FOUNDATIONS OF MECHANICS OF BRITTLE FRACTURE OF MATERIALS WITH INITIAL STRESSES

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ABSTRACT

The results are presented of the study to lay down the foundations of the linearized mechanics of the brittle fracture of materials with initial stresses, including the following aspects. Formulation of main statements and problems for bodies with arbitrary elastic potential. Derivation of strict solutions for a group of main classes of plane and three-dimensional problems under static and dynamic loading. Formulations of brittle fracture criteria for materials with initial stresses. The study of main effects of initial stresses in mechanics of brittle fracture of materials with initial stresses.

KEYWORDS

Initial stresses; linearized mechanics; arbitrary elastic potential; static and dynamic loading; plane and three-dimentional problems.

1. FORMULATION OF THE PROBLEM

In most structure elements initial of resting stresses occur that are caused by technological processes of the production or assembly of structures. In some case the level of those initial or resting stresses may exceed the operating stresses. On the basis of these considerations, taking account of initial stresses in various problems of the brittle fracture mechanics becomes an important problem.

Among the initial stresses applied to various planes, of special intrest are stresses located along planes in which cracks have occurred.

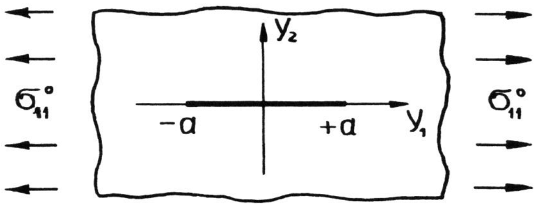

Fig.1

It should be pointed out that within the framework of the classical linear brittle fracture mechanics (Cherepanov, 1974) it is not possible to take account of these stresses. Indeed, within the framework of the classical linear mechanics of the brittle fracture (solutions of the classical linear elasticity theory) these stresses have no effect on the coefficients of stress intensities, consequently, these stresses are not included in fracture criteria also.

On the basis of the foregoing considerations in this article the foundations of the brittle fracture mechanics are presented for materials with initial stresses applied along planes in which cracks are located. List of references on the problem considered and published in Russian language will be presented Guz (1983). The mechanics of brittle fracture of materials with initial stresses is based on linearized elasticity theory at finite and small initial deformations. In the theory of mechanics of brittle fracture for materials with initial stresses the following four assumptions were made that are essential in the theory presented and consequently restrict its application. The following assumptions are made:

1. In the initial stressed-deformed state, the body containing cracks is loaded in such a manner that on planes in which cracks are located stresses do not arise.

2. Upon application to the body of additional arbitrary loads (additional relative to the initial state) the values of disturbances of the stressed-deformed state are considerably lower than the respective values of the initial state.

3. The initial stressed-deformed states posses such a structure that in the vicinity of the crack that state could be assumed a homogenous one, approximately with a sufficient degree of accuracy.

4. The solution of the linearized elasticity theory, that is used within the framework of the mechanics of the brittle fracture of materials with initial stresses presented here, is a unique one.

It should be pointed out that the first and the second assumptions are essential ones in the theory presented. The third

assumption is an auxiliary one in the problem considered it plays an important role in the solution of problems arising in that approach as the use of the third assumption allows to derive equations with constant coefficients.

In the foregoing approach for study the problem considered the linearized elasticity theory was used. The studies were made in the general manner for compressible and incompressible elastic bodies with elastic potentials of arbitrary structure for the theory of great (finite) initial deformations and for two variants of the theory of small initial deformations. For some materials with elastic potentials with specific structure the main mechanical effects of the influence of initial stresses on the main regularities of the brittle fracture mechanics were found. In the study various representations of general solutions were used for systems of differential equations derived. It should be noted that the stress tensor has been used, the components of which are referred to dimensions of respective planes in the initial stressed-deformed state. The use of that stress tensor concomitant with the use of coordinate system in the initial shessed-deformed state makes it possible in the problems of mechanics of brittle fracture of materials with initial stresses to construct boundary conditions, completely coinciding with analogous boundary conditions of the classical linear mechanics of brittle fracture. That approach was used in all problems of the mechanics of brittle fracture of materials with initial stresses considered here. Below in a brief outline main results are presented of the mechanics of brittle fracture of materials with initial stresses, these results are applied to some problems, the essential mechanical effects of initial stresses are pointed out.

