PATH DEPENDENCE OF STABLE CRACK GROWTH

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ABSTRACT

The path dependence of stable crack growth process is investigated by the stra energy density criterion. Considered is a center cracked plane specimen and a tensile load applied in a direction perpendicular to the crack plane. The stabl crack growth process is simulated by predicting a series of finite increments of crack extension which are assumed to occur when material elements ahead of the crack absorb a critical strain energy density, (dW/dV). In this way for a stepwise increasing load the successive crack growth increments are determined Unstable crack growth leading to global instability takes place when the critical crack growth increment becomes equal to r_c, which is a material constant Thus, the whole crack growth process including initiation, stable and unstable growth is characterized by two material parameters. The applied load is impose at different loading rates and the corresponding crack growth increments are determined. It is demonstrated that the critical stress for global instability increases as the loading rate also increases. From the whole study the path dependent nature of stable crack growth is demonstrated.

KEYWORDS

Stable , unstable crack growth; strain energy density criterion; crack initiation; global instability; loading rate.

INTRODUCTION

A large amount of effort has been spent studying the phenomenon of stable crack growth. The energy release rate approach of Griffith and Irwin has had reasonable success for predicting brittle fracture but failed to recognize the importance of stable crack growth prior to onset of rapid unstable crack propagation. Attempts have been made to use the R-curve and J-integral to study the complete history of crack growth from initiation to instability. Although these criteria have had some success in predicting and corroborating experimental observation the critical parameters entered are geometry dependent and not material constants.

In a series of recent publications Sih (1981) made an attempt to clarify the phenomenon of ductile fracture using the strain energy density concept Sih, (1973). This criterion is suited for analyzing subcritical crack growth accompanied by irreversible plastic deformation because it studies the whole fracture process in a unique and consistent fashion. Two material constants are used to study crack initiation, stable crack growth and final crack instability.

An attempt is made in this communication to study the stable growth process of a central crack in a rectangular sheet. Particular emphasis is given to the dependence of stable crack growth process on the rate of loading. Although the importance of this phenomenon has been recognized it is far from well been understood. The analysis is based on the strain energy density fracture criterion and takes place in the framework of small-scale yielding, where linear elastic fracture mechanics analysis of the crack tip stress field is applicable. Although it is customarily beleived that the phenomenon of stable crack growth takes place at large plastic deformations near the crack, the scale of yielding has very little to do with this phenomenon. Indeed, for a given material the occurence of small-or large-scale yielding is inherently related to the linear dimensions of the plastic zone compared to the crack length. Thus, the basic difference between small-scale and large-scale yielding is to be found in the crack lengths and not in the plastic zones. Thus, the fracture process in small-scale and large-scale yielding is essentially the same, while the response of the body is quite different.

THE STRAIN ENERGY DENSITY CRITERION FOR STABLE CRACK GROWTH

The strain energy density criterion can be used to address the problem of material damage with or without the presence of initial defects. The fundamental quantity is the strain energy density function dW/dV which in general can be calculated for any material from the relation

$$\frac{dW}{dV} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \qquad (1)$$

where σ_{ij} and ε_{ij} are the stress and strain components, respectively. Only in the linear elastic range $dW/dV = \sigma_{ij}\varepsilon_{ij}/2$.

First attention is focused on material elements at a finite distance r_o from the point of failure initiation, let say the crack tip (Fig.1). The circular region of radius r_o represents the core region in which the continuum model fails to describe in detail the state of stress and deformation. The values of the strain energy density dW/dV are calculated along the boundary of the core region and attention is concentrated on the stationary values of dW/dV. Thus, it is assumed that the direction of the element that initiates fracture or yielding corresponds to the minimum or maxinum value of the strain energy density function, $(dW/dV)_{min}$ or $(dW/dV)_{max}$. The critical values of dW/dV for yielding and fracture are obviously different. Referring to the true stress-true strain curve of the material the critical value of dW/dV for yielding, $(dW/dV)^c_{max}$, is equal to the area under this diagram up to the point of yielding, while the critical value of dW/dV for fracture, $(dW/dV)^c_{min}$, is equal to the area of the diagram up to the point of fracture. For brittle materials the values of $(dW/dV)^c_{max}$ and $(dW/dV)^c_{min}$ are almost equal, while for ductile materials $(dW/dV)^c_{min}$ is always greater than $(dW/dV)^c_{max}$. This fact combined with the previous observation that yielding and fracture is associated with the maximum and minimum value of dW/dV respectively suggests that in ductile materials yielding always precedes fracture initiation.

