

# FRACTURE ASSESSMENT IN THE PRESENCE OF RESIDUAL STRESSES

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## ABSTRACT

The problems associated with the inclusion of residual stresses in fracture mechanics analyses are discussed. Methods for calculating the linear elastic stress intensity due to residual stresses are reviewed, including linearization, the initial strain method and the crack face stress method, in combination with standard published LEFM solutions or finite element analyses. The elastic stress intensity due to residual stresses is used as a basis for including their effects in the CEGB "R6" elastic plastic assessment procedure. Similar methods for including residual stresses in the CTOD design curve or J-estimation procedures are proposed in this paper. A direct comparison of the various assessment procedures is made by using them to calculate the toughness required to prevent fracture in a centre-cracked panel containing a longitudinal weld.

## KEYWORDS

Residual stresses; fracture assessment; LEFM; elastic plastic fracture mechanics; stress intensity; CTOD; J-estimation; centre cracked panel; longitudinal weld.

## INTRODUCTION

It is well known that welding induces residual stresses whose peak values are usually of yield magnitude. In the absence of any detailed knowledge of the distribution of residual stresses in a particular welded joint, it is necessary to assume that any defect is subject to yield tensile residual stresses in addition to any applied or thermal stresses which may be present. This assumption may lead to an unacceptable degree of conservatism in cases where the defect lies in a region where the residual stresses are small or compressive, or where the effects of the residual stresses are reduced by plastic deformation around the crack tip. However, there are a number of practical, computational and conceptual difficulties involved in moving away from the all-embracing assumption of yield-magnitude residual stresses and hence reducing the level of conservatism.

The first is the difficulty in predicting the residual stress field at a particular welded joint. Despite the long-term and ongoing studies in many research centres, there is still an insufficient experimental database to cover all or indeed most of the weldment configurations of practical interest. It is relatively simple (Okerblom, 1958; Leggatt, 1983) to produce an upper bound estimate of the residual stresses parallel to the welding direction, based on a consideration of the maximum temperatures reached. These, however, are generally of less interest than the stresses transverse to the weld, which are more difficult to predict with any degree of precision. One joint configuration which has received some attention is the multipass butt weld, particularly in heavy section components (Ferril, 1966; Ueda, 1977; Fidler, 1977) and at hoop welds in pipes (EPRI Seminar, 1980; Shack, 1982; Leggatt, 1983). A related difficulty is that of ensuring that residual stresses measured in the laboratory are applicable in real structures. This uncertainty is caused by their sensitivity to the assembly sequence of the structure, which is seldom subject to rigorous procedural control.

The second difficulty associated with residual stresses in welds is that of separating their effects from those of material variability and inhomogeneity. Many of the fracture tests in which residual stress effects have been reported have been primarily concerned with other factors: residual stresses have been invoked retrospectively to account for otherwise inexplicable results. Investigations of this type are seldom accompanied by detailed residual stress measurements. There is a clear requirement for fracture tests designed specifically to investigate residual stress effects.

The final problem is associated with the variety of methods available for the assessment of defects in the post-yield regime, and the corresponding variety of methods for including residual stresses in those procedures. In this paper, some commonly used post-yield assessment methods will be compared using non-dimensional fracture toughness parameters. These provide a useful basis for comparison of the various procedures, with or without residual stresses.

LINEAR ELASTIC SOLUTIONS

Linearization

Linearization is a technique which enables the results of elastic fracture calculations for membrane and bending loads to be applied to non-linear distributions of concentrated, thermal or residual stresses in a conservative manner.

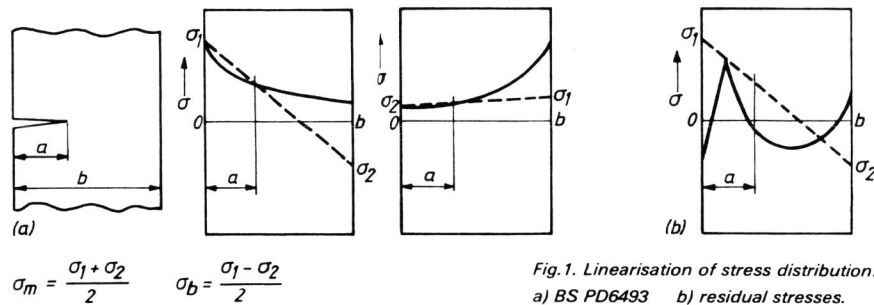


Fig.1. Linearisation of stress distribution: a) BS PD6493 b) residual stresses.

Figure 1a shows the linearization of various stress distributions at a surface defect, taken from British Standard PD 6493 (1980). Any linearized distribution of stress is acceptable provided that it is greater than or equal to the residual stresses over the crack surface. A similar technique is recommended in ASME Boiler & Pressure Vessel Code, Section XI, Appendix A (1980). This method gives a reasonably accurate solution for the types of stress distributions shown, but not for the high local peak stresses which may be found in a weld (Fig. 1b).