2. STATIC PLANE AND ANTI-PLANE PROBLEMS

In the general case of compressible and incompressible bodies with elastic potentials of arbitrary structure, for the theory of finite initial deformations and for two variants of the small initial deformations theory the representations is obtained of components of tensor of stresses $\tilde{Q}$ and of displacements u_k in complex potentials. As an example, we present that representation for a plane problem. In the case of unequal roots it has the following form

$$\tilde{Q}_{22}=2\Re\left[\Phi_1'(z_1)+\Phi_2'(z_2)\right]; \quad z_j=y_1+\mu_j y_2; \quad \mu_1\neq\mu_2;$$

$$\tilde{Q}_{21}=-2\Re\left[\gamma_{21}^{(1)}\mu_1\Phi_1'(z_1)+\gamma_{21}^{(2)}\mu_2\Phi_2'(z_2)\right];$$

$$\tilde{Q}_{11}=2\Re\left[\gamma_{11}^{(1)}\mu_1^2\Phi_1'(z_1)+\gamma_{11}^{(2)}\mu_2^2\Phi_2'(z_2)\right];$$

$$\tilde{Q}_{12} = -2\Re\left[m_1\Phi_1'(z_1) + m_2\Phi_2'(z_2)\right];$$

$$u_k = 2\Re\left[\gamma_k^{(1)}\Phi_1(z_1) + \gamma_k^{(2)}\Phi_2(z_2)\right]; \qquad k = 1,2 .$$

In (2.1): $\Phi_1(z_1)$ and $\Phi_2(z_2)$ - are analytical functions of complex variables. Values m_j, $\gamma_{2j}^{(j)}$, $\gamma_{1j}^{(j)}$ and $\gamma_k^{(j)}$ are represented by initial stresses and by elastic potential.

In similar form the representation is obtained for a plane problem in the case of equal roots $m_1 = m_2$ and for anti-plane problems.

The foregoing representation in complex potentials in the absence of initial stresses transforms into classical representation of the linear elasicity theory. Thus in the case of equal roots for the plane problem the Kolosov-Muskhelishvili representation is obtained, and in the case of unequal roots for the plane problem the representation of S.G.Lekhnitsky is obtained for a linear elastic orthotropic body. It should be noted that the transformation mentioned occurs when initial stresses tend to zero.

The foregoing representations in complex potentials of the (2.1) type are used as relations on the basis of which accurate solutions are derived in the mechanics of the brittle fracture of materials with initial stresses.

Using the foregoing representation accurate solutions are obtained for a single crack in the case of a crack with a normal separation, for transverse and longitudinal shear and for the problem of crack opening under the action of absolutely stiff wedge. These results were obtained in the general form under arbitrary respective load; the asymptotic distribution of stresses and displacements is also derived for the region near the crack tip, with introduction of stress coefficients. On the basis of the analysis of results with application to specific materials conclusions were drawn on the effects of initial stresses in the brittle fracture mechanics.

Conclusions for cracks with normal separation and transverse shear. 1) For materials with elastic potentials with the simplest structure the order of singularity at the crak tipwi thin the framework of the mechanics of brittle fracture for materials with initial stresses-coincide with the order of singularity at the crack tip within the framework of the mechanics of brittle fracture of materials without initial stresses.
2) Stresses in the body on the crack line near the crack tip are independent of initial stresses and coincide wholly with the corresponding results of the classical theory. However the opening of crack lips is closely dependent on initial stresses. The foregoing singularities are noted in "free" cracks only, i.e. in cracks on lips of which only the stresses are prescribed. 3) When initial stresses tend to values corresponding to the surface instability of the half-space, phenomena of "resonance" character arise consisting of marked change in stresses and displacements near the crack tip. As an exception, only stresses in the body on the crack line should be mentioned, in the case of "free" cracks. 4) The initial stresses have a more essential effect of quantitative character in highly elastic materials compared with stiffer materials, while the qualitative effect is indential in both types of materials.