Based on the above arguments the basic hypotheses of the strain energy density criterion for fracture and/or yielding may be stated as:

Hypothesis (1): The location of fracture coincides with the location of relative minimum strain energy density, $(dW/dV)_{min}$, and yielding with relative maximum strain energy density, $(dW/dV)_{max}$.

Hypothesis (2): Failure by stable fracture or yielding occurs when $(dW/dV)_{min}$ or $(dW/dV)_{max}$ reach their respective critical values.

Hypothesis (3): The amount of incremental growth $r_1, r_2, \ldots, r_j \ldots r_c$ for a rising load that is increased incrementally is governed by

$$\left(\frac{dW}{dV}\right)_c = \frac{S_1}{r_1} = \frac{S_2}{r_2} = \ldots = \frac{S_j}{r_j} = \ldots = \frac{S_c}{r_c} = \text{constant} \qquad (2)$$

There is unstable fracture or yielding when the critical ligament size r_c is reached.

The quantity S in equation (2) defined by

$$S = \frac{dW}{dV} r \qquad (3)$$

represents the local energy release for a segment of crack growth r. Note that equation (3) is independent of the constitutive relations of the material.

Consider the variation of the strain energy density function dW/dV versus distance r along the direction of expected crack growth (Fig.1). Crack initiation starts for that load for which the dW/dV-r curve passes from the point defined by the radius r_o of the core region and $dW/dV = (dW/dV)^c_{min} = (dW/dV)_c$. Thus in Fig. 1a the crack starts to propagate for $q_{cr}=q_2$, while for $q=q_1$ the crack does not grow and for $q=q_3$ it grows an amount $r=r_3$. Unstable crack growth leading to global instability takes place when $r=r_c$, where the critical distance r_c is defined from relation (2). This is equivalent to $S=S_c$ where S_c represents the fracture toughness of the material. Values of the quantities $(dW/dV)_c$, S_c and r_c for various metal alloys can be found in references by Sih (1981).

To summarize the process of crack initiation and slow stable crack growth is controlled by the critical level of strain energy density, $(dW/dV)_c$, while crack instability occurs when the strain energy density factor S reached a critical value $S_c = r_c(dW/dV)_c$. The value S_c is a geometry and load independent material constant and is indicative of the fracture toughness of the material.

STABLE CRACK GROWTH

The process of slow stable growth of a center crack in a rectangular sheet is studied using the previously presented strain energy density theory. Considered is a plate of height 2h and width 2b with a center crack of length 2a. The plate is subjected to a uniform uniaxial tensile stress σ perpendicular to the crack plane. The influence of plate's geometry on the process of stable crack growth is analyzed. Although stable growth of a crack is usually accompanied by plastic deformation the stress and deformation field around the crack tip is computed from a purely elastic analysis. This procedure cannot be considered unrealistic for non-ductile materials. Indeed, in such materials the amount of plastic deformation around the crack tip is small and as it was established from an elastic-plastic analysis the plastic zones are inclined to the crack axis. Thus, the crack grows in elastic material.

The strain energy density function dW/dV computed from the linear elastic solution of the problem based on the stress intensity factor is given by (Rooke and Cartwright, 1976)

$$\frac{dW}{dV} = \frac{1}{r}\;\frac{1-2\nu}{4\mu}\;k_1^2 \tag{4}$$

where k_I is the opening-mode stress intensity factor, ν the Poisson's ratio and μ the shear modulus.