Crack Face Stress Methods

The application of crack face stresses to model the effects of residual stresses depends on the principle of superposition illustrated in Fig. 2.

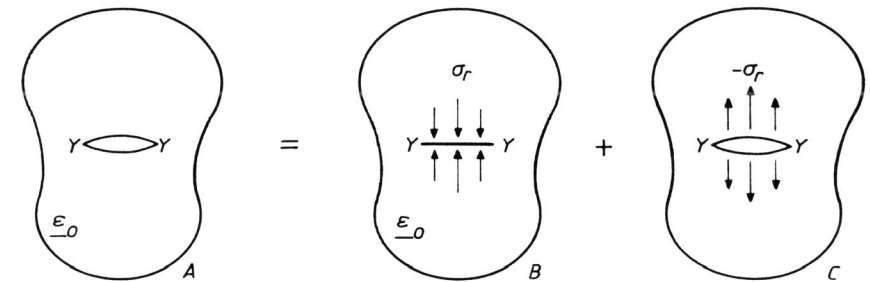


Fig.2. The principle of superposition.

Residual stresses in the cracked body, load case A, are caused by incompatible initial strains  $\epsilon_0$ . The crack face stresses necessary to close the crack (load case B) are  $\sigma_r$ , and are equal to the residual stresses on the crackline YY in an equivalent uncracked body also subject to  $\epsilon_0$ . Since the crack is closed, the stress intensity  $K_{IB} = 0$ . Hence the stress intensity due to  $\epsilon_0$  in the cracked body A is equal to that due to crack face stresses  $-\sigma_r$  in C, where  $-\sigma_r$  are equal and opposite to the residual stresses on the crackline in the uncracked body.

Hence  $K_I^r$ , the stress intensity due to residual stresses, may be calculated from LEFM solutions for arbitrary crack face stresses. If  $F(a/b, y/a)$  is the stress intensity due to unit point load at position  $y$  on a crack length  $a$  in a body of reference dimension  $b$ , then

$$K_I^r = \int_a^b \sigma_r \left( \frac{y}{a} \right) \cdot F \left( \frac{a}{b}, \frac{y}{a} \right) \cdot dy \quad [1]$$

Tada (1973) has given solutions for point crack face loads applied to a range of two dimensional geometries. Shah and Kobayashi (1971) have given a solution for a third order polynomial stress distribution applied to an embedded elliptical crack.

In bodies of complex geometry where no standard solutions are available, a numerical method such as finite elements must be adopted. The residual stresses are applied as point or distributed loads on the crack face. Since loads are applied close to the crack tip, accurate representation of the residual stress distribution is essential.

The problem may be encountered where LEFM solutions are available for applied stresses but not for an arbitrary distribution of residual or crack face stresses. In this case, the available solutions may be used to generate the function  $F$  in [1] using weight function techniques (Parker, 1982).

### Initial Strains

A more fundamental approach to residual stress problems is to consider the incompatible initial strains  $\epsilon_0$  which cause residual stresses. These may (in principle) be calculated from an analysis of the thermal and mechanical process to which the body has been subjected, or they may be related to residual stresses measured or assumed to be present in the body. The three principal initial strains at any point in a body are in general independent, and cannot necessarily be modelled by isotropic thermal strains. The initial strains  $\epsilon_0$  may be used to calculate  $K_I^r$  using a finite element system which has the facility of accepting initial strains as a form of loading. This method is probably unnecessarily complicated under linear elastic conditions, for which the superposition principle is applicable. However, for numerical elastic plastic analysis, initial strains are the only rigorous approach. The presence of the crack will give rise to additional plastic straining, and consideration should be given to the sequence of events involved in the generation of initial strains, crack growth, and the development of crack tip plasticity.

### POST-YIELD FRACTURE ASSESSMENT PROCEDURES

The value of  $K_I^r$  calculated using the procedures described above may be used as the basis for incorporating residual stresses in post-yield fracture assessment procedures.

### CEGB "R6" Procedure

In the R6 procedure (Harrison and colleagues, 1980), failure is assessed in terms of the fracture parameter  $K_r$  and the collapse parameter  $S_r$ , where:

$$K_r = K_I^D/K_{IC} + K_I^r/K_{IC} + \rho \quad [2]$$

$$S_r = \text{applied load/collapse load} \quad [3]$$

and  $K_I^D$  is the stress intensity due to primary (i.e. applied) stress  $\sigma_p$ ,  $K_I^r$  is the stress intensity due to residual stresses (thermal stresses should also be included in this term), and  $\rho$  is a correction factor for the effect of plasticity on  $K_I^r$ .