Conclusions for cracks in longitudinal shear. 1) In the general case the order of singularity at the crack tip within the framework of the mechanics of brittle fracture for materials with initial stresses coincide with the order of singularity at the crack tip within the framework of the mechanics of brittle fracture in materials without initial stresses. 2) The second conclusion is completely identical with the analogous conclusio for cracks in normal separation and in transverse shear. 3) Whe initial stresses tend to values, corresponding to the surface instability, phenomena of "resonance" character do not occur ir contrast to the cracks in normal separation and in transverse shear. 4) The fourth conclusion is completely identical with analogous conclusion for cracks in normal separation and in transverse shear.

Conclusions for problems of cracks opening in bodies under the action of absolutely stiff wedge. 1) The first conclusion is completely identical with the analogous conclusion for cracks in normal separation and in transverse shear. 2) The stresses in the body on the crack line near the crack tip are depending closely on initial stresses and do not coincide with the corres ponding result of the mechanics of brittle fracture of materials without initial stresses. This demonstrates an important difference from "free" cracks as in the present problem on separate parts of the crack boundary conditions in stresses and displacements are prescribed. 3) When initial stresses tend to values corresponding to the surface instability of the half-space, the coefficients of stress intensity tend to zero.
4) The fourth conclusion coincides completely with analogous conclusion for cracks in normal separation and in transverse shear.

3. DYNAMIC PLANE PROBLEMS

The representation of stresses is derived for plane dynamic li nearized problems in complex potentials for the case when initial dynamic problems can be reduced to stationary problems in movable cartesian system of coordinates that moves uniformly and rectangularly relative to cartesian coordinate system intr duced in the initial state. That representation in complex potentials in the case of unequal roots is of the type (2.1); when initial stresses tend to zero the complex representation obtained is transformed into representation of L.A.Galin of th classical linear elasticity theory. It should be noted also th this complex representation has been obtained in the general form for compressible and incompressible bodies with an arbitrary form elastic potential for the theory of finite initial deformations and for two variants of the theory of small initial deformations. These representations formed the basis for deriving accurate solutions in mechanics of brittle fracture of materials with initial stresses; in particular, accurate solutions were obtained of dynamic problems corresponding to foregoing static problems.

Conclusions for dynamic problems: 1) In the case of "free" cracks the coefficients of stress intensity are independent of initial stresses, while the opening of crack lips is depending closely upon initial stresses. 2) In the case of "not free" cracks the coefficients of stress intensity depend closely on initial stresses. 3) The velocity of crack propagation in an elastic body with initial stresses cannot exceed the velocity of propagation of Rayleigh waves in an elastic body with initial stresses. It should be pointed out that under compression the velocity of propagation of Rayleigh waves is reduced. 4) When cracks move at speed that corresponds to the velocity of Rayleigh waves in an elastic body with initial stresses, effects of "resonance" character of the various types occur that correspond to the case of surface instability in analogous static problems. Since the propagation velocity of Rayleigh waves depends continuously on initial stresses, a continuous (in velocity) spectrum exists in which phenomena of "resonance" character can occur.

4. STATIC THREE-DIMENSIONAL PROBLEMS

For axisymmetrical initial states the representation of general solutions is obtained in coordinates of the initial state in the general form for compressible and incompressible elastic bodies with arbitrary structure of elastic potentials in the case of equal and unequal roots. In the case of equal roots, the stresses and displacements at $y_3 = \mathrm{const}$ (the crack lies in the plane $y_3 = 0$) can be represented in the following form (Guz,1983)

$$u_1 = \frac{\partial}{\partial y_1}(\varphi_1 + \varphi_2) + y_3 \frac{\partial^2}{\partial y_1 \partial y_3}\varphi_2 - \frac{\partial}{\partial y_2}\varphi_3 \; ; \quad \ldots$$

In (4.1) φ_j - harmonic functions; values c_{44} , n_j , m_j and ℓ_j - depend on initial stresses and on the structure of elastic potential.