The stress intensity factor k_I is calculated from the equation

$$k_I = k_{Io}\;\sigma\sqrt{a} \tag{5}$$

where $k_{Io} > 1$ for a plate of finite dimension and $k_{Io} = 1$ for the infinite plate. Values of k_{Io} were obtained from the work of Rooke and Cartwright as a function of the plate dimensions and the crack length.

According to the strain energy density theory the crack starts to grow when the strain energy density dW/dV in a material element along the direction of crack extension and at a distance r_o from the crack tip reaches the critical level $(dW/dV)_c$. The critical stress σ_i for crack initiation determined from relations (4) and (5) is given by:

$$\sigma_i\left[\frac{a}{16\mu(dW/dV)_c\;r_o}\right]^{1/2} = \frac{1}{2}\left(\frac{1}{1-2\nu}\right)^{1/2}\frac{1}{k_{Io}} \tag{6}$$

The value r_o of the radius of the core region reflects a basic material property in the microstructural level and is considered to be a material constant.

In order to obtain numerical results the width 2b of the panel is obtained equal to 20in, while the panel height 2h and the crack length varied. The material of the panel is a steel with the following constants:

$$E = 3x10^7 \text{ psi},\; \nu = 0.3,\; \sigma_{ys} = 7.5x10^4 \text{ psi},\; k_{Ic} = 9.42x10^4 \text{ psi}\sqrt{\text{in}},$$

$$S_c = 7.7x10 \text{ lb/in},\; (dW/dV)_c = 26{,}684 \text{ in-lb/in}^3$$

The value r_c of the critical ligament size determined from equation (2) is equal to $2.885x10^{-3}$ in. The value r_o is obtained equal to $1.2x10^{-3}$ in. Fig.2a presents the variation of σ_i versus the initial crack length a for various values of the panel height h. Note that σ_i decreases with the initial crack length and as the height of the panel decreases. Thus, longer plates require higher load for crack initiation.

After crack initiation, the crack grows stably until global instability is reached. In order to study the process of stable crack growth the applied stress σ is increased by constant intervals $\Delta\sigma$ and the corresponding crack increments Δa are determined. Thus, for an applied stress $\sigma_1 = \sigma_i + \Delta\sigma_1$ the crack grows by the amount r_1 and the new crack length becomes $2(a+r_1)$. If now the stress σ_1 is increased by $\Delta\sigma_2$ a new crack increment r_2 results and the new crack length is equal to $2(a+r_1+r_2)$. This process is continued until the crack increment r becomes equal to the critical size r_c which corresponds to global instability. The final crack increment r_c corresponds to a critical applied stress σ_c. The critical crack length at instability is equal to a_c.

Table I presents the successive crack increments r and lengths a for a rising stress at intervals $\Delta\sigma = 0.2$ ksi for the first and last ten steps of a growing crack of length 2a = 2in, and b = 20in, h = 16in. The first value of the applied stress in the table $\sigma_i = 39.8$ for which $r = 1.2x10^{-3}$ in corresponds to the critical stress for crack initiation, while the last value $\sigma_c = 58.2$ for wh[...] $r = 2.89$ in corresponds to the critical applied stress for global instability. Note that for constant stress increment the crack increments r increase unt[...] the critical size r_c is reached. Thus, the critical applied stress σ_c and t[...] final crack length a_c are obtained. Fig. 2b presents the variation of the st[...] σ_c versus the initial crack length a for various values of the panel height [...] It is observed that σ_c decreases as a increases and h decreases.

