MODELLING THE UNLOCKING AND SLIP OF CRACK SURFACES UNDER MODE II LOADING

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ABSTRACT

A simple Coulomb frictional model of the interaction between the flanks of a crack under Mode II loading is developed. It is shown to reproduce the observed unlocking and slip behaviour of Mode I fatigue cracks under cyclic Mode II loading.

KEYWORDS

Fatigue crack propagation, Mode II, crack locking, crack slip.

INTRODUCTION

Because the flanks of a crack subjected to Mode II loading are always in contact their interaction is likely to have an important effect upon its respons to both static and cyclic loading. A series of experiments has been carried out to monitor relative movement of the flanks of existing Mode I fatigue cracks under cyclic Mode II loading (Smith, 1984). This paper reports some important results and develops a mechanical model which is able to reproduce the observed behaviour.

Experiments were performed on a low carbon structural steel (BS 4360 grade 50 in the form of 3.2 mm thick hot-rolled sheet, using the single edge notched specimen developed by Richard (1981). Displacements in the vicinity of the crack tip were measured using a combination of clip gauges, and acetate replicas of finely scribed lines. Displacements parallel to the crack (in the Mode II direction) of ∿ 0.25 μm could be detected. Pure Mode II loading was applied so that the crack flanks, although in contact, were supposedly free of compressive normal stresses. The behaviour observed was as follows:

(i) On first loading, slip (relative movement of the crack flanks) too place progressively from the free surface to the crack tip, and di not always reach the crack tip at maximum load.

(ii) At maximum load, Mode II displacements of the crack flanks were much lower than predicted from the nominal applied stress intensity, even when slip had reached the crack tip.

(iii) On unloading, the extent of reversed slip was often much less than that of static slip, especially in tests conducted at high load ratio. Ahead of the effective crack tip (the maximum extent of reversed slip) the flanks of the crack remained locked with non-zero relative displacements and a residual stress intensity was left at the slip/no slip boundary.

(iv) Cyclic loading resulted in a slow spread of both static (non-reversed) slip and reversed slip towards the crack tip. Crack growth only occurred if reversed slip reached the crack tip, because only then were cyclic stress intensities developed there.

These observations are summarised in Fig. 1.

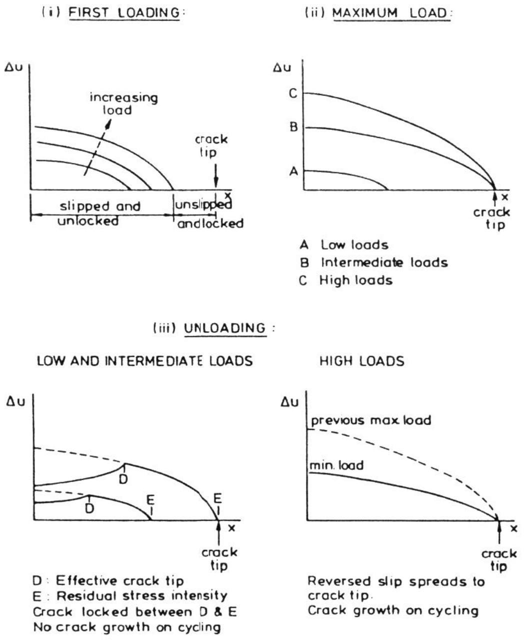

Fig. 1: Observed Mode II displacement profiles (Δu v. x) for Mode I pre-cracks under Mode II loading

This behaviour must be a consequence of frictional shear stresses opposing slip acting across the crack surfaces. At first sight the lack of an externally applied compressive normal stress across the faces of the crack suggests that no frictional resistance to slip should exist. However, there are several possible sources for this resistance.

(i) Compressive residual stresses exist behind the crack tip due to residual tensile deformations left in the plastically deformed wake of the pre-crack (Williams and Stouffer, 1979; Fleck, 1983). These cause frictional shear stresses opposing slip along the crack flanks

(ii) Although macroscopically flat, the crack surfaces actually follow a zig-zag path. Under Mode II loading the crack surfaces wedge open over surface irregularities (Smith, 1984) thus generating both a positive Mode I stress intensity at the crack tip and compressive normal stresses along the crack flanks. These in turn cause frictional shear stresses opposing slip.

(iii) Plastic deformation of interlocking surface asperities under Mode II loading produces shear stresses opposing slip.

The compressive stresses due to pre-cracking (Fleck, 1983) and wedging will increase as the pre-crack tip is approached, and hence the frictional shear stresses will follow the same pattern. The shear stresses due to asperity deformation will probably be reasonably constant along the crack flanks, as the surface roughness is constant in the tests conducted.

These frictional shear stresses appear to control the behaviour of cracks under Mode II loading, so it is important to be able to model their effects on slip between crack flanks.