For arbitrary number of cracks located in one plane a method is proposed for reducing the three-dimensional static problems of the mechanics of brittle fracture of materials with initial stresses to mixed problems of the theory of harmonic potential for a half-space in the case of one and two functions; cracks in normal separation and shear were studied. An accurate solution is obtained for problems of crack in the form of a circular disk as applied to axisymmetrical problems for cracks in normal separation and radial shear as well as to problem of torsion crack;

Conclusions for static three-dimensional problem. 1) The first

conclusion coincides completely with analogous conclusion for plane static problems. 2) In axisymmetric problems in the case of crack in the form of circular disc the stresses in the crack plane are independent of initial stresses and coincide completely with analogous result of the mechanics of brittle fracture of materials without initial stresses, while the opening of crack lips depends significantly on initial stresses. These results were obtained for "free" cracks. 3) In the general case of the shear crack of arbitrary form it is impossible to demonstrate that the stress distribution in the crack plane in independent of initial stresses even in the case of "free" cracks. Herein lies an important difference between three-dimensional and plane problem. In the particular case of an axisymmetrical problem of the radial shear crack as applied to the discformed circular crack that special characteristic is not noted.
4) When the initial stresses tend to values corresponding to the surface instability of the half-space, the effects of "resonance" character occur for cracks of normal separation and radial shear in the same form as in plane problems. However in case of the problem of torsion crack the effects of "resonance" character are not occuring as in the case of a crack at longitudinal shear (anti-plane problem).

5. CRITERIA OF BRITTLE FRACTURE OF MATERIALS WITH INITIAL STRESSES (CRITERIA OF THE GRIFFITS-IRVIN)

These are the energy criteria of the brittle fracture, constructed on the basis of Griffits' ideas and of the power approach of Irvin generalized for the case of materials with initial stresses. These criteria are obtained for the case of plane crack in a unified general form for compressible and incompressible bodies with arbitrary form of the elastic potential in the case of equal and unequal roots. When the initial stresses tend to zero the fracture criteria formulated transform into the classical criterion of Griffits-Irvin of the mechanics of the brittle fracture of materials without initial stresses. For example, in the case of unequal roots the criterion of brittle fracture of materials with initial stresses has the form

$$\mathcal{K}_I^2(\gamma_2^{(1)} m_2 \gamma_{21}^{(2)} - \gamma_2^{(2)} m_1 \gamma_{21}^{(1)}) + \mathcal{K}_{II}^2(\gamma_1^{(1)} - \gamma_2^{(2)}) + \tag{5.1}$$
$$+ \mathcal{K}_{III}^2(\tilde{\omega}_{1331}\tilde{\omega}_{2332})^{-\frac{1}{2}} i^{-1}(m_2\gamma_{21}^{(2)} - m_1\gamma_{21}^{(1)}) = 4\gamma i^{-1}(m_2\gamma_{21}^{(2)} - m_1\gamma_{21}^{(1)}).$$

In (5.1) $\mathcal{K}_I$, $\mathcal{K}_{II}$ and $\mathcal{K}_{III}$ - are coefficients of stress intensity, respectively, for cracks in normal separation, transverse and longitudinal shear; values m_j , $\gamma_1^{(j)}$, $\gamma_2^{(j)}$, $\gamma_{21}^{(j)}$, $\tilde{\omega}_{1331}$ and $\tilde{\omega}_{2331}$ - depend on initial stresses and on the structure of elastic potential; γ - effective specific surface energy for material with initial stresses. Analogous results were obtained for unequal roots. As a result of analysis for materials with specific elastic potential conclusions were drawn on the effect of initial stresses on fracture criteria.

Conclusions for fracture criteria: 1) The increase of compressive stresses along the cracks in materials with initial stresses leads to a reduction of fracture loads corresponding to brittle fracture for cracks in normal separation and transverse shear. 2) When initial stresses reach values corresponding to the surface instability of the half-space (within the framework of plane deformation) fracture loads go to zero for cracks in normal separation and transverse shear.

The mechanical phenomenon corresponding to the second conclusion has the following explanation. As the linearized theory is applied, at the foregoing values of initial stresses, the body with the crack is in the state of neutral equilibrium; in that case the additional load of finite values need not be applied to eliminate that state of the body.

Some other results on the mechanics of brittle fracture of materials with initial stresses are presented in publications mentioned in review article Guz (1983).

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