Table I Values of relevant parameters for the first and last ten steps of a growing crack (b=20in, h=16in, a=2in)

Number of Crack Increment	Applied Stress σ(ksi)	Crack Increment $r(in) X10^{-3}$	Half Crack Length a(in)	Number of Crack Increment	Applied Stress σ(ksi)	Crack Increment $r(in) X10^{-3}$	Half Crac[k] Length a(in)
1	39.8	1.2002	2.0011988	85	56.6	2.7014	2.159517[...]
2	40.0	1.2133	2.0024118	86	56.8	2.7254	2.162241[...]
3	40.2	1.2265	2.0036383	87	57.0	2.7496	2.164992[...]
4	40.4	1.2398	2.0048771	88	57.2	2.7739	2.167765[...]
5	40.6	1.2532	2.0061312	89	57.4	2.7985	2.170563[...]
6	40.8	1.2667	2.0073977	90	57.6	2.8232	2.173387[...]
7	41.0	1.2802	2.0086775	91	57.8	2.8482	2.176235[...]
8	41.2	1.2939	2.0099716	92	58.0	2.8733	2.179108[...]
9	41.4	1.3077	2.0112801	93	58.2	2.8986	2.182007[...]
10	41.6	1.3216	2.0126009				

The previous analysis referred to the case when the applied stress was incre[...]ased incrementally at intervals $\Delta\sigma = 0.2$. The applied stress was then increase[...] at various intervals $\Delta\sigma$ and the same analysis was repeated. Fig. 3a presents the variation of the critical stress σ_c versus the loading increment $\Delta\sigma$ for various values of the panel height h. All curves of the figure correspond t[...] b=10in and a=3in. It is observed that as the loading increment $\Delta\sigma$ increases the stress σ_c also increases. This means that the stress σ_c increases when t[...] loading rate also increases. This result is in accordance with experimental findings of Kolsky and Rader (1968) who concluded that "the tensile strength of many brittle solids depend very markedly on the time for which the tensil[...] stress is maintained and is found to increase with decreasing loading time". In Fig. 3b the variation of the critical crack length a_c versus $\Delta\sigma$ is plotte[...] It is observed that a_c decreases with increasing loading interval $\Delta\sigma$.

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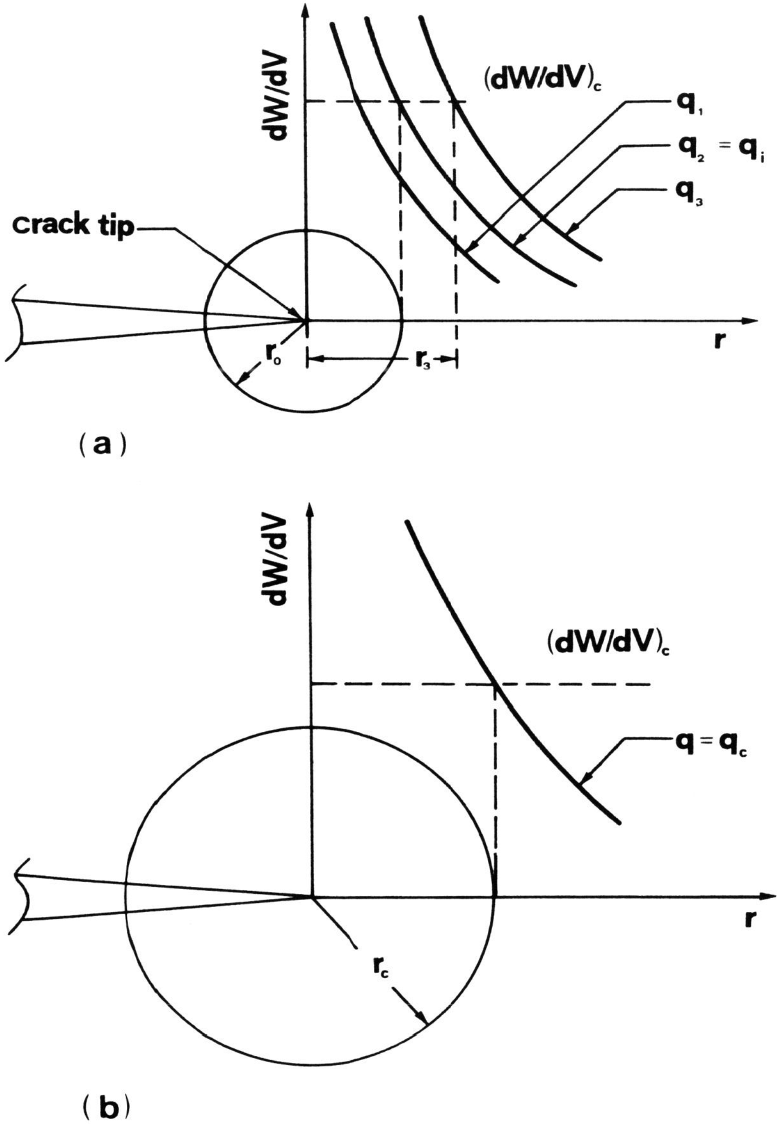