DESCRIPTION OF THE MODEL (see Fig. 2)

A Coulomb frictional shear stress distribution $\tau(x)$ opposing slip is assumed to act along the crack, and assumed to be unchanged by slip. A simple linear distribution is used, such that:

$$\tau(x) = \tau_\ell x + \tau_s \qquad (1)$$

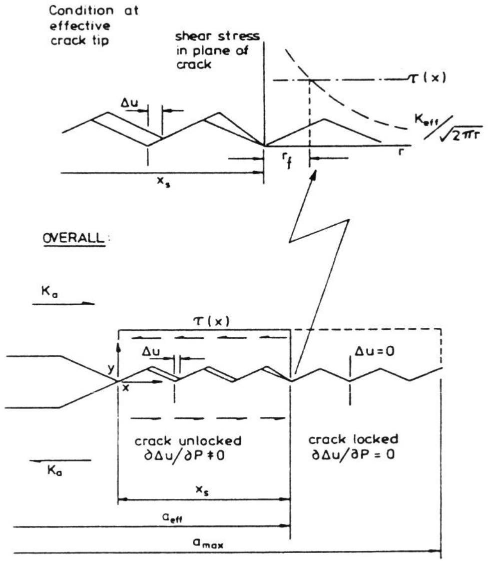

Fig. 2: Unlocking behaviour on first loading

where x is the distance along the crack from its origin at a free surface.

On loading, slip is assumed to spread along the crack from a free surface such that[1]:

$$K_a = K_f + K_{eff} \tag{2}$$

where K_a - The stress intensity that would act at the boundary between the slipped and unslipped portions of the crack, if no frictional retardation takes place along the unlocked portion of the crack, and the locked portion does not exist.

K_f - The attenuation in applied stress intensity caused by the frictional shear stresses acting along the slipped portion of the crack, summed using the influence function for a semi-infinite crack in an infinite sheet (Tada, Paris and Irwin, 1973). Thus

$$K_f = \int_0^{x_s} \frac{2\tau(x)dx}{\sqrt{2\pi x}} \tag{3}$$

K_{eff} - The effective stress intensity at the current effective crack tip a_{eff}. When $a_{eff} < a_{max}$, the actual crack length, K_{eff} is the stress intensity required to cause further slip at a_{eff}. It is assumed that for this to occur the shear stresses ahead of the crack tip along the crack plane, τ_{xy}, must exceed $\tau(x)$ over a critical distance, r_f (see Fig. 2). Thus

$$\tau_{xy} = \frac{K_{eff}}{\sqrt{2\pi r_f}} = \tau(x_s + r_f) \tag{4}$$

During first loading (Figure 2) slip spreads progressively from the free surface towards the pre-crack tip. Ahead of the current effective crack tip, a_{eff}, the crack faces are locked, with zero slip, as the limiting frictional stresses for slip have not yet been exceeded. Here the material behaves as if no crack were present. Behind the effective tip the crack flanks are free to slip, with Coulomb frictional stresses acting between their faces.

On unloading (Figure 3) the behaviour is more complex. As soon as the applied load begins to fall the entire slipped portion of the crack locks with the full frictional stress distribution developed along its flanks and a residual stress intensity, K_{res}, at its tip, a_{res}. Further reduction in the applied load causes reversed slip to spread progressively from the free surface, with one important difference to the first loading case. Because the flanks of the crack are locked with the full frictional stresses acting to resist slip in the loading direction, the effective frictional resistance to slip in the unloading direction is doubled. This leads to the extent of reversed slip being much less than that of static slip.

Relative displacements of the crack flanks are calculated using the near-tip stress field equations (Tada, Paris and Irwin, 1973) and superimposing displacements due to the various effective and residual stress intensities

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[1] All stress intensities are Mode II

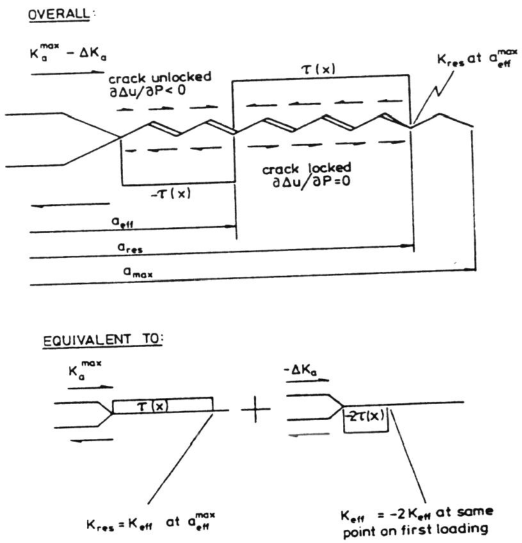

Fig. 3: Unlocking behaviour during unloading

present along the crack.

No provision is made to simulate the slow spread of slip on cyclic loading, which is caused by surface wear in the region of reversed slip.

COMPARISON WITH EXPERIMENT

Three tests which illustrate the three major classes of slip behaviour were compared with the model:

(i) reversed slip extending to the crack tip, with eventual crack growth (ΔK_{nom} = 19 MPa$\sqrt{m}$, P_{min}/P_{max} = 0.05)[2].