The critical values of  $K_r$  and  $S_r$  are related by:

$$K_r = S_r \{8\pi^{-2} \ln \sec(S_r \cdot \pi/2)\}^{-1/2} \quad [4]$$

The required toughness may be obtained by rearranging [2]:

$$K_{IC} = (K_I^D + K_I^r)/(K_r - \rho) \quad [5]$$

Milne (1981) has proposed a modified version of R6 which takes account of strain hardening behaviour.  $S_r$  in [4] is replaced by

$$\{S_r - (1 - K_r^{1/2}) \cdot (\sigma_u/\sigma_y - 1)\}, \text{ where } \sigma_u \text{ is the ultimate tensile stress.}$$

### Modified CTOD Procedure

In BS PD 6493 (1980), the stresses in the region of the defect are characterised by the parameter  $\epsilon/\epsilon_y$ :

$$\epsilon/\epsilon_y = \sigma/\sigma_y = (P_m + P_b + Q + F)/\sigma_y \quad [6]$$

where  $P_m$  is the membrane stress,  $P_b$  the bending stress,  $Q$  is the secondary (i.e. thermal plus residual stress) and  $F$  is the peak stress. In the terminology of R6,  $P_m + P_b + F = \sigma_p$  and  $Q = \sigma_s$ . If the structure is in the as-welded condition, PD 6493 recommends that the residual stresses should be assumed equal to the yield or proof stress of the material in which the defect lies. Hence:

$$\epsilon/\epsilon_y = (\sigma_p + \sigma_y)/\sigma_y \quad [7]$$

In order to make an allowance for residual stresses based on their actual distribution, the following modified version of [7] is proposed:

$$\epsilon/\epsilon_y = (K_I^D + K_I^r)/(\sigma_y \sqrt{\pi a}) \quad [8]$$

The total peak stress ( $\sigma_p + \sigma_y$ ) has been replaced by the LEFM equivalent stress  $(K_I^D + K_I^r)/\sqrt{\pi a}$ , where  $a$  is the half-length of a through-thickness defect, the half-depth of an embedded defect, or the depth of a surface defect. This formulation conforms with PD 6493 for low residual stresses (where  $(\sigma_p + \sigma_r) \ll \sigma_y$ ) and, at the other extreme, where  $\sigma_r = \sigma_y$  over the entire crack face. It provides a logical method of interpolating for intermediate cases. Additionally, by including  $K_I^D$ , it makes allowance for geometric effects (such as panel size in a centre-cracked panel) which are not included in the current version of PD 6493. For a defect in ferritic material, the required CTOD is given by:

$$\delta_c/\epsilon_y = \bar{a} \cdot 2\pi(\epsilon/\epsilon_y)^2 \quad \text{if } \epsilon/\epsilon_y \leq 0.5 \quad [9]$$

$$\delta_c/\epsilon_y = \bar{a} \cdot 2\pi(\epsilon/\epsilon_y - 0.25) \quad \text{if } \epsilon/\epsilon_y > 0.5 \quad [10]$$

### Extended J-Estimation Procedure

In the J-estimation procedure developed by Kumar, German and Shih (1981), the J-integral is obtained as the summation of elastic and fully plastic contributions:

$$J = J^e(a_e) + J^p(a, n) \quad [11]$$

where  $J^e(a_e)$  is the elastic contribution based on  $a_e$ , Irwin's effective crack length modified to take account of strain hardening.  $J^p(a, n)$  is the plastic contribution based on the material hardening exponent  $n$  in the Ramberg-Osgood stress-strain formulation. In small scale yielding, the plastic contribution

is small compared to the elastic contribution and hence [11] reduces to the well known elastic solution modified by Irwin's effective crack length. At the other extreme in the fully plastic range, the plastic contribution is the dominant term. Since residual stresses which are balanced within the section are eliminated under fully plastic conditions, it would appear to be consistent with the published procedure to include residual stresses only in the elastic term. The following extended formulation is proposed for  $J^e$  in the presence of residual stresses:

$$J^e = f_1(a_e) P_e^2 / E \quad (\text{plane stress}) \quad [12]$$

where  $f_1(a_e) = \{K_I^D(a_e) / P\}^2$  ;  $P_e = 2b\sigma_p (K_I^D + K_I^R) / K_I^D$  ;  $P = 2b\sigma_p$

$$a_e = a + \phi r_y \quad ; \quad r_y = \frac{1}{2\pi} \cdot \frac{n-1}{n+1} \cdot \left[ \frac{K_I^D + K_I^R}{\sigma_0} \right]^2 \quad ; \quad \phi = 1 / (1 + P^2 / P_0^2)$$

In this formulation, the effects of residual stress are included in both the load term,  $P_e$ , in which the primary stress is scaled up to take account of  $K_I^R$ , and in the calculation of the effective crack size,  $a_e$ .