Fig. 1 Crack tip core region and strain energy density variations.

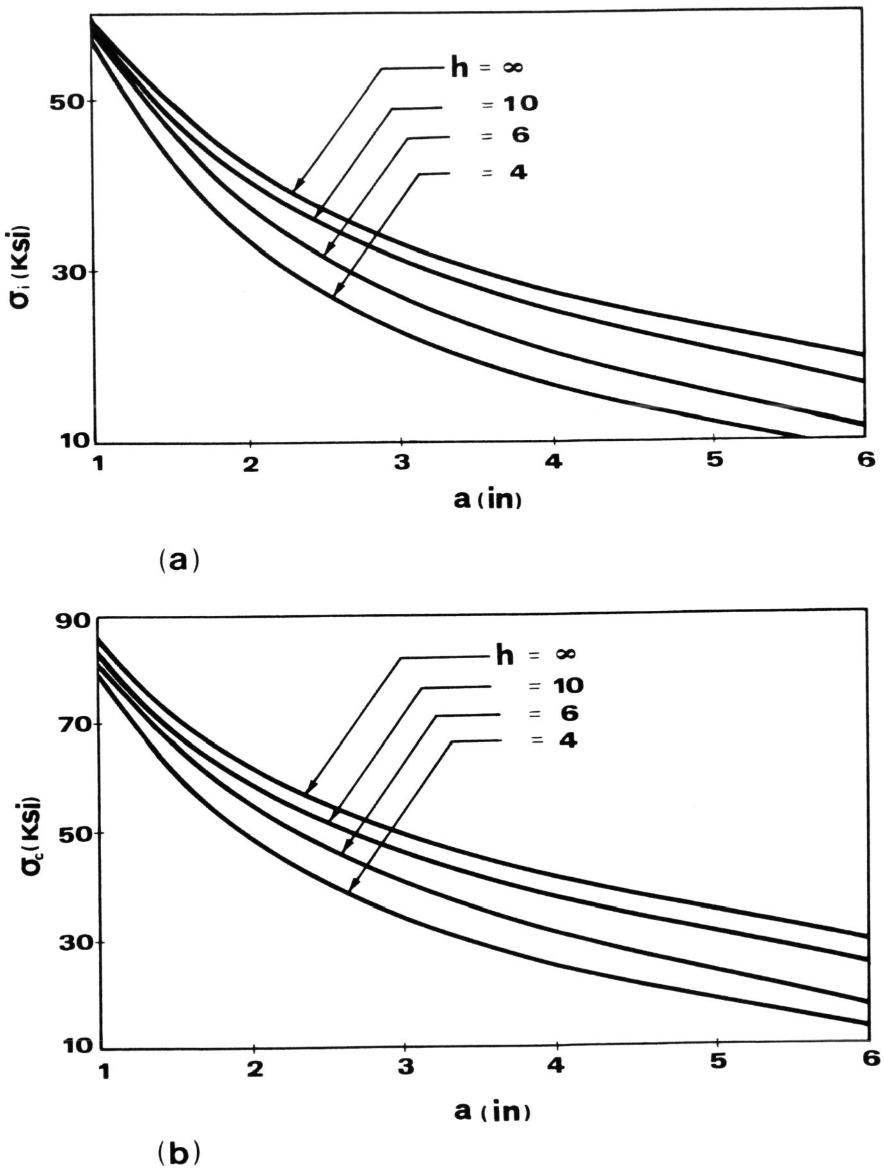
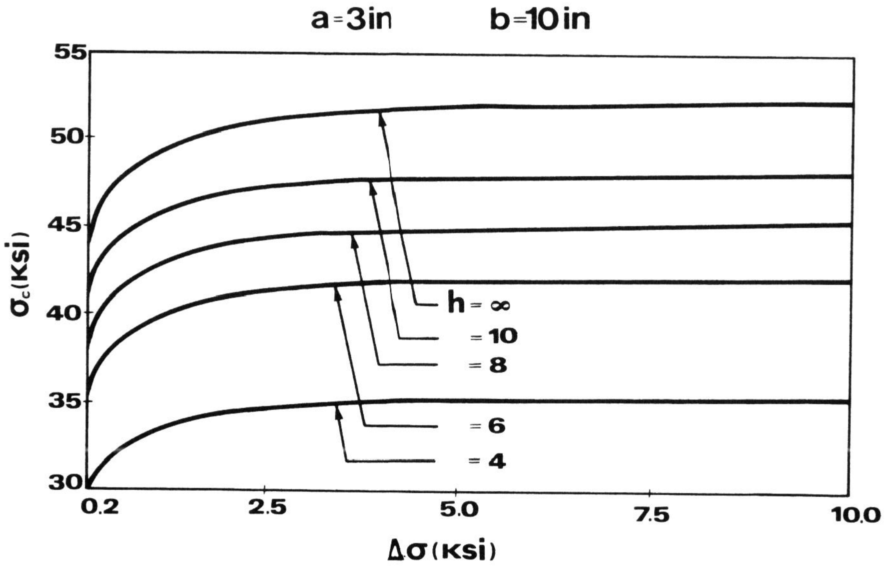
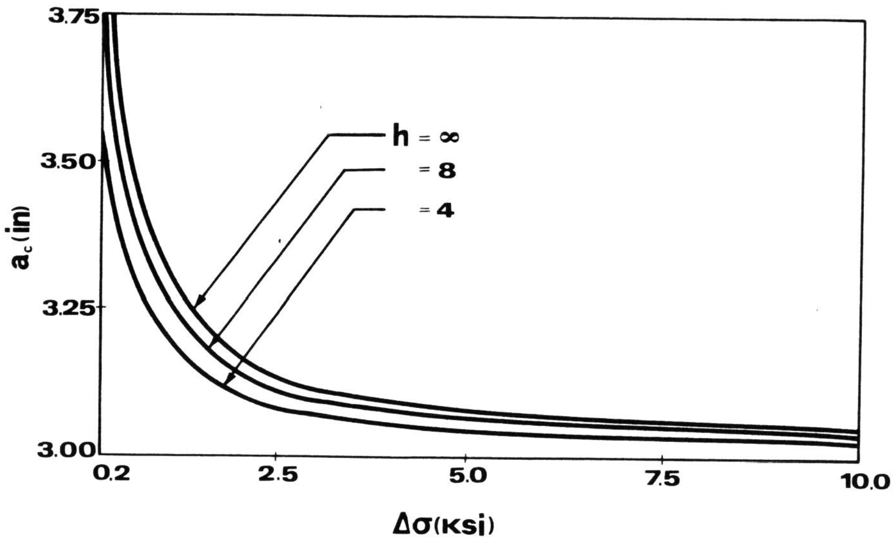


Fig. 2 Critical stress for crack initiation σ_i, and global instability, σ_c, versus initial half crack length, a, for various plate heights.



(a)



(b)

Fig.3 Critical stress σ_c(a) and critical crack length a_c(b) versus loading increment $\Delta\sigma$ for various plate heights.