(ii) static slip extending to the crack tip, but not reversed slip, with no crack growth (ΔK_{nom} = 9 MPa$\sqrt{m}$, P_{min}/P_{max} = 0.5)

(iii) neither static nor reversed slip extending to the crack tip, with no crack growth (ΔK_{nom} = 9 MPa$\sqrt{m}$, P_{min}/P_{max} = 0.05).

All three tests were performed on specimens which were not stress-relieved after pre-cracking in Mode I. Qualitative agreement between model and experiment was obtained (see Figures 4 and 5) if reasonable values for the frictional shear stresses were assumed. In particular, the following observations were reproduced well:

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[2] The subscript 'nom' indicates that stress intensity which would be developed at the pre-crack tip, a_{max}, if there was no frictional attenuation in K.

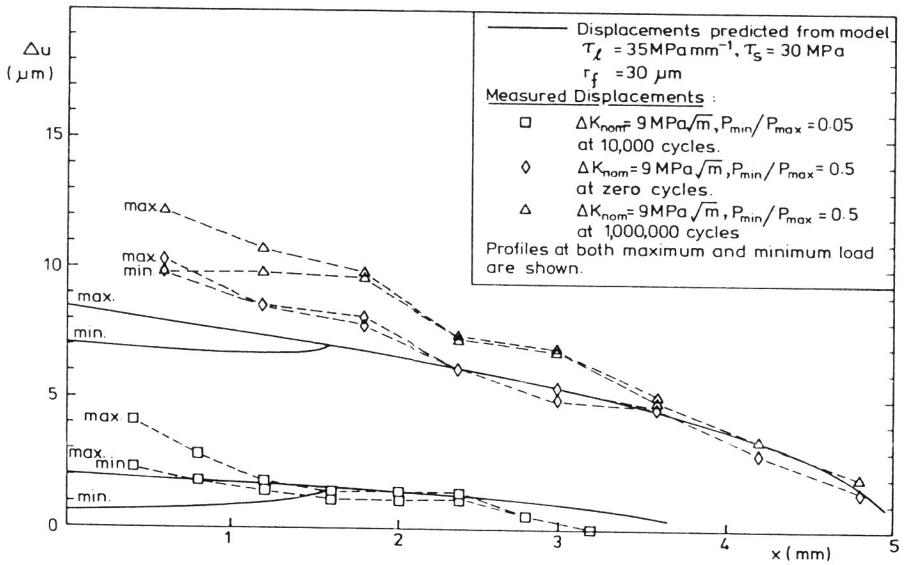

Fig. 4: Comparison of model with experiment (I)

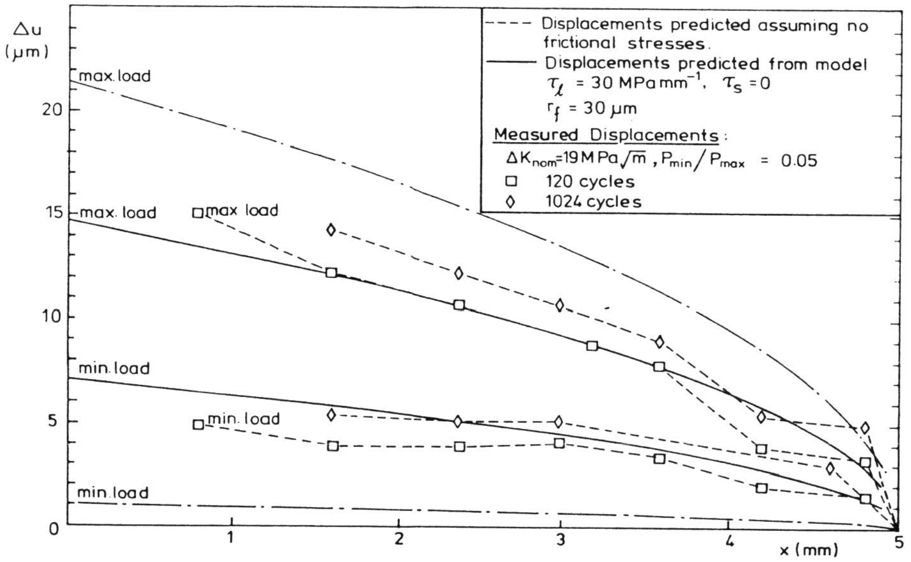

Fig. 5: Comparison of model with experiment (II)

(i) partial slip along the flanks of a crack (Fig. 4)

(ii) the much greater extent of static slip on first loading, compared with reversed slip on unloading and subsequent cycles (Fig. 4)

(iii) the exaggerated effect of a large load ratio on the differences between static and reversed slip (Fig. 4)

(iv) displacements at maximum load that were less than predicted by ignoring frictional effects, and displacements at minimum load that were larger, even when reversed slip reached the crack tip (Fig. 5).

Figures 4 and 5 also show the change in crack flank displacements due to crack surface wear during testing, which the model could not simulate. If this limitation is accepted, the simple frictional model successfully describes the unlocking and slip behaviour of Mode I fatigue cracks under Mode II loading.

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