alpha)=\frac{\cos\alpha/2+3\cos 3\alpha/2}{2\pi G^+(-1/2)}, \quad \Lambda=K_{IIc}/K_{II} \tag{9}$$

$$\Lambda_1=c_1/\tau_s$$

Here $G^+(p)$ is determined by the formula (4), where

$$G(p)=1-\left(\frac{\cos p\alpha\cos\alpha-p\sin p\alpha\sin\alpha}{\cos p\pi}\right)^2$$

It is seen from (9) that the angle of inclination of the branch to the extension of the slip-line should satisfy the condition $\mathcal{D}(\alpha)>0$.

We shall assume that in a homogeneous and isotropic body the slip-lines develop in the direction of the positive maximum of the function $\mathcal{D}(\alpha)$ (see Cherepanov, 1976; Kipnis and Cherepanov, 1984).

The existence of the single positive maximum of the function $\mathcal{D}(\alpha)$ would correspond to the presence of a corner point on the contour of the slip-line (deviation of the slip-line), and the point of maximum would represent the angle of inclination of the branch to its extension. The existence of two or more positive maximums would correspond to the presence of the point of branching on the contour of the slip-line (twinning or branching of the slip-line).

However the investigation shows that the function $\mathcal{D}(\alpha)$ does not have any positive maximums at the sections of its continuity. This circumstance enables us to come to the conclusion that in homogeneous and isotropic bodies corner points and points of branching on the contour of the slip-line are impossible.

Thus, a deviation of a slip-line may be only an open mode crack which, in turn, may deviate as a slip-line. This results in a saw form shear rupture (Cherepanov, 1983).

CONCLUSIONS

The problem of formation of cleavage crack in metals according to the Cottrell mechanism is strictly analysed.

Kinking of slip-lines is shown to be impossible. Hence, kink on the contour of a slip-line can appear only due to local mode I (or mixed mode) rupture.

REFERENCES

Cherepanov, G.P. (1976). Slip-lines at the end of the crack. PMM, v.40, N 4, 720-728.

Cherepanov, G.P. (1979). Mechanics of Brittle Fracture. New York, Mc Graw Hill, 1-950.

Cherepanov, G.P. (1983). A model of a saw form rupture in the Earth crust. Dokl. Akad. Nauk SSSR, v.269, N 4, 835-838.

Cottrell, A.H. (1958). Theory of brittle fracture in steel and similar metals. Trans. AIME, v.212, 192-203.

Honeycombe, R.W.K. (1968). The Plastic Deformation of metals (Chapter 15). Cambridge: Edward Arnold (Publishers), 427-458

Kipnis, L.A., and G.P. Cherepanov (1982). The contact problem of the theory of elasticity for the wedge. PMM, v.46, N 1, 141-147.

Kipnis, L.A., and G.P. Cherepanov (1983). To the theory of Cottrell's mechanism of crack formation in metals. Izvestia Akad. Nauk SSSR. MTT, N 3, 109-114.

Kipnis, L.A., and G.P. Cherepanov (1984). Slip-lines near apex of a wedge-form cut. PMM, v.48, N 1, 144-148.

Stroh, A.N. (1954). The formation of cracks as a result of plastic flow. Proc. R. Soc., A 223, 404-414.

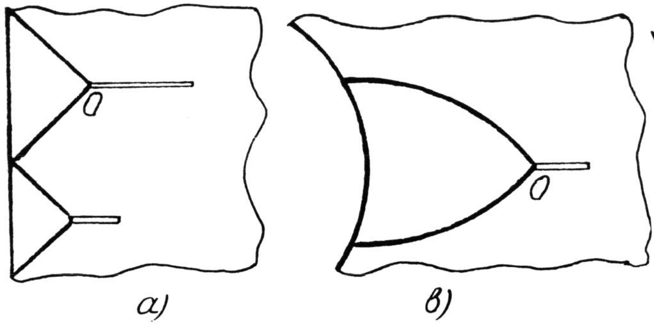

Fig. 1. Meeting of slip-lines.

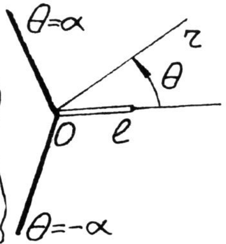

Fig. 2. Singular small scale approach.

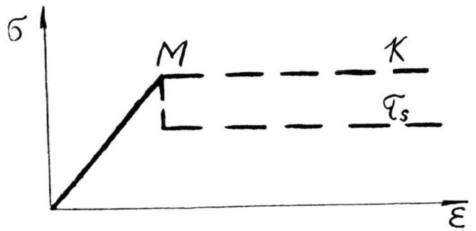

Fig. 3. Diagram σ-ε with delayed plasticity.

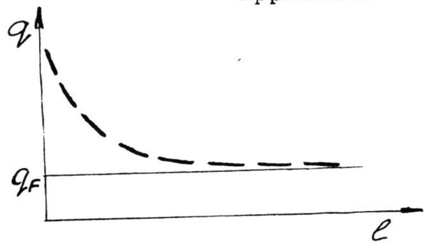

Fig. 4. Dependence of q on ℓ according to (8)

$\alpha°$	95	105	115	125	135	145	155	165	175
$\gamma(\alpha)$	1,8344	1,7710	1,7117	1,6574	1,6088	1,5633	1,5171	1,4645	1,3853

Table γ as function of α

Fig. 5. Slip-line kinked.