EXAMPLE

The inset sketch on Fig. 3 shows an idealized distribution of longitudinal residual stresses in a long panel containing a longitudinal weld. The required toughnesses  $K_{Ic}$ ,  $\delta_c$  and  $J$  have been calculated as a function of crack size using the methods described above and are plotted in Fig. 3 in non-dimensional form. The required toughnesses may in this context be regarded as driving forces, as crack growth may occur if they exceed the actual material toughness. Non-dimensionalization permits a direct comparison of the various assessment procedures, though it should be noted that the relative magnitude of the characterizing parameters depends on the parametric groups chosen, which give direct equivalence for small scale yielding under plane stress conditions, i.e. when:

$$K_{Ic}^2 = JE = \sigma_y E \delta \quad [13]$$

The lower curve gives the linear elastic stress intensity under residual stress only, calculated using Tada's (1973) solution for a long centre-cracked panel. A finite element solution using initial strain loading gave virtually identical results. It should be noted that  $K_I^R$  remains positive for all crack sizes, even where the residual stresses are negative beyond  $a/b = 0.167$ . A similar result was obtained by Tada and Paris (1983) using a residual stress distribution fitted to experimental data.

The remaining curves consider the combined effect of the residual stresses shown and an applied stress of 60% of yield. The standard PD 6493 procedure assumes  $\sigma_r = \sigma_y$  for all crack sizes, and produces a straight line which is realistic only for very short crack lengths where the residual stresses are equal to yield. The remaining methods produce a transition between residual stress dominated behaviour for small crack sizes and applied stress dominated behaviour for longer cracks with plastic collapse at about  $a/b = 0.5$ . The degree of agreement between the post-yield methods which use  $K_I^R$  is quite good.

The procedure used in R6 for calculating the plasticity correction factor for residual stresses produces an artificial double bump in the curve. This

problem is avoided with the procedures proposed here for CTOD and J-estimation. At  $a/b = 0.51$ , the ligament stress is equal to the flow stress and the R6 curve rises vertically, indicating plastic collapse. The R6 (strain hardening) and J-estimation procedures rise steeply in this region, but with finite gradient. The CTOD methods provide no mechanism for a gradual transition between fracture dominated behaviour and plastic collapse

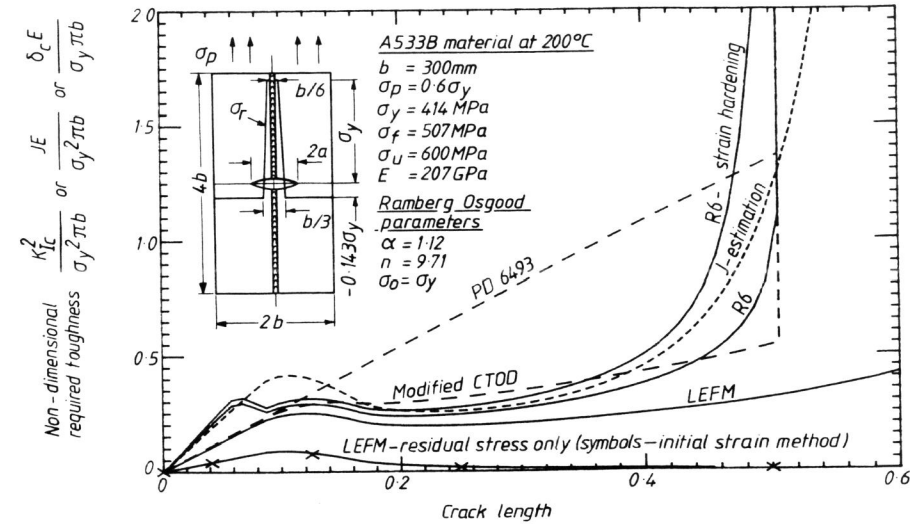


Fig.3. Non-dimensional required toughness curves for welded CCP.

CONCLUDING REMARKS

Methods for including residual stresses in linear elastic and post-yield fracture mechanics analyses have been discussed. The elastic stress intensity due to residual stresses may be calculated approximately using linearization techniques in cases where the residual stress distribution happens to be linear, or where a conservative result is acceptable. Accurate values be obtained using the crack face stress method, in combination with published solutions for simple geometries, or with finite element techniques for more complex configurations.

The stress intensity factor due to residual stresses is used as the basis for including their effects in the CEBG "R6" elastic plastic assessment procedure. Similar methods for including residual stresses in the CTOD design curve and the J-estimation procedure are proposed in this paper.

Standard and modified assessment procedures have been applied to the case of a centre-cracked panel containing residual stresses caused by a longitudinal weld. Results from different procedures were compared by constructing non-dimensional "required toughness" curves. An encouraging degree of agreement was found between those methods which including residual stresses in a realistic manner, namely R6, R6 (strain hardening version), extended J-estimation and the modified CTOD design curve procedure